SURDS AND INDICES

* **Indices** :- An Indices is a rational number with a rational power.

घातांक:- एक परिमेय संख्या जिसकी घात भी परिमेय होती है।

Ex:- 3^5, 10^{10}, $4^{-2} = \frac{1}{4^2}$

* **Surds** :- An Irrational number with a power.

करणी :- एक अपरिमेय संख्या जिसकी कोई घात होती है।

Ex:- $\sqrt{5}$, $\sqrt{3}$, $\sqrt{10}$

$a^{m} \rightarrow$ Power

a → base

LAWS OF INDICES (घातांक के नियम)

1. $(a^m)^n = a^{m\times n}$
2. $a^{m^n} = a^{m \times m \times m \times \cdots \times n}$ बार — Ex:- $2^{3^{10}}$ = $2^{3\times3\times3 \cdots 10}$ बार
3. $a^m \times a^n = a^{m+n}$ — Ex:- $2^3 \times 2^4 = 2^7$
4. $a^m \times b^m = (a\times b)^m$ — Ex:- $2^4 \times 3^4 = (2\times3)^4 = 6^4$
5. $a^0 = 1$ — Ex:- $3^0 = 1$, $1000^0 = 1$
6. $a^{-b} = \frac{1}{a^b} \Rightarrow \frac{1}{a^{-b}} = a^{+b}$ — Ex:- $5^{-3} = \frac{1}{5^3}$
7. $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$ — Ex:- $\frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$
8. $\frac{a^m}{a^n} = a^{m-n}$ — Ex:- $\frac{4^5}{4^3} = 4^{5-3} = 4^2$
9. $a^m = a^n \Rightarrow m = n$ — Ex:- $2^x = 2^3$, $x = 3$

Important formulae about indices:
घातांक से सम्बन्धित महत्वपूर्ण सूत्र:
If $a > 0, a \neq 1$, m and n are integers then
यदि $a > 0, a \neq 1$, तथा m और n पूर्णांक हो तो

(a) $a^m \times a^n = a^{m+n}$
(b) $a^m \times a^n \times a^p = a^{m+n+p}$
(c) $(a^m)^n = a^{mn}$
(d) $\frac{a^m}{a^n} = a^{m-n}$
(e) $a^0 = 1$
(f) $a^{-m} = \frac{1}{a^m}$
(g) $a^{m^n} = a^{(m^n)}$
(h) $(ab)^n = a^n b^n, (abc)^n = a^n b^n c^n$

Laws Of Surds (करणी के नियम)

1. $\sqrt[n]{a} = (a)^{\frac{1}{n}}$

   Ex:- $\sqrt[2]{2} = (2)^{1/2}$ $\quad \sqrt[2]{3} = (3)^{\frac{1}{2}}$
   $\sqrt[3]{5} = (5)^{1/3}$ $\quad \sqrt[9]{7} = (7)^{\frac{1}{9}}$

2. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

   Ex:- $\frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$

3. if, $a^x = b$
   then $a = b^{1/x}$

   Ex:- $x^3 = 216$
   $x = (216)^{1/3}$

4. $a^x = b^y$
   $a = b^{\frac{y}{x}}$

5. $a^n = y$
   $a = y^{1/n}$

6. $a^x = b^y$
   $a^{1/y} = b^{1/x}$

Important formula for surd:
करणी से सम्बन्धित महत्वपूर्ण सूत्रः

(a) If (यदि) $a^n = y$ then (तो) $a = y^{1/n}$
(b) If (यदि) $a^x = b^y$ then (तो) $a = b^{y/x}$
(c) If (यदि) $a^x = b^y$ then (तो) $\boxed{a^{1/y} = b^{1/x}}$ etc.
(d) $x^n = a \Leftrightarrow x = \sqrt[n]{a}, (a \in R, a \geq 0)$
(e) If n is an odd positive integer and (यदि n एक विषम धन पूर्णांक है तथा) $a > 0$ then (तो) $\sqrt[n]{-a} = -\sqrt[n]{a}$ (here $m, n \geq 2$, and $a, b > 0$ then)
(f) $\sqrt[n]{a} = a^{\frac{1}{n}}$
(g) $(\sqrt[n]{a})^m = \sqrt[n]{a^m} = a^{\frac{m}{n}}$
(h) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$
(i) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
(j) $\sqrt[n]{\sqrt[m]{a}} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{mn}} = \sqrt[mn]{a}$
(k) $\sqrt[n]{a} \cdot \sqrt[m]{a} = a^{\frac{1}{n}} \times a^{\frac{1}{m}} = a^{\frac{1}{n}+\frac{1}{m}} = a^{\frac{m+n}{mn}} = \sqrt[mn]{a^{m+n}}$
(l) $\frac{\sqrt[n]{a}}{\sqrt[m]{a}} = \frac{a^{\frac{1}{n}}}{a^{\frac{1}{m}}} = a^{\frac{1}{n}-\frac{1}{m}} = a^{\frac{m-n}{mn}} = \sqrt[mn]{a^{m-n}}$

TYPE-I

Q) $3^{\sqrt{x}} + 4^{\sqrt{x}} = 5^{\sqrt{x}}$. find x.

$3^2 + 4^2 = 5^2$

$\sqrt{x} = 2 \Rightarrow$ Square both side

$x = 2^2 = 4$

$x^{1/2} = 2$

$x = 2^{\frac{2}{1}} = 2^2 = 4$

Q) $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$ find x

$5^2 + 12^2 = 13^2$

$\sqrt{x} = 2$

$x = 2^2 = 4$

Q) If $3^{x+y} = 81$ and $81^{x-y} = 3$ then the value of $\frac{x}{y}$ is

$3^{x+y} = 3^4$

$\boxed{x+y=4}$

$(3^4)^{(x-y)} = 3^1$

$3^{4(x-y)} = 3^1$

$4(x-y) = 1$

$\boxed{x-y=\frac{1}{4}}$

$a^m = a^n$

$\boxed{m=n}$

$$\frac{x+y}{x-y} = \frac{4}{\frac{1}{4}} = \frac{16}{1}$$

C & D Rule $\frac{x}{y} = \frac{16+1}{16-1} = \frac{17}{15}$

Q) If $3^{x-1} + 3^{3-x} = 6$, then what is $2^{x-1} + 2^{3-x}$ equal

$$\frac{3^x}{3^1} + \frac{3^3}{3^x} = 6$$

① $a^{m-n} = \frac{a^m}{a^n}$

② $a^{m+n} = a^m \times a^n$

$$\frac{3^{2x} + 3^4}{3 \times 3^x} = 6$$

$$3^{2x} + 81 = 18 \times 3^x$$

$x = 2$

$3^4 + 81 \qquad 18 \times 3^2$

$81 + 81 \qquad 18 \times 9$

$162 = 162$

$2^{x-1} + 2^{3-x}$

$2^{2-1} + 2^{3-2}$

$2^1 + 2^1$

4

Q) If $5^{x+1} - 5^{x-1} = 600$, then what is the value of 10^{2x}?

$$5^1 \times 5^x - \frac{5^x}{5^1}$$

$$5^x\left(5 - \frac{1}{5}\right) = 600$$

$$5^x\left(\frac{\cancel{24}^{\,2}}{5}\right) = \cancel{600}\ \cancel{50}\ 25$$

$$5^x = 125 = 5^3$$

$$\boxed{x = 3}$$

10^{2x}

$10^{2 \times 3} = 10^6$

1000000

Q) If $3^{x+3} \times 9^{2x-5} = 3^{3x+7}$ then the value of x is :

$3^{x+3} \times (3^2)^{2x-5}$

$3^{x+3} \times 3^{4x-10}$

$3^{5x-7} = 3^{3x+7}$

$5x-7 = 3x+7$

$\cancel{2}x = \cancel{14}\,7$

$x = 7$

Q) If $\frac{4^x \times 16^8 \times 8^{\frac{2}{3}}}{2^{3x+2} \times (256)^3} = 32^{\frac{x}{2}+2}$ then the value of x is

$$\frac{2^{2x} \times (2^4)^8 \times (2^{\cancel{3}})^{\frac{2}{\cancel{3}}}}{2^{3x+2} \times (2^8)^3} = (2^5)^{\frac{x}{2}+2}$$

