# Foundation Batch

# MATHS

## Surds and Indices

## (घातांक एवं करणी)

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## Indices (घातांक)

↳ An Indices is a rational number with a rational power.

ex → $3^5$, $10^{10}$, $4^{-2} = \frac{1}{4^2}$

## Surds (करणी)

↳ An Irrational number with a power

ex $\sqrt{5}$, $\sqrt{3}$, $\sqrt{10}$

# Laws of Indices (घातांक के नियम)

① $(a^m)^n = a^{m \times n}$

② $a^{m^n} = a^{m \times m \times m \times \cdots \text{n बार}}$

$2^{3^{10}}$

$2^{3 \times 3 \times 3 \cdots \text{10 बार}}$

③ $a^m \times a^n = a^{m+n}$

$2^3 \times 2^4 = 2^7$

④ $a^m \times b^m = (a \times b)^m$

$2^4 \times 3^4 = (2 \times 3)^4$

$= 6^4$

⑤ $a^0 = 1$

⑥ $a^{-b} = \frac{1}{a^b} \Rightarrow \frac{1}{a^{-b}} = a^{+b}$

$5^{-3} = \frac{1}{5^3}$

⑦ $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

$\frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$

⑧ $\frac{a^m}{a^n} = a^{m-n}$

$\frac{4^5}{4^3} = 4^{5-3} = 4^2$

⑨* $a^m = a^n \Rightarrow \boxed{m=n}$

$2^x = 2^3$

$\boxed{x=3}$

**Important formulae about indices:**

**घातांक से सम्बन्धित महत्वपूर्ण सूत्रः**

**If $a > 0, a \neq 1$, m and n are integers then**

**यदि $a > 0, a \neq 1$, तथा m और n पूर्णांक हो तो**

**(a)** $a^m \times a^n = a^{m+n}$

**(b)** $a^m \times a^n \times a^p = a^{m+n+p}$

**(c)** $(a^m)^n = a^{mn}$

**(d)** $\frac{a^m}{a^n} = a^{m-n}$

**(e)** $a^0 = 1$

**(f)** $a^{-m} = \frac{1}{a^m}$

**(g)** $a^{m^n} = a^{(m^n)}$

**(h)** $(ab)^n = a^n b^n, (abc)^n = a^n b^n c^n$

# laws of Surds (करणी के नियम)

① $\sqrt[n]{a} = (a)^{\frac{1}{n}}$

② $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

③ if, $a^x = b$

then $\boxed{a = b^{\frac{1}{x}}}$

ex

$x^3 = 216$

$x = (216)^{1/3}$

$$a^x = b^y$$

$$a = b^{\frac{y}{x}}$$

**Important formula for surd:**

**करणी से सम्बन्धित महत्वपूर्ण सूत्र:**

(a) If (यदि) $a^n = y$ then (तो) $a = y^{1/n}$

(b) If (यदि) $a^x = b^y$ then (तो) $a = b^{y/x}$

(c) If (यदि) $a^x = b^y$ then (तो) $a^{1/y} = b^{1/x}$ etc.

(d) $x^n = a \Leftrightarrow x = \sqrt[n]{a}, (a \in R, a \geq 0)$

(e) If n is an odd positive integer and (यदि n एक विषम धन पूर्णांक है तथा) $a > 0$ then (तो) $\sqrt[n]{-a} = -\sqrt[n]{a}$ (here $m, n \geq 2$, and $a, b > 0$ then)

(f) $\sqrt[n]{a} = a^{\frac{1}{n}}$

(g) $(\sqrt[n]{a})^m = \sqrt[n]{a^m} = a^{\frac{m}{n}}$

(h) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

(i) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

(j) $\sqrt[n]{\sqrt[m]{a}} = \left(a^{\frac{1}{m}}\right)^{\frac{1}{n}} = a^{\frac{1}{mn}} = \sqrt[mn]{a}$

(k) $\sqrt[n]{a} \cdot \sqrt[m]{a} = a^{\frac{1}{n}} \times a^{\frac{1}{m}} = a^{\frac{1}{n}+\frac{1}{m}} = a^{\frac{m+n}{mn}} = \sqrt[mn]{a^{m+n}}$

(l) $\frac{\sqrt[n]{a}}{\sqrt[m]{a}} = \frac{a^{\frac{1}{n}}}{a^{\frac{1}{m}}} = a^{\frac{1}{n}-\frac{1}{m}} = a^{\frac{m-n}{mn}} = \sqrt[mn]{a^{m-n}}$

# TYPE- I

## Basic

1. $3^{\sqrt{x}} + 4^{\sqrt{x}} = 5^{\sqrt{x}}$ find $x$.

