

SURDS AND INDICES

Q) $(\sqrt{5} + \sqrt{4})$, $(\sqrt{7} + \sqrt{2})$, $(\sqrt{6} + \sqrt{3})$ find the value of largest and smallest number.

$(\sqrt{5} + \sqrt{4})$, $(\sqrt{7} + \sqrt{2})$, $\sqrt{6} + \sqrt{3}$ सबसे बड़ी और सबसे छोटी संख्या का मान ज्ञात करें।

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(\sqrt{5} + \sqrt{4})^2 = 9 + 2\sqrt{20} \text{ --- ①}$$

$$(\sqrt{7} + \sqrt{2})^2 = 9 + 2\sqrt{14} \text{ --- ③}$$

$$(\sqrt{6} + \sqrt{3})^2 = 9 + 2\sqrt{18} \text{ --- ②}$$

largest: $(\sqrt{5} + \sqrt{4})$

Smallest: $(\sqrt{7} + \sqrt{2})$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

* $\sqrt{a} + \sqrt{b}$, $\sqrt{c} + \sqrt{d}$, $\sqrt{e} + \sqrt{f}$

where $(a+b) = (c+d) = (e+f)$

then जिस surd के root के अन्दर वाली संख्याओं का Multiplication बड़ा होगा वो surd बड़ा हो जाएगा।

TYPE-III

$$\bullet (a+b)^2 = a^2 + b^2 + 2ab \longrightarrow (7+3)^2 = 58 + 2 \times 21$$

$$\bullet (a-b)^2 = a^2 + b^2 - 2ab$$

$$\bullet (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$$

$$\bullet (a+b)^2 - (a-b)^2 = 4ab$$

$$\bullet (\sqrt{a})^2 = a$$

$$\begin{array}{l} a^2 + b^2 \quad 2 \quad a \times b \\ 7^2 + 3^2 = 58 \end{array}$$

Q) What is the square root of $16 + 6\sqrt{7}$?

$16 + 6\sqrt{7}$ का वर्गमूल क्या है?

$$\begin{aligned} & \sqrt{16 + 6\sqrt{7}} \\ & \downarrow \\ & (a^2 + b^2) + 2 \times \underbrace{3\sqrt{7}}_{\substack{3 \times \sqrt{7} \\ 9 + 7 = 16}} - (ab) \\ & \sqrt{(3 + \sqrt{7})^2} \\ & = 3 + \sqrt{7} \end{aligned}$$

Q) What is the square root of $15 - 4\sqrt{14}$?
 $15 - 4\sqrt{14}$ का वर्गमूल क्या है ?

$$\begin{aligned} & \sqrt{15 - 4\sqrt{14}} \\ & a^2 + b^2 - 2 \times \underbrace{2\sqrt{14}}_{a \times b} = \frac{\sqrt{2}}{\sqrt{7}} \\ & = 2\sqrt{2} - \sqrt{7} \\ & 8 + 7 = 15 \end{aligned}$$

Q) If $x = \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$ then find the value of x . Where $x > 0$

यदि $x = \sqrt{-\sqrt{3} + \sqrt{3 + 8\sqrt{7 + 4\sqrt{3}}}}$ तो x का मान ज्ञात करें जहाँ $x > 0$

$$\begin{aligned} & \begin{array}{c} 2 \times 2\sqrt{3} \\ \swarrow \searrow \\ 2 \times \sqrt{3} \end{array} \\ & 8\sqrt{(2 + \sqrt{3})^2} = 8(2 + \sqrt{3}) \\ & = 16 + 8\sqrt{3} \\ & \sqrt{19 + 8\sqrt{3}} \\ & 2 \times 4\sqrt{3} = \sqrt{(4 + \sqrt{3})^2} = 4 + \sqrt{3} \\ & \begin{array}{c} 4 \times \sqrt{3} \end{array} \\ & \sqrt{-\sqrt{3} + 4 + \sqrt{3}} \\ & \sqrt{4} = 2 \end{aligned}$$

Q) Find the positive square root of $92 - 24\sqrt{6 - 4\sqrt{2}}$.

