TRIGONOMETRY

Q) Find: $\tan\left(\frac{5\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)$

ज्ञात कीजिये: $\tan\left(\frac{5\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)$?

$$\tan\left(\pi+\left(\frac{\pi}{4}+\theta\right)\right)-\tan\left(\frac{\pi}{4}-\theta\right)$$

$$\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)$$

$$\sec 2\theta+\tan 2\theta-(\sec 2\theta-\tan 2\theta)$$

$$\cancel{\sec 2\theta}+\tan 2\theta-\cancel{\sec 2\theta}+\tan 2\theta$$

$$2\tan 2\theta$$

Q) If $A+B+C=\pi$, then the value of $\tan(A+B)+\tan C$ is:

यदि $A+B+C=\pi$, तो $\tan(A+B)+\tan C$ का मान है:

$$C=\pi-(A+B)$$

$$\tan C=\tan(\pi-(A+B))$$

$$\tan C=-\tan(A+B)$$

$$\tan C+\tan(A+B)=0$$

Q) If $\tan A+\tan B+\tan C=\tan A\tan B\tan C$, the $(A+B+C)$ will be equal to:

यदि $\tan A+\tan B+\tan C=\tan A\tan B\tan C$, तो $(A+B+C)$ बराबर होगा:

$$A+B+C=180^\circ/\pi$$

$$\tan A+\tan B+\tan C=\tan A\cdot\tan B\cdot\tan C$$

\# $\boxed{A+B+C=180^\circ}$

$$A+B=180-C$$

$$\tan(A+B)=\tan(180-C)$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$$

$$\tan A+\tan B=-\tan C+\tan A\tan B\tan C$$

$$\boxed{\tan A+\tan B+\tan C=\tan A\tan B\tan C}$$

$A = B + C$

$$\tan A = \tan(B+C)$$

$$\tan A = \frac{\tan B + \tan C}{1 - \tan B \tan C}$$

$$\tan A - \tan A \tan B \tan C = \tan B + \tan C$$

$$\tan A - \tan B - \tan C = \tan A \tan B \tan C$$

Ex :- $\tan 80^\circ - \tan 70^\circ - \tan 10 = ?$

$= \tan 80 \tan 70 \tan 10$

Q) If $\tan A - \tan B - \tan C = \tan A \tan B \tan C$, what is the value of A in terms of B and C?

यदि $\tan A - \tan B - \tan C = \tan A \tan B \tan C$ है, तो B और C के संदर्भ मे A का मान क्या है?

$$A = B + C$$

Q) $\tan 13x - \tan 9x - \tan 4x =$

$\tan 13x \cdot \tan 9x \cdot \tan 4x$

$a \sin x + b \cos x = m$
$a \cos x - b \sin x = n$

$$(a^2+b^2)(\underbrace{\sin^2 x + \cos^2 x}_{1}) = m^2 + n^2$$

$$a^2 + b^2 = m^2 + n^2$$

$a \sec x - b \tan x = m$
$a \tan x - b \sec x = n$

$$a^2 - b^2 = m^2 - n^2$$

$a \operatorname{cosec} x - b \cot x = m$
$a \cot x - b \operatorname{cosec} x = n$

$$a^2 - b^2 = m^2 - n^2$$

Q) If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then the value of x^2+y^2 is:

यदि $x = a\cos\theta + b\sin\theta$ तथा $y = a\sin\theta - b\cos\theta$ है, तो x^2+y^2 का मान ज्ञात करें।

$$a^2+b^2 = x^2+y^2$$

Q) यदि $x = m\sec A + n\tan A$ और $y = m\tan A + n\sec A$ है, तो x^2-y^2 का मान क्या है?

