

TRIGONOMETRY

$$\bullet a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$$

$$\bullet a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$$

Q) What is $(\sin^2 x - \cos^2 x)(1 - \sin^2 x \cos^2 x)$ equal to ?

$(\sin^2 x - \cos^2 x)(1 - \sin^2 x \cos^2 x)$ किसके बराबर है ?

$$(\sin^2 x - \cos^2 x)((\sin^2 x + \cos^2 x)^2 - \sin^2 x \cos^2 x)$$

$$(\sin^4 x + \cos^4 x + 2 \sin^2 x \cdot \cos^2 x - \sin^2 x \cos^2 x)$$

$$(\sin^2 x - \cos^2 x)(\sin^4 x + \cos^4 x + \sin^2 x \cdot \cos^2 x)$$

$$(\sin^2 x)^3 - (\cos^2 x)^3$$

$$\sin^6 x - \cos^6 x$$

Q) Consider the following equations:

निम्नलिखित समीकरणों पर विचार करें:

I. $\operatorname{cosec}^2 x + \sec^2 x = \operatorname{cosec}^2 x \sec^2 x$

II. $\sec^2 x + \tan^2 x = \sec^2 x \tan^2 x$

III. $\operatorname{cosec}^2 x + \tan^2 x = \cot^2 x + \sec^2 x$

Which of the above statements are correct?

उपर्युक्त में से कौन से कथन सही हैं ?

a) I and II

b) II and III

☒ c) I and III

d) All of these

☒ I. $\operatorname{cosec}^2 x + \sec^2 x = \operatorname{cosec}^2 x \sec^2 x$

$$\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$$

$$\frac{\cos^2 x + \sin^2 x}{\sin^2 x \cdot \cos^2 x}$$

$$\frac{1}{\sin^2 x \cdot \cos^2 x}$$

$$= \operatorname{cosec}^2 x \cdot \sec^2 x$$

II) $\sec^2 x + \tan^2 x = \sec^2 x \tan^2 x$

$$\frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{1 + \sin^2 x}{\cos^2 x}$$

III) $\operatorname{cosec}^2 x + \tan^2 x = \cot^2 x + \sec^2 x$

$$1 + \cot^2 x + \sec^2 x - 1$$

$$\cot^2 x + \sec^2 x$$

$$\therefore \sec^2 x - \tan^2 x = 1$$

$$\tan^2 x = \sec^2 x - 1$$

Q) If $\sin^2 \alpha = \cos^3 \alpha$, then the value of $(\cot^6 \alpha - \cot^2 \alpha)$ is -

यदि $\sin^2 \alpha = \cos^3 \alpha$, तो $(\cot^6 \alpha - \cot^2 \alpha)$ का मान है -

$$\sin^2 \alpha = \cos \alpha \cos^2 \alpha \quad \cot^2 \alpha (\cot^4 \alpha - 1)$$

$$\frac{\cos^2 \alpha}{\sin^2 \alpha} = \frac{1}{\cos \alpha}$$

$$\cot^2 \alpha (\sec^2 \alpha - 1)$$

$$\cot^2 \alpha \cdot \tan^2 \alpha$$

$$= 1$$

$$\boxed{\cot^2 \alpha = \sec \alpha}$$

$$\cot^4 \alpha = \sec^2 \alpha$$

Q) If $a^2 = \frac{1+2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta}$, then what is the value of $\frac{a+1}{a-1}$?

यदि $a^2 = \frac{1+2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta}$ है, तो $\frac{a+1}{a-1}$ का मान क्या है ?

$$\frac{\sin^2 \theta + \cos^2 \theta + 2\sin\theta\cos\theta}{\sin^2 \theta + \cos^2 \theta - 2\sin\theta\cos\theta}$$

$$a^2 = \frac{(\sin\theta + \cos\theta)^2}{(\sin\theta - \cos\theta)^2}$$

$$\boxed{\frac{a}{1} = \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}}$$

$$\frac{a+1}{a-1} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\begin{aligned} \bullet (a+b)^3 &= a^3 + b^3 + 3a^2b + 3ab^2 \\ \bullet (a-b)^3 &= a^3 - b^3 - 3a^2b + 3ab^2 \end{aligned}$$

Q) If $\sin \theta + \sin^2 \theta = 1$, then what is the value of $\cos^{12} \theta + 3\cos^{10} \theta + 3\cos^8 \theta + \cos^6 \theta - 1$?