$92 - 24\sqrt{6 - 4\sqrt{2}}$ का धनात्मक वर्गमूल ज्ञात कीजिए।

$$\begin{array}{c} 2 \times 2\sqrt{2} \\ / \quad \backslash \\ 2 \times \sqrt{2} \end{array}$$

$$\sqrt{(2 - \sqrt{2})^2} = (2 - \sqrt{2})$$

$$92 - 24(2 - \sqrt{2})$$

$$92 - 48 + 24\sqrt{2}$$

$$44 + 24\sqrt{2}$$

$$\sqrt{44 + 24\sqrt{2}}$$

$$\begin{array}{c} 2 \times 12\sqrt{2} \\ / \quad \backslash \\ 6 \times 2\sqrt{2} \end{array}$$

$$\sqrt{(6 + 2\sqrt{2})^2}$$

$$= 6 + 2\sqrt{2}$$

Q) $\sqrt{28 + 10\sqrt{3}} - \sqrt{7 - 4\sqrt{3}}$ का मान किसके निकटतम है ?

$$\begin{array}{c} 2 \times 5\sqrt{3} \\ / \quad \backslash \\ 5 \times \sqrt{3} \end{array}$$

$$\begin{array}{c} 2 \times 2\sqrt{3} \\ / \quad \backslash \\ 2 \times \sqrt{3} \end{array}$$

$$\sqrt{(5 + \sqrt{3})^2} - \sqrt{(2 - \sqrt{3})^2}$$

$$5 + \sqrt{3} - (2 - \sqrt{3})$$

$$5 + \sqrt{3} - 2 + \sqrt{3}$$

$$3 + 2\sqrt{3} \Rightarrow 3 + 2 \times 1.73$$

$$3 + 3.46$$

$$= 6.46$$

$$= 6.5$$

Q) The value of $\sqrt{5 + 2\sqrt{6}} - \frac{1}{\sqrt{5 + 2\sqrt{6}}}$ is :

$$\sqrt{5 + 2\sqrt{6}} - \frac{1}{\sqrt{5 + 2\sqrt{6}}} \text{ का मान है ;}$$

$$\sqrt{5+2\sqrt{6}}$$

$$\swarrow \searrow$$

$$\sqrt{3} \times \sqrt{2}$$

$$\sqrt{(\sqrt{3}+\sqrt{2})^2}$$

$$= (\sqrt{3}+\sqrt{2})$$

$$(\sqrt{3}+\sqrt{2}) - \frac{1}{(\sqrt{3}+\sqrt{2})}$$

$$(\sqrt{3}+\sqrt{2}) - (\sqrt{3}-\sqrt{2})$$

$$\downarrow$$

$$\cancel{\sqrt{3}} + \sqrt{2} - \cancel{\sqrt{3}} + \sqrt{2}$$

$$= 2\sqrt{2}$$

$$Q) \sqrt{30+12\sqrt{6}} - \sqrt{4+2\sqrt{3}} - \sqrt{19-6\sqrt{2}} = ?$$

$$\begin{array}{ccc} 2 \times 6\sqrt{6} & \swarrow \searrow & 2 \times 3\sqrt{2} \\ \swarrow \searrow & \sqrt{3} \times 1 & \swarrow \searrow \\ 2 \times 3 \times \sqrt{6} & & 3\sqrt{2} \times 1 \\ \swarrow \searrow & & \\ 2\sqrt{3} & 3\sqrt{2} & \end{array}$$

$$(3\sqrt{2}+2\sqrt{3}) - (\sqrt{3}+1) - (3\sqrt{2}-1)$$

$$3\sqrt{2} + 2\sqrt{3} - \sqrt{3} - \cancel{1} - 3\sqrt{2} + \cancel{1}$$

$$2\sqrt{3} - \sqrt{3}$$

$$= \sqrt{3}$$

$$Q) \text{ If } x = 8 + 2\sqrt{15}, \text{ then find } \sqrt{x} + \frac{2}{\sqrt{x}}$$

$$\sqrt{x} = \sqrt{8+2\sqrt{15}}$$

$$\swarrow \searrow$$

$$\sqrt{5} \times \sqrt{3}$$

$$\sqrt{x} = \sqrt{(\sqrt{5}+\sqrt{3})^2}$$

$$= \sqrt{5} + \sqrt{3}$$

$$\sqrt{5} + \sqrt{3} + \frac{2}{\sqrt{5}+\sqrt{3}}$$

$$\sqrt{5} + \sqrt{3} + \frac{\cancel{2}(\sqrt{5}-\sqrt{3})}{\cancel{2}}$$

$$\sqrt{5} + \cancel{\sqrt{3}} + \sqrt{5} - \cancel{\sqrt{3}}$$

$$= 2\sqrt{5}$$

$$Q) \left(\frac{2}{\sqrt{6} + \sqrt{4}} + \frac{1}{\sqrt{7} + \sqrt{6}} + \frac{1}{\sqrt{8} - \sqrt{7}} + 2 - 2\sqrt{2} \right) = ?$$