If $x = m\sec A + n\tan A$ and $y = m\tan A + n\sec A$, then what is x^2-y^2 equal to

$$m\sec A + n\tan A = x$$
$$m\tan A + n\sec A = y$$
$$m\sec A - (-n)\tan A = x$$
$$m\tan A - (-n)\sec A = y$$
$$m^2-(-n)^2 = x^2-y^2$$
$$m^2-n^2 = x^2-y^2$$

Q) If $a\cot\theta + b\operatorname{cosec}\theta = p$ and $b\cot\theta + a\operatorname{cosec}\theta = q$ then p^2-q^2 is equal to:

यदि $a\cot\theta + b\operatorname{cosec}\theta = p$ और $b\cot\theta + a\operatorname{cosec}\theta = q$ है, तो p^2-q^2 निम्नलिखित में से किसके बराबर होगा?

$$b\operatorname{cosec}\theta + a\cot\theta = p$$
$$b\cot\theta + a\operatorname{cosec}\theta = q$$
$$b\operatorname{cosec}\theta - (-a)\cot\theta = p$$
$$b\cot\theta - (-a)\operatorname{cosec}\theta = q$$
$$b^2-a^2 = p^2-q^2$$

Q) If $3\sin\theta + 5\cos\theta = 5$, then what is the value of $5\sin\theta - 3\cos\theta$?

यदि $3\sin\theta + 5\cos\theta = 5$, तो $5\sin\theta - 3\cos\theta$ का मान क्या है?

$3\cos\theta - 5\sin\theta = n$

$3^2 + 5^2 = 5^2 + n^2$

$\boxed{n = \pm 3}$

$5\sin\theta - 3\cos\theta$

$-(3\cos\theta - \sin\theta)$

-3

Q) If $\sin x + \cos x = \sqrt{2}\sin x$, then the value of $\sin x - \cos x$.

यदि $1\sin x + 1\cos x = \sqrt{2}\sin x$ है, तो $\sin x - \cos x$ का मान ज्ञात करें।

$1\sin x + 1\cos x = \sqrt{2}\sin x$

$1\cos x - 1\sin x = n$

$1^2 + 1^2 = (\sqrt{2}\sin x)^2 + n^2$

$2 - 2\sin^2 x = n^2$

$2(1 - \sin^2 x) = n^2$

$2\cos^2 x = n^2$

$\boxed{n = \sqrt{2}\cos x}$

$\sin x - \cos x$

$-(\cos x - \sin x)$

$-(\sqrt{2}\cos x)$

$-\sqrt{2}\cos x$

Q) If $\cos x + \sin x = \sqrt{2}\cos x$, what will be the value of $(\cos x - \sin x)^2 + (\cos x + \sin x)^2$?

यदि $\cos x + \sin x = \sqrt{2}\cos x$ है, तो $(\cos x - \sin x)^2 + (\cos x + \sin x)^2$ का मान क्या होगा?

$\sin x - \cos x = n$

$1^2 + 1^2 = (\sqrt{2}\cos x)^2 + n^2$

$2 - 2\cos^2 x = n^2$

$2(1 - \cos^2 x)$

$2\sin^2 x = n^2$

$\boxed{n = \sqrt{2}\sin x}$

$(\cos x - \sin x)^2 + (\cos x + \sin x)^2$

$(\sqrt{2}\sin x)^2 + (\sqrt{2}\cos x)^2$

$2\sin^2 x + 2\cos^2 x$

$2(\underbrace{\sin^2 x + \cos^2 x}_{1})$

$2 \times 1 = 2$

Q) If $\sin\theta + \cos\theta = \frac{\sqrt{11}}{3}$, Find the value of $(\cos\theta - \sin\theta)$.

यदि $\sin\theta + \cos\theta = \frac{\sqrt{11}}{3}$ है, तो $(\cos\theta - \sin\theta)$ का मान ज्ञात कीजिए।

$$\cos\theta - \sin\theta = n$$

$$1^2 + 1^2 = \left(\frac{\sqrt{11}}{3}\right)^2 + n^2$$

$$2 = \frac{11}{9} = n^2$$

$$n^2 = 2 - \frac{11}{9} = \frac{7}{9}$$

$$n = \sqrt{\frac{7}{9}} = \frac{\sqrt{7}}{3}$$

Q) If $\sin\theta + \cos\theta = \frac{1}{29}$, find the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$.