यदि $\sin \theta + \sin^2 \theta = 1$ हो, तो $\cos^{12} \theta + 3\cos^{10} \theta + 3\cos^8 \theta + \cos^6 \theta - 1$ का मान क्या होगा?

$$\sin \theta = 1 - \sin^2 \theta$$

$$\boxed{\sin \theta = \cos^2 \theta}$$

$$\boxed{\sin^2 \theta = \cos^4 \theta}$$

$$[(\cos^4 \theta)^3 + (\cos^2 \theta)^3 + 3\cos^8 \theta \cdot \cos^2 \theta + 3 \times \cos^4 \theta \times \cos^4 \theta] - 1$$

$$[\cos^4 \theta + \cos^2 \theta]^3 - 1$$

$$[\sin^2 \theta + \cos^2 \theta]^3 - 1$$

$$1 - 1 = 0$$

Q) If $\cos A = 1 - \sin^2 (90^\circ - A)$, then $\sin^{12} A + 3\sin^{10} A + 3\sin^8 A + \sin^6 A + 2\sin^4 A + 2\sin^2 A - 2 = ?$

यदि $\cos A = 1 - \sin^2 (90^\circ - A)$, तो $\sin^{12} A + 3\sin^{10} A + 3\sin^8 A + \sin^6 A + 2\sin^4 A + 2\sin^2 A - 2 = ?$

$$\cos A = 1 - \cos^2 A$$

$$\boxed{\cos A = \sin^2 A}$$

$$\cos^2 A = \sin^4 A$$

$$[\sin^4 A + \sin^2 A]^3 + 2[\sin^4 A + \sin^2 A - 1]$$

$$[\cos^2 A + \sin^2 A]^3 + 2[\cos^2 A + \sin^2 A - 1]$$

$$1^3 + 2(1 - 1)$$

$$1 + 2 \times 0 = 1$$

$\boxed{\sin \theta + \operatorname{cosec} \theta = 2}$

$$\downarrow$$

$$\downarrow$$

$$\boxed{\theta = 90^\circ}$$

$$\boxed{\sin^n \theta + \operatorname{cosec}^n \theta = 2}$$

$\boxed{\cos \theta + \sec \theta = 2}$

$$\downarrow$$

$$\downarrow$$

$$\boxed{\theta = 0}$$

$$\boxed{\cos^n \theta + \sec^n \theta = 2}$$

$\boxed{\tan \theta + \cot \theta = 2}$

$$\downarrow$$

$$\downarrow$$

$$\boxed{\theta = 45^\circ}$$

$$\boxed{\tan^n \theta + \cot^n \theta = 2}$$

Q) If $\cos \theta + \sec \theta = 2$, the value of $\cos^6 \theta + \sec^6 \theta$ is :

यदि $\cos \theta + \sec \theta = 2$, तो $\cos^6 \theta + \sec^6 \theta$ का मान है :

$$\boxed{\theta = 0^\circ}$$

$$\frac{11}{2}$$

Q) If $\sin \theta + \operatorname{cosec} \theta = 2$, then what is the value of $\sin^4 \theta + \cos^4 \theta$?

यदि $\sin \theta + \operatorname{cosec} \theta = 2$ है, तो $\sin^4 \theta + \cos^4 \theta$ का मान क्या है ?

$$\boxed{\theta = 90^\circ}$$

$$\sin^4 90^\circ + \cos^4 90^\circ$$

$$1 + 0$$

$$1$$

Q) If θ is an acute angle and $\tan \theta + \cot \theta = 2$, then find the value of $\tan^3 \theta + \cot^3 \theta + 6 \tan^3 \theta \cdot \cot^2 \theta$

यदि θ एक न्यूनकोण है और $\tan \theta + \cot \theta = 2$ है, तो $\tan^3 \theta + \cot^3 \theta + 6 \tan^3 \theta \cdot \cot^2 \theta$ का मान ज्ञात कीजिए

$$\tan^3 \theta + \cot^3 \theta + 6 \tan^3 \theta \cdot \cot^2 \theta$$

$$\downarrow$$

$$\downarrow$$

$$\downarrow$$

$$1$$

$$1$$

$$6 \times 1 \times 1$$

$$1 + 1 + 6$$

$$8$$

$$\boxed{\theta = 45^\circ}$$

Q) If $\sin \theta + \operatorname{cosec} \theta = 2$, then what will be the value of $\sin^n \theta + \operatorname{cosec}^n \theta$?