$$\begin{aligned} & \frac{2(\sqrt{6} - \sqrt{4})}{2} + \frac{\sqrt{7} - \sqrt{6}}{1} + \frac{\sqrt{8} + \sqrt{7}}{1} + 2 - 2\sqrt{2} \\ & \frac{\cancel{\sqrt{6}} - \cancel{\sqrt{4}}}{2} + \sqrt{7} - \cancel{\sqrt{6}} + \sqrt{8} + \sqrt{7} + 2 - 2\sqrt{2} \\ & \frac{2}{2} + 2\sqrt{7} = 2\sqrt{7} \end{aligned}$$

$$Q) \text{ What is } \frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{2020} + \sqrt{2021}}$$

$$\begin{aligned} & \frac{1}{1 + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{2020} + \sqrt{2021}} \text{ किसके बराबर है?} \\ & \frac{1}{\sqrt{2} + 1} + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{4} + \sqrt{3}} + \dots + \frac{1}{\sqrt{2021} + \sqrt{2020}} \\ & \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \dots + \sqrt{2021} - \sqrt{2020} \\ & \sqrt{2021} - 1 \\ & = \sqrt{2021} - 1 \end{aligned}$$

$$Q) \frac{1}{\sqrt{4} + \sqrt{5}} + \frac{1}{\sqrt{5} + \sqrt{6}} + \dots + \frac{1}{\sqrt{224} + \sqrt{225}} = ?$$

$$\begin{aligned} & \frac{1}{\sqrt{5} + \sqrt{4}} + \frac{1}{\sqrt{6} + \sqrt{5}} + \dots + \frac{1}{\sqrt{225} + \sqrt{224}} \\ & \sqrt{5} - \sqrt{4} + \sqrt{6} - \sqrt{5} + \sqrt{7} - \sqrt{6} + \dots + \sqrt{225} - \sqrt{224} \\ & \sqrt{225} - \sqrt{4} \\ & 15 - 2 = 13 \end{aligned}$$

$$Q) \text{ What is the value of } \frac{1}{4 - \sqrt{15}} - \frac{1}{\sqrt{15} - \sqrt{14}} + \frac{1}{\sqrt{14} - \sqrt{13}} - \frac{1}{\sqrt{13} - \sqrt{12}} +$$

$$\frac{1}{\sqrt{12} - \sqrt{11}} - \frac{1}{\sqrt{11} - \sqrt{10}} + \frac{1}{\sqrt{10} - 3} - \frac{1}{3 - \sqrt{8}}$$

$$\frac{1}{\sqrt{16} - \sqrt{15}} - \frac{1}{\sqrt{15} - \sqrt{14}} + \frac{1}{\sqrt{14} - \sqrt{13}} - \frac{1}{\sqrt{13} - \sqrt{12}} + \frac{1}{\sqrt{12} - \sqrt{11}} - \frac{1}{\sqrt{11} - \sqrt{10}} +$$

$$\frac{1}{\sqrt{10} - 3} - \frac{1}{3 - \sqrt{8}} \text{ का मान क्या है?}$$

$$\begin{aligned} & \sqrt{16} + \sqrt{15} - (\sqrt{15} + \sqrt{14}) + \sqrt{14} + \sqrt{13} - (\sqrt{13} + \sqrt{12}) + \sqrt{12} + \sqrt{11} - (\sqrt{11} + \sqrt{10}) + \\ & \quad \sqrt{10} + \sqrt{9} - (\sqrt{9} + \sqrt{8}) \\ & \sqrt{16} + \cancel{\sqrt{15}} - \cancel{\sqrt{15}} - \cancel{\sqrt{14}} + \cancel{\sqrt{14}} + \sqrt{13} - \cancel{\sqrt{13}} - \cancel{\sqrt{12}} + \cancel{\sqrt{12}} + \sqrt{11} - \cancel{\sqrt{11}} - \cancel{\sqrt{10}} + \cancel{\sqrt{10}} \\ & \quad + \cancel{\sqrt{9}} - \cancel{\sqrt{9}} - \sqrt{8} \end{aligned}$$