$$\frac{2^{2x} \times 2^{32} \times 2^2}{2^{3x+2} \times 2^{24}} = \underline{2^{\frac{5x}{2}+10}}$$

$$\frac{2^{2x} \times 2^8 \times 2^2}{2^{3x+2}} = \frac{2^{2x+10}}{2^{3x+2}}$$

$2^{2x+10-3x-2}$

$= 2^{8-x}$

$2^{8-x} = \underline{2^{\frac{5x}{2}+10}}$

$8-x = \frac{5x}{2}+10$

$-2 = \frac{5x}{2}+x$

$-2 = \frac{7x}{2}$

$x = \frac{-4}{7}$

Q) If $\sqrt{6}^5 \times (36)^2 = \sqrt[3]{6} \times 6^{\frac{x}{2}} \times 6\sqrt{6}$ then find the value

$$\left(6^{\frac{1}{2}}\right)^5 \times \left(6^2\right)^2 = 6^{\frac{1}{3}} \times 6^{\frac{x}{2}} \times 6^1 \times 6^{1/2}$$

$$6^{\frac{5}{2}} \times 6^4 = 6^{\frac{1}{3}+\frac{x}{2}+1+\frac{1}{2}}$$

$$\frac{3}{2}+\frac{1}{3} = \frac{11}{6}+\frac{x}{2}$$

$$\frac{22+6x}{12}$$

$$6^{\frac{13}{2}} = 6^{\frac{22+6x}{12}}$$

$$\frac{13}{\cancel{2}} = \frac{22+6x}{\cancel{12}\ 6}$$

$$22+6x = 78$$

$$6x = 56$$

$$x = \frac{\cancel{56}}{\cancel{6}}\ \frac{28}{3}$$

$$\frac{28}{3}$$

1. $3^{\sqrt{x}} + 4^{\sqrt{x}} = 5^{\sqrt{x}}$ find x.
(a) 2
(b) 3
(c) 4
(d) 5

2. $3^x - 3^{x-1} = 486$. Find x
(a) 7
(b) 9
(c) 5
(d) 6

3. If $2^{n-1} + 2^{n+1} = 320$ then the value of n is
यदि $2^{n-1} + 2^{n+1} = 320$ तो n का मान है
(a) 6
(b) 8
(c) 5
(d) 7

4. $9^{(5x-2)} = 27^4$ find $x =$?
(a) 2
(b) 3.2
(c) 4
(d) 1.6

5. If $27^{2n-1} = (243)^3$ then the value of n is :
यदि $27^{2n-1} = (243)^3$ तो n का मान है :
(a) 3
(b) 6
(c) 7
(d) 9

6. If $3^{x+y} = 81$ and $81^{x-y} = 3$ then the value of $\frac{x}{y}$ is
यदि $3^{x+y} = 81$ तथा $81^{x-y} = 3$ तो $\frac{x}{y}$ का मान क्या होगा?
(a) $\frac{15}{17}$ (b) $\frac{17}{30}$
(c) $\frac{15}{34}$ (d) $\frac{17}{15}$

7. If $11\sqrt{n} = \sqrt{112} + \sqrt{343}$ then the value of n is:
यदि $11\sqrt{n} = \sqrt{112} + \sqrt{343}$ तो n का मान है
(a) 3
(b) 13
(c) 11
(d) 7

8. simplify $\frac{2^{3a}\times3^{2a}\times5^{a}\times6^{a}}{8^{a}\times9^{3a/2}\times10^{a}}$

(a) 5

(b) 3

(c) 0

(d) 1

9. $\frac{18^4\times27^3\times36^5}{(1024)\times(243)^4}=\left(2^{1/2}\right)^8\times3^{x/2}$, find the value of x

$\frac{18^4\times27^3\times36^5}{(1024)\times(243)^4}=\left(2^{1/2}\right)^8\times3^{x/2}$, x का मान ज्ञात करें