यदि $\sin \theta + \operatorname{cosec} \theta = 2$ हो, तो $\sin^n \theta + \operatorname{cosec}^n \theta$ का मान क्या होगा ?

$$\frac{11}{2}$$

Q) If $\cos \theta + \sec \theta = 2$, then $(\cos^{17} \theta + \sec^{17} \theta)$ is equal to :

यदि $\cos \theta + \sec \theta = 2$ है, तो $(\cos^{17} \theta + \sec^{17} \theta)$ बराबर है :

$$\frac{11}{2}$$

Q) If θ is an acute angle and $\cot \theta + \tan \theta = 2$ then find the value of $\tan^{12} \theta + \cot^{12} \theta + 2 \tan^5 \theta \cot^7 \theta$.

यदि θ एक न्यून कोण है और $\cot \theta + \tan \theta = 2$ है तो $\tan^{12} \theta + \cot^{12} \theta + 2 \tan^5 \theta \cot^7 \theta$ का मान ज्ञात कीजिए।

$$1 + 1 + 2 \times 1 \times 1$$

$$1 + 1 + 2$$

$$4$$

$$\boxed{\theta = 45^\circ}$$

#

- $\sin(A+B) = \sin A \cos B + \cos A \sin B$

- $\boxed{B \rightarrow (-B)}$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

- $\boxed{A \rightarrow (90+A)}$ $\cos(A+B) = \cos A \cos B - \sin A \sin B$

- $\boxed{B \rightarrow (-B)}$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

- $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

- $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

- $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$

- $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

Q) Find the value of $\sin 15^\circ$.

$\sin 15^\circ$ का मान ज्ञात कीजिए।

$$\sin(15^\circ) = \sin(45^\circ - 30^\circ)$$

$$\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

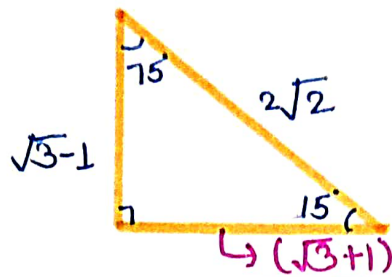
$$\frac{1}{\sqrt{2}} \left[\frac{\sqrt{3}}{2} - \frac{1}{2} \right]$$

$$\frac{1}{\sqrt{2}} \left(\frac{\sqrt{3}-1}{2} \right)$$

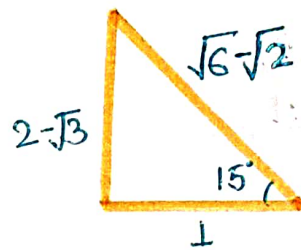
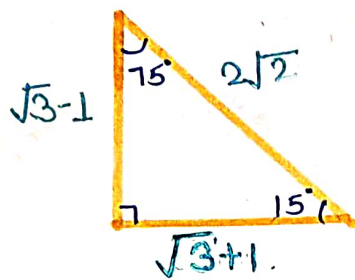
$$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$\sin(90+A+B)$
 $\cos(A+B)$

- $\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$



$$\begin{aligned} & \frac{\sqrt{8 - (\sqrt{3}-1)^2}}{\sqrt{8 - (3+1-2\sqrt{3})}} \\ & \frac{\sqrt{8 - (4-2\sqrt{3})}}{\sqrt{8-4+2\sqrt{3}}} \\ & \frac{\sqrt{4+2\sqrt{3}}}{\sqrt{4+2\sqrt{3}}} = 1 \end{aligned}$$



- $\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

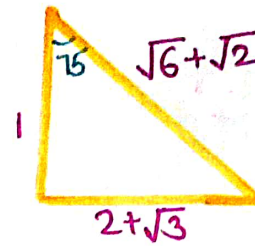
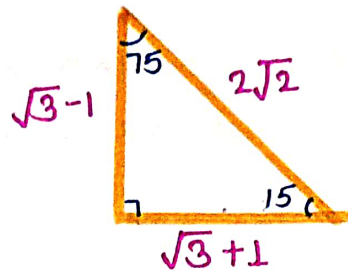
- $\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