$$\begin{aligned} & \sqrt{16} - \sqrt{8} \\ & = 4 - 2\sqrt{2} \end{aligned}$$

$$\textcircled{8} \quad \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{2}+1}{\sqrt{2}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{2}-1}{\sqrt{2}+1} = ?$$

$$\begin{aligned} & \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{3-1} + \frac{(\sqrt{2}+1)(\sqrt{2}+1)}{2-1} + \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{3-1} + \frac{(\sqrt{2}-1)(\sqrt{2}-1)}{2-1} \\ & \frac{(\sqrt{3}+1)^2}{2} + \frac{(\sqrt{3}-1)^2}{2} + \frac{(\sqrt{2}+1)^2}{1} + \frac{(\sqrt{2}-1)^2}{1} \\ & \frac{1}{2} (2(3+1) + 2(2+1)) \\ & \quad 4 + 2 \times 3 \\ & \quad 4 + 6 = 10 \end{aligned}$$

$$\textcircled{9} \quad \left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \right)^2 + \left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \right)^2 \text{ is equal to:}$$

$$\begin{aligned} & \left[\frac{(\sqrt{5}+\sqrt{3})^2}{2} \right]^2 + \left[\frac{(\sqrt{5}-\sqrt{3})^2}{2} \right]^2 \\ & \frac{(8+2\sqrt{15})^2}{4} + \frac{(8-2\sqrt{15})^2}{4} \\ & \frac{1}{4} [2(8^2 + (2\sqrt{15})^2)] \\ & \frac{1}{2} [2 \times (64 + 60)] \\ & \frac{1}{2} (124) \\ & = 62 \end{aligned}$$

$$\begin{aligned} \textcircled{10} \quad & (\sqrt{3} + \sqrt{2})^2 - (\sqrt{3} - \sqrt{2})^2 = ? \\ & 4 \times \sqrt{3} \times \sqrt{2} \\ & = 4\sqrt{6} \end{aligned}$$

$$(a+b)^2 - (a-b)^2 = 4ab$$

Q) If $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$, find $a+b$

$$\frac{(\sqrt{3}+\sqrt{2})^2}{1}$$

$$3+2+2\sqrt{6} = a+b\sqrt{6}$$

$$5+2\sqrt{6} = a+b\sqrt{6}$$

$$a=5 \quad b=2$$

$$a+b = 5+2 = 7$$

Q) $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} = ?$

$$\frac{\cancel{3}\sqrt{2}(\sqrt{6}-\sqrt{3})}{\cancel{3}} - \frac{\cancel{4}\sqrt{3}(\sqrt{6}-\sqrt{2})}{\cancel{4}} + \frac{\sqrt{6}(\sqrt{3}-\sqrt{2})}{1}$$

$$\sqrt{12}-\sqrt{6} - (\sqrt{18}-\sqrt{6}) + \sqrt{18}-\sqrt{12}$$

$$\cancel{\sqrt{12}} - \sqrt{6} - \cancel{\sqrt{18}} + \sqrt{6} + \sqrt{18} - \cancel{\sqrt{12}}$$

$$= 0$$

1. What will be the positive square root of $19 + 4\sqrt{21}$

$19 + 4\sqrt{21}$ का धनात्मक वर्गमूल क्या होगा?

- (a) $\sqrt{7} + 2\sqrt{3}$
- (b) $\sqrt{3} + 2\sqrt{7}$
- (c) $\sqrt{2} + 3\sqrt{7}$
- (d) $\sqrt{7} + 3\sqrt{3}$

2. What is the square root of $14 + 6\sqrt{5}$

$14 + 6\sqrt{5}$ का धनात्मक वर्गमूल क्या है?

- (a) $3 + \sqrt{5}$
- (b) $3 - \sqrt{5}$
- (c) $5 + \sqrt{3}$
- (d) $5 - \sqrt{3}$

3. What is the value of

I. $(2 + \sqrt{2}) + \left(\frac{1}{2+\sqrt{2}}\right) + \left(\frac{1}{2-\sqrt{2}}\right) + (2 - \sqrt{2})$?

I. $(2 + \sqrt{2}) + \left(\frac{1}{2+\sqrt{2}}\right) + \left(\frac{1}{2-\sqrt{2}}\right) + (2 - \sqrt{2})$ का मान क्या है?