(a) 14

(b) 21

(c) 7

(d) 16

10. The value of $\frac{(243)^{\frac{n}{5}}\times3^{2n+1}}{9^{n}\times3^{n-1}}$ is

$\frac{(243)^{\frac{n}{5}}\times3^{2n+1}}{9^{n}\times3^{n-1}}$ का मान है

(a) 3

(d) 9

(c) 6

(d) 12

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
C	D	D	D	A	D	D	D	A	B

Worksheet solution

Sol 1

$$3^{\sqrt{x}} + 4^{\sqrt{x}} = 5^{\sqrt{x}}$$

we know

$$3^2 + 4^2 = 5^2$$

$$\therefore \sqrt{x} = 2$$

$$x = 4$$

Sol 2

$$3^x - 3^{x-1} = 486$$

$$3^x\left[1 - \frac{1}{3}\right] = 486$$

$$3^x\left[\frac{2}{3}\right] = 486$$

$$3^x = \frac{486 \times 3}{2}$$

$$3^x = 3^6$$

$$\therefore x = 6$$

Sol 3

$$2^{x-1} + 2^{x+1} = 320$$

$$2^x\left[\frac{1}{2} + 2\right] = 320$$

$$2^x = \frac{320 \times 2}{5}$$

$$2^x = 2^7$$

$$\therefore x = 7$$

Sol 4

$$9^{5x-2} = 27^4$$

$$3^{10x-4} = 3^{12} \quad \left[27^4 = (3^3)^4\right]$$

$$10x - 4 = 12$$

$$10x = 16$$

$$x = 1.6$$

Sol 5

$$27^{2x-1} = (243)^3$$

$$(3^3)^{2x-1} = (3^5)^3$$

$$3^{6x-3} = 3^{15}$$

$$6x - 3 = 15$$

$$6x = 18$$

$$x = 3$$

Sol 6

$$3^{x+y} = 81 \ \& \ 81^{x-y} = 3$$

$$3^{x+y} = 3^4, \quad x + y = 4$$

$$\&$$

$$(3^4)^{x-y} = 3^1, \quad 4x - 4y = 1$$

$$4x + 4y = 16$$

$$4x - 4y = 1$$

$$8x = 17$$

and

$$8y = 15$$

$$\therefore \frac{x}{y} = \frac{17}{15}$$

$\boxed{\text{Sol-7}}$

$11\sqrt{x} = \sqrt{112} + \sqrt{343}$

$11\sqrt{x} = 4\sqrt{7} + 7\sqrt{7}$

$11\sqrt{x} = 11\sqrt{7}$

$\therefore x = 7$

$\boxed{\text{Sol 8}}$

$$\frac{2^{3a} \times 3^{2a} \times 5^{a} \times 6^{a}}{8^{a} \times 9^{3a/2} \times 10^{a}}$$

$$\frac{2^{3a} \times 3^{2a} \times 30^{a}}{2^{3a} \times (3^{2})^{3a/2} \times 10^{a}}$$

$$\frac{2^{3a} \times 3^{2a} \times 30^{a}}{2^{3a} \times 3^{3a} \times 10^{a}}$$

$3^{-a} \times 3^{a} = 1$

$\boxed{\text{Sol 9}}$

$$\frac{18^{4} \times 27^{3} \times 36^{5}}{(1024) \times (243)} = (2^{1/2})^{8} \times 3^{x/2}$$

$$\frac{(2 \times 3^{2})^{4} \times (3^{3})^{3} \times (2^{2} \times 3^{2})^{5}}{2^{10} \times (3^{5})^{4}} = 2^{4} \times 3^{\frac{x}{2}}$$

$$\frac{2^{4} \times 2^{10} \times 3^{8} \times 3^{9} \times 3^{10}}{2^{10} \times 3^{20}} = 2^{4} \times 3^{\frac{x}{2}}$$

$2^{4} \times 3^{7} = 2^{4} \times 3^{\frac{x}{2}}$

$3^{7} = 3^{\frac{x}{2}}$

$7 = \frac{x}{2}$

$x = 14$

$\boxed{\text{Sol 10}}$

$$\Rightarrow \frac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^{n} \times 3^{n-1}}$$

$$= \frac{(3^{5})^{\frac{n}{5}} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$$

$$= \frac{3^{n} \times 3^{2n+1}}{3^{2} \times 3^{n-1}}$$

$$= \frac{3^{3n} \times 3}{3^{3n} \times 3^{-1}} = 9$$