- $\cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$$\begin{aligned} \tan 15^\circ &= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} \\ &= \frac{(\sqrt{3}-1)^2}{3-1} = \frac{4-2\sqrt{3}}{2} \\ &= \frac{2(2-\sqrt{3})}{2} \end{aligned}$$

$$\tan 15^\circ = 2-\sqrt{3}$$

Q) Find the value of $\tan 75^\circ$
 $\tan 75^\circ$ का मान ज्ञात कीजिए।



$$\begin{aligned}\tan 75^\circ &= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2}{3-1} \\ &= \frac{4+2\sqrt{3}}{2} \\ &= \frac{2(2+\sqrt{3})}{2} \\ &= (2+\sqrt{3})\end{aligned}$$

I	II	III

1. For $0^\circ < \theta < 90^\circ$, $\tan \theta + \cot \theta = 2$ then θ is equal to:

$0^\circ < \theta < 90^\circ$, के लिए $\tan \theta + \cot \theta = 2$ तो θ बराबर है :

- (a) 30°
- (b) 60°
- (c) 45°
- (d) 0°

2. If $\tan A + \cot A = 2$, then $\tan 3A + \cot 3A = ?$

यदि $\tan A + \cot A = 2$ है, तो $\tan 3A + \cot 3A = ?$

- (A) -2
- (B) 1
- (C) 0
- (D) 3

3. If $\cos \theta + \sec \theta = 2$, then find the value of $\sin^6 \theta + \cos^6 \theta$.

यदि $\cos \theta + \sec \theta = 2$ है, तो $\sin^6 \theta + \cos^6 \theta$ का मान ज्ञात कीजिए।

- (a) $\frac{1}{3}$
- (b) 1
- (c) $\frac{1}{2}$
- (d) 2

4. Find the value of $\sin 25^\circ \cos 35^\circ + \cos 25^\circ \sin 35^\circ$.

$\sin 25^\circ \cos 35^\circ + \cos 25^\circ \sin 35^\circ$ का मान ज्ञात कीजिए।

- (a) $\frac{\sqrt{3}}{2}$
- (b) $\frac{1}{\sqrt{2}}$
- (c) 1
- (d) $\frac{1}{2}$

5. If $0^\circ \leq \theta \leq 90^\circ$, and $\sec^{107} \theta + \cos^{107} \theta = 2$, then, $(\sec \theta + \cos \theta)$ is equal to:

यदि $0^\circ \leq \theta \leq 90^\circ$ और $\sec^{107} \theta + \cos^{107} \theta = 2$ है, तो $(\sec \theta + \cos \theta)$ का मान ज्ञात कीजिए।

- (a) 2^{-107}
- (b) 2
- (c) $1/2$
- (d) 1

6. If $\tan \theta + \cot \theta = 2$, θ is an acute angle, then find the value of $2\tan^{25} \theta + 3\cot^{20} \theta + 5\tan^{30} \theta \cot^{15} \theta$.

यदि $\tan \theta + \cot \theta = 2$ है, θ एक न्यून कोण है, तो $2\tan^{25} \theta + 3\cot^{20} \theta + 5\tan^{30} \theta \cot^{15} \theta$ का मान ज्ञात कीजिए।

- a. 12
- b. 10
- c. 8
- d. 6

7. If $\sin \theta + \operatorname{cosec} \theta = 2$, then what is the value of $(\sin^{153} \theta + \operatorname{cosec}^{253} \theta)$?

यदि $\sin \theta + \operatorname{cosec} \theta = 2$ है, तो $(\sin^{153} \theta + \operatorname{cosec}^{253} \theta)$ का मान है?

(a) $\frac{1}{153 \times 253}$

(b) $\frac{253}{153}$

(c) 2

(d) $\frac{153}{253}$

8. If $\sin(A + B) = \frac{\sqrt{3}}{2}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, then $(2A + 3B)$ is equal to:

यदि $\sin(A + B) = \frac{\sqrt{3}}{2}$ और $\tan(A - B) = \frac{1}{\sqrt{3}}$ तो $(2A + 3B)$ बराबर है:

(a) 120°

(b) 135°

(c) 130°

(d) 125°

9. What is the value of $\frac{\sin(A+B)}{\sin A \cos B}$?

$\frac{\sin(A+B)}{\sin A \cos B}$ का मान क्या है?