- (a) 2
- (b) 4
- (c) 8
- (d) 6

4. Simplify: $\frac{1}{\sqrt{100}-\sqrt{99}} - \frac{1}{\sqrt{99}-\sqrt{98}} + \frac{1}{\sqrt{98}-\sqrt{97}} - \frac{1}{\sqrt{97}-\sqrt{96}} + \dots + \frac{1}{\sqrt{2}-\sqrt{1}}$

- (a) 10
- (b) 9
- (c) 13
- (d) 11

5. $\left(\frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}}\right) + \frac{\sqrt{3}-1}{\sqrt{3}+1} = ?$

- (a) 16
- (b) $13 - \sqrt{3}$
- (c) $16 - \sqrt{3}$
- (d) $16 + \sqrt{3}$

6. $\left(\frac{1}{\sqrt{9}-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}} \right) = ?$

(a) 0

(b) 1

(c) 5

(d) $\frac{1}{3}$

7. $(3 + 2\sqrt{2})^2 + (3 - 2\sqrt{2})^2 = ?$

(a) 14

(b) 24

(c) 17

(d) 34

8. $\left(\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} \right)^2 + \left(\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} \right)^2 = ?$

(a) 142

(b) 140

(c) 71

(d) 60

9. If $x = 19 + 8\sqrt{3}$, then find $\sqrt{x} + \frac{13}{\sqrt{x}}$

(a) 38

(b) 8

(c) 19

(d) 16

10. If $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = a + b\sqrt{15}$, then find $a + b$

(a) 2

(b) 8

(c) 3

(d) 5

11 (If) यदि $x = \sqrt{-\sqrt{3} + \sqrt{3 + 8(2 + \sqrt{3})}}$ (find the value of x) तो x का

मान क्या होगा?

(a) 3 (b) 4

(c) 1 (d) 2

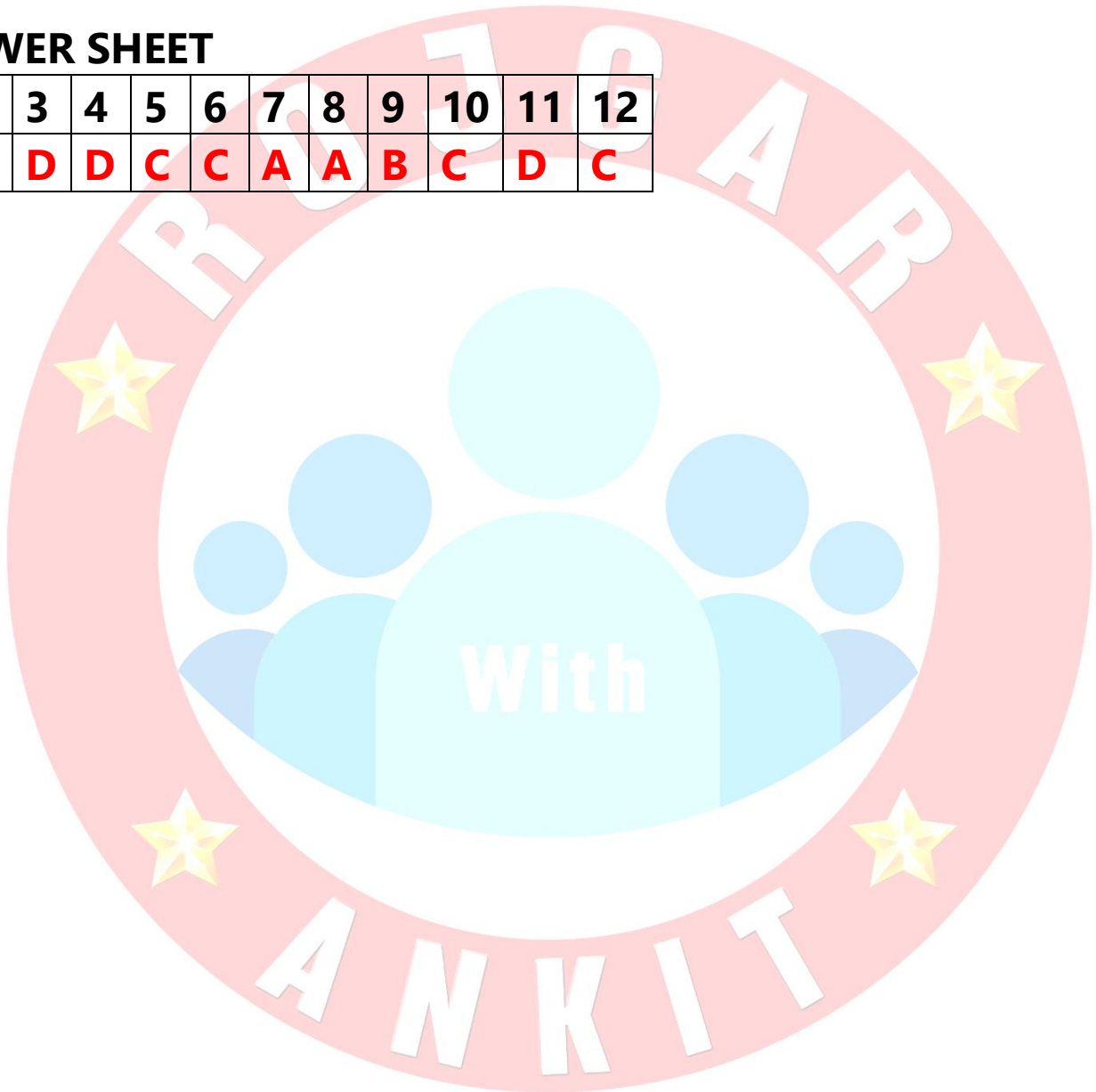
12. If $\sqrt{86 - 60\sqrt{2}} = a - b\sqrt{2}$, what is the correct value of $\sqrt{a^2 + b^2}$ upto one decimal place?