(a) 2

(b) 3

(c) ~~4~~

(d) 5

$3^2 + 4^2 = 5^2$

$\sqrt{x} = 2 \Rightarrow$ Square both side

$x = 2^2 = 4$

$x^{1/2} = 2$

$x = 2^{\frac{2}{1}} = 2^2 = 4$

2. $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$ find $x$

(a) 3

(b) 4

(c) 2

(d) 1

$5^2 + 12^2 = 13^2$

$\sqrt{x} = 2$

$x = 2^2 = 4$

**3.** If $3^{x+y} = 81$ and $81^{x-y} = 3$ then the value of $\frac{x}{y}$ is

(a) $\frac{15}{17}$

(b) $\frac{17}{30}$

(c) $\frac{15}{34}$

(d) $\frac{17}{15}$

$3^{x+y} = 3^4$

$x+y=4$

$(3^4)^{(x-y)} = 3^1$

$3^{4(x-y)} = 3^1$

$4(x-y)=1$

$x-y = \frac{1}{4}$

$a^m = a^n$

$m=n$

$\frac{x+y}{x-y} = \frac{4}{\frac{1}{4}} = \frac{16}{1}$

C&D Rule $\frac{x}{y} = \frac{16+1}{16-1} = \frac{17}{15}$

**4.** If $3^{x-1} + 3^{3-x} = 6$, then what is $2^{x-1} + 2^{3-x}$ equal to?

(a) 4
(b) 3
(c) 2
(d) 1

$\frac{3^x}{3^1} + \frac{3^3}{3^x} = 6$

$\frac{3^{2x} + 3^4}{3 \times 3^x} = 6$

$3^{2x} + 81 = 18 \times 3^x$

$x = 2$

$3^4 + 81$ | $18 \times 3^2$

$81 + 81$ | $18 \times 9$

$162 = 162$ | $1$

$2^{2-1} + 2^{3-2}$

$2^1 + 2^1$

$= 4$

① $a^{m-n} = \frac{a^m}{a^n}$

$a^{m+n} = a^m \times a^n$

CDS [2023-II]

**5.** If $5^{x+1} - 5^{x-1} = 600$, then what is the value of $10^{2x}$?

(a) 1
(b) 1000
(c) 100000
(d) 1000000

$5^1 \times 5^x \quad \frac{5^x}{5^1}$

$5^x\left(5 - \frac{1}{5}\right) = 600$

$5^x\left(\frac{24}{5}\right) = 600 \times 25$

$5^x = 125 = 5^3$

$x = 3$

$10^{2x3} = 10^6$

CDS [2020-I]

**6.** If $3^{x+3} \times 9^{2x-5} = 3^{3x+7}$ then the value of $x$ is :

(a) 5

(b) 6

~~(c) 7~~

(d) 8

$$3^{x+3} \times (3^2)^{2x-5}$$

$$3^{x+3} \times 3^{4x-10}$$

$$3^{5x-7} = 3^{3x+7}$$

$$5x-7 = 3x+7$$

$$2x = 14$$

$$x = 7$$

**7.** If $\frac{4^x \times 16^8 \times 8^{\frac{2}{3}}}{2^{3x+2} \times (256)^3} = 32^{\frac{x}{2}+2}$ then the value of x is

(a) 3/2
(b) 9/2
(c) ~~−4/7~~
(d) 4/7

$$\frac{2^{2x} \times (2^2)^8 \times (2^3)^{\frac{2}{3}}}{2^{3x+2} \times (2^8)^3} = (2^5)^{\frac{x}{2}+2}$$

$$\Rightarrow \frac{2^{2x} \times 2^{32} \times 2^2}{2^{3x+2} \times 2^{24}} = 2^{\frac{5x}{2}+10}$$

$$\frac{2^x \times 2^8 \times 2^2}{2^{3x+2}} = \frac{2}{2^{3x+2}} = 2^{2x+10} \quad 2^{2x+10-3x-2} = 2^{8-x}$$

$$\therefore 2^{8-x} = 2^{\frac{5x}{2}+10}$$

$$8-x = \frac{5x}{2}+10$$

$$-2 = \frac{5x}{2}+x$$

$$-2 = \frac{7x}{2}$$

$$x = -\frac{4}{7}$$

$2^{32}$ / $2^{24}$

$\frac{a^m}{a^n} = a^{m-n}$

**8.** If $\sqrt{6}^{5} \times (36)^{2} = \sqrt[3]{6} \times 6^{\frac{x}{2}} \times 6\sqrt{6}$ then find the value of $x$ :

(a) 25/3

(b) 28/3

(c) −25/3

(d) -28/3

$$\left(6^{\frac{1}{2}}\right)^{5} \times (6^{2})^{2} = 6^{\frac{1}{3}} \times 6^{\frac{x}{2}} \times 6^{1} \times 6^{1/2}$$

$$6^{\frac{5}{2}} \times 6^{4} = 6^{\frac{1}{3}+\frac{x}{2}+1+\frac{1}{2}}$$

$$\frac{3}{2}+\frac{1}{3} = \frac{11}{6}+\frac{x}{2}$$

$$\frac{22+6x}{12}$$

$$6^{\frac{13}{2}} = 6^{\frac{22+6x}{12}}$$

$$\frac{13}{2} = \frac{22+6x}{12_{6}}$$

$$22+6x = 78$$

$$6x = 56$$

$$x = \frac{28}{3}$$

H.W.

11. If $\frac{4^n \times 4^8 - 4^{(n+1)}}{4 \times 4^{n+7}} = x$, then find $x \times 4^7$

(a) $(4^7 - 1)$

(b) $(4^6 - 1)$

(c) $(4^5 - 1)$

(d) $4^{10} - 1$