(a) $1 + \cot A \tan B$

(b) $1 + \tan A \cot B$

(c) $1 - \sin A \cos B$

(d) $1 - \cot A \tan B$

10. Using trigonometric formulas, find the value of

$\left(\frac{\sin(x-y)}{\sin(x+y)} \right) \left(\frac{\tan x + \tan y}{\tan x - \tan y} \right)$.

त्रिकोणमितीय सूत्रों का उपयोग करके $\left(\frac{\sin(x-y)}{\sin(x+y)} \right) \left(\frac{\tan x + \tan y}{\tan x - \tan y} \right)$ का मान ज्ञात करें।

करें।

(a) -2

(b) 2

(c) 0

(d) 1

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
C	A	B	A	B	B	C	B	A	D

Worksheet solution

sol 1

$$\tan \theta + \cot \theta = 2$$

$$\therefore \theta = 45^\circ \quad (\text{Property})$$

sol 2

$$\tan A + \cot A = 2 \quad (\because A = 45^\circ)$$

$$\tan 3A + \cot 3A = ?$$

$$\tan 3 \times 45 + \cot 3 \times 45$$

$$\tan 135^\circ + \cot 135^\circ = -2$$

sol 3

$$\cos \theta + \sec \theta = 2$$

$$\theta = 0^\circ$$

$$= \sin^6 \theta + \cos^6 \theta$$

$$= \sin^6 0^\circ + \cos^6 0^\circ$$

$$= 0 + (1)^6 = 1$$

sol 4

$$\sin A \cdot \cos B + \cos A \cdot \sin B = \sin(A+B)$$

$$\sin 25^\circ \cos 35^\circ + \cos 25^\circ \sin 35^\circ = \sin(25^\circ + 35^\circ)$$

$$= \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2}$$

sol 5

$$\sec^{107} \theta + \cos^{107} \theta = 2$$

$$\theta = 0^\circ$$

$$(\because \sec^n \theta + \cos^n \theta = 2)$$

$$= \sec \theta + \cos \theta$$

$$= \sec 0^\circ + \cos 0^\circ$$

$$= 1 + 1 = 2$$

Sol-6

$$\tan \theta + \cot \theta = 2$$

$$\theta = 45^\circ$$

$$= 2 \tan^{25} \theta + 3 \cot^{20} \theta + 5 \tan^{30} \theta \cot^{15} \theta$$

$$= 2 \tan^{25} 45^\circ + 3 \cot^{20} 45^\circ + 5 \tan^{30} 45^\circ \cot^{15} 45^\circ$$

$$= 2 (1)^{25} + 3 (1)^{20} + 5 (1)^{30} \cdot (1)^{15}$$

$$= 2 + 3 + 5 = 10$$

Sol-7

$$\sin \theta + \operatorname{cosec} \theta = 2$$

$$\theta = 90^\circ$$

$$= \sin^{153} \theta + \operatorname{cosec}^{253} \theta$$

$$= \sin^{153} 90^\circ + \operatorname{cosec}^{253} 90^\circ$$

$$= (1)^{153} + (1)^{253} = 2$$

Sol-8

$$\sin(A+B) = \frac{\sqrt{3}}{2}, (A+B) = 60^\circ \dots (i)$$

$$\tan(A-B) = \frac{1}{\sqrt{3}}, (A-B) = 30^\circ \dots (ii)$$

$$A+B = 60^\circ$$

$$A-B = 30^\circ$$

$$\hline 2A = 90^\circ$$

$$A = 45^\circ, B = 15^\circ$$

$$\therefore (2A+3B) = 90^\circ + 45^\circ = 135^\circ$$

Sol-9

$$\frac{\sin(A+B)}{\sin A \cos B}$$

$$= \frac{\sin A \cdot \cos B + \cos A \cdot \sin B}{\sin A \cdot \cos B}$$

$$= 1 + \cot A \tan B$$

Sol 10

$$= \left(\frac{\sin(x-y)}{\sin(x+y)} \right) \times \left[\frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}} \right]$$

$$= \frac{\sin(x-y)}{\sin(x+y)} \times \frac{\frac{\sin x \cdot \cos y + \sin y \cos x}{\cos x \cos y}}{\frac{\sin x \cos y - \sin y \cos x}{\cos x \cos y}}$$

$$= \frac{\sin(x-y)}{\sin(x+y)} \times \frac{\sin(x+y)}{\sin(x-y)}$$

$$= 1$$