यदि $\sqrt{86 - 60\sqrt{2}} = a - b\sqrt{2}$ है तो $\sqrt{a^2 + b^2}$ का एक दशमलव तक शुद्ध मान क्या होगा?

- (a) 8.4
- (b) 8.2
- (c) 7.8
- (d) 7.2

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10	11	12
A	A	D	D	C	C	A	A	B	C	D	C



Worksheet solution

Sol 1

$$19 + 4\sqrt{21}$$

$$= 19 + 2\sqrt{4 \times 21}$$

$$= 7 + 12 + 2\sqrt{7 \times 12}$$

$$= (\sqrt{7})^2 + (\sqrt{12})^2 + 2\sqrt{7} \sqrt{12}$$

$$= (\sqrt{7} + \sqrt{12})^2$$

वर्ग मूल = $\sqrt{(\sqrt{7} + \sqrt{12})^2}$

$$= \sqrt{7} + 2\sqrt{3}$$

Sol 2

$$14 + 6\sqrt{5}$$

$$= \sqrt{14 + 2 \times 3\sqrt{5}}$$

$$= \sqrt{9 + 5 + 2 \times 3\sqrt{5}}$$

$$= \sqrt{(3)^2 + (\sqrt{5})^2 + 2 \times 3\sqrt{5}}$$

$$= \sqrt{(3 + \sqrt{5})^2}$$

$$= 3 + \sqrt{5}$$

Sol 3

$$(2 + \sqrt{2}) + \left(\frac{1}{2 + \sqrt{2}} \right) +$$

$$\left(\frac{1}{2 - \sqrt{2}} \right) + (2 - \sqrt{2})$$

$$= \frac{4 + 2\sqrt{2} + 2 - \sqrt{2} + 2 + \sqrt{2} + 4 - 2\sqrt{2}}{2}$$

$$= \frac{12}{2} = 6$$

Sol 4

$$\frac{1}{\sqrt{100} - \sqrt{99}} - \frac{1}{\sqrt{99} - \sqrt{98}} + \dots$$

$$- \dots - \frac{1}{\sqrt{2} - \sqrt{1}}$$

$$\left[\frac{1}{\sqrt{100} - \sqrt{99}} \times \frac{\sqrt{100} + \sqrt{99}}{\sqrt{100} + \sqrt{99}} \right]$$

$$= \frac{\sqrt{100} + \sqrt{99}}{1}$$

$$= (\sqrt{100} + \sqrt{99}) - (\sqrt{99} + \sqrt{98})$$

$$- \dots - (\sqrt{2} + \sqrt{1})$$

$$= \sqrt{100} + \sqrt{1}$$

$$= 10 + 1 = 11 \text{ ans}$$

Sol 5

$$\left(\frac{2 + \sqrt{3}}{2 - \sqrt{3}} + \frac{2 - \sqrt{3}}{2 + \sqrt{3}} \right) + \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \left[\frac{(4+2\sqrt{3}+2\sqrt{3}+3) + (4-2\sqrt{3}-2\sqrt{3}+3)}{(2-\sqrt{3})(2+\sqrt{3})} \right] + \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \left[\frac{7+4\sqrt{3}+7-4\sqrt{3}}{1} \right] + \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= 14 + \frac{(\sqrt{3}-1)^2}{2}$$

$$= 14 + 2 - \sqrt{3} = 16 - \sqrt{3}$$

Sol 6

$$\frac{1}{\sqrt{9}-\sqrt{8}} = \frac{1}{\sqrt{9}-\sqrt{8}} \times \frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}} = \frac{\sqrt{9}+\sqrt{8}}{1}$$

$$= (\sqrt{9}+\sqrt{8})$$

similarly ; $\frac{1}{(\sqrt{8}-\sqrt{7})} = (\sqrt{8}+\sqrt{7})$, $\frac{1}{(\sqrt{7}-\sqrt{6})} = (\sqrt{7}+\sqrt{6})$

$$\frac{1}{(\sqrt{6}-\sqrt{5})} = (\sqrt{6}+\sqrt{5}) \quad \frac{1}{(\sqrt{5}-\sqrt{4})} = \sqrt{5}+\sqrt{4}$$

Given expression = $(\sqrt{9}+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+\sqrt{4})$

$$= 5(\sqrt{9}+\sqrt{4}) = (3+2) = 5 \underline{\text{Ans}}$$

Sol 7

$$\begin{aligned}
 & (3+2\sqrt{2})^2 + (3-2\sqrt{2})^2 \\
 &= 7-4\sqrt{2} + 7+4\sqrt{2} \\
 &= 14
 \end{aligned}$$

Sol 8

$$\begin{aligned}
 & \left[\frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} - \sqrt{5}} \right]^2 + \left[\frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}} \right]^2 \\
 &= \frac{12 + 2\sqrt{35}}{12 - 2\sqrt{35}} + \frac{12 - 2\sqrt{35}}{12 + 2\sqrt{35}} \\
 &= \frac{284 + 284}{4} = 142
 \end{aligned}$$

Sol 9

$$\begin{aligned}
 x &= 19 + 8\sqrt{3} \\
 x &= 16 + 3 + 8\sqrt{3} \\
 x &= (4 + \sqrt{3})^2 \\
 \sqrt{x} &= \sqrt{(4 + \sqrt{3})^2} \\
 \sqrt{x} &= 4 + \sqrt{3}
 \end{aligned}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}}$$

$$\frac{1}{\sqrt{x}} = \frac{4 - \sqrt{3}}{13}$$

$$\frac{13}{\sqrt{x}} = 4 - \sqrt{3}$$

$$\begin{aligned}
 \sqrt{x} + \frac{13}{\sqrt{x}} &= 4 + \sqrt{3} + 4 - \sqrt{3} \\
 &= 8
 \end{aligned}$$

Sol 10

$$\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} = a + b\sqrt{15}$$

$$\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} \times \frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} - \sqrt{3}}$$

$$= a + b\sqrt{15}$$

$$\frac{8 - 2\sqrt{15}}{2} = a + b\sqrt{15}$$

$$\therefore a = 4, b = -1$$

$$4 - \sqrt{15} = 4 - \sqrt{15}$$

$$\therefore a + b = 4 - 1 = 3$$

Sol 11

$$\begin{aligned}
 x &= \sqrt{-\sqrt{3} + \sqrt{3 + 8(2 + \sqrt{3})}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{3 + 16 + 8\sqrt{3}}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{19 + 8\sqrt{3}}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{(4)^2 + (\sqrt{3})^2 + 2 \cdot 4 \cdot \sqrt{3}}} \\
 &= \sqrt{-\sqrt{3} + \sqrt{(4 + \sqrt{3})^2}} \\
 &= \sqrt{-\sqrt{3} + 4 + \sqrt{3}} = 2
 \end{aligned}$$

Sol 12

$$\sqrt{86 - 60\sqrt{2}} = a - b\sqrt{2}$$

$$\sqrt{50 + 36 - 2 \times 6 \times 5\sqrt{2}} = a - b\sqrt{2}$$

$$\sqrt{(5\sqrt{2} - 6)^2} = a - b\sqrt{2}$$

$$a - b\sqrt{2} = -6 + 5\sqrt{2}$$

$$a = -6 \text{ and } b = -5$$

$$\sqrt{(-6)^2 + (-5)^2} = \sqrt{36 + 25}$$

$$= \sqrt{61} = 7.8$$