यदि $\sin\theta + \cos\theta = \frac{1}{29}$ है, तो $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$ का मान ज्ञात कीजिए।

$$\cos\theta + \sin\theta = \frac{1}{29}$$

$$\sin\theta - \cos\theta = n$$

$$1^2 + 1^2 = \left(\frac{1}{29}\right)^2 + n^2$$

$$n^2 = \frac{1681}{841}$$

$$n = \sqrt{\frac{1681}{841}} = \frac{41}{29} \Rightarrow \sin\theta - \cos\theta$$

$$\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$\frac{\frac{1}{29}}{\frac{41}{29}} = \frac{1}{41}$$

Q) If $\sin\theta - \cos\theta = \frac{7}{17}$, then find the value of $\sin\theta + \cos\theta$

यदि $\sin\theta - \cos\theta = \frac{7}{17}$ है, तो $\sin\theta + \cos\theta$ का मान ज्ञात कीजिए।

$$\cos\theta + \sin\theta = n$$

$$1^2 + 1^2 = \left(\frac{7}{17}\right)^2 + n^2$$

$$2 = \frac{49}{289} + n^2$$

$$n^2 = 2 - \frac{49}{289}$$

$$\frac{578 - 49}{289} = \frac{529}{289}$$

$$n = \sqrt{\frac{529}{289}} = \frac{23}{17}$$

Q) If $3\sin x + 4\cos x = 2$, then find the value of $3\cos x - 4\sin x$ is:

यदि $3\sin x + 4\cos x = 2$ तो $3\cos x - 4\sin x$ का मान ज्ञात करे।

$$3\cos x - 4\sin x = n$$

$$3^2 + 4^2 = 2^2 + n^2$$

$$9 + 16 = 4 + n^2$$

$$21 = n^2$$

$$n = \sqrt{21}$$

✱ Questions Based On Some Other Basic Identities like :-

(i) $\dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \dfrac{1}{\sec A - \tan A}$

$= \sec A + \tan A$

$= \dfrac{1 + \sin A}{\cos A}$

(ii) $\dfrac{\cos A + \sin A - 1}{\cos A - \sin A + 1} = \operatorname{cosec} A - \cot A$

$= \dfrac{1 - \cos A}{\sin A}$

(iii) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

$= \dfrac{1 + \cos A}{\sin A}$

Q) If $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = p\sec\theta + q\tan\theta$ where $0<\theta<\frac{\pi}{2}$ then what is the value of $p+q$?

यदि $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = p\sec\theta + q\tan\theta$ जहाँ $0<\theta<\frac{\pi}{2}$ है तो $p+q$ का मान क्या है?

$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$$

$$\sec\theta+\tan\theta$$

$p=1 \quad q=1$

$p+q=2$

1. If $A + B + C = 180^\circ$ and $\tan A = \frac{1}{2}$, $\tan B = 3$, then what will be the value of $\tan C$?

यदि $A + B + C = 180^\circ$ और $\tan A = \frac{1}{2}$, $\tan B = 3$, तो $\tan C$ का मान क्या होगा?

(A) $-\frac{5}{3}$
(B) $\frac{5}{3}$
(C) $\frac{7}{2}$
(D) 7

2. If $\tan A = \frac{2}{3}, \tan B = \frac{3}{4}$, and $\tan C = k$, and $A + B + C = 90^\circ$, then what will be the value of k?

यदि $\tan A = \frac{2}{3}, \tan B = \frac{3}{4}$, और $\tan C = k$, तथा $A + B + C = 90^\circ$, तो k का मान क्या होगा?

(A) $17/6$
(B) $15/6$
(C) $12/5$
(D) $19/6$

3. If $\cot\theta + \cos\theta = x$ & $\cot\theta - \cos\theta = y$, then the value of $(x^2 - y^2)$ is:
यदि $\cot\theta + \cos\theta = x$ और $\cot\theta - \cos\theta = y$, तो $(x^2 - y^2)$ का मान है:

(a) $4\sqrt{xy}$ (b) $2xy$
(c) $2\sqrt{xy}$ (d) $4xy$

4. If $\tan x + \sin x = m$ and $\tan x - \sin x = n$, then $(m^2 - n^2)$ is equal to:

यदि $\tan x + \sin x = m$ और $\tan x - \sin x = n$, तो $(m^2 - n^2)$ बराबर है:

(a) $4\sqrt{mn}$ (b) $\sqrt{mn}$
(c) $2\sqrt{mn}$ (d) $4mn$

5. If $3\sin x + 5\cos x = 5$, then the value of $(3\cos x - 5\sin x)^2$ is:
यदि $3\sin x + 5\cos x = 5$, तो $(3\cos x - 5\sin x)^2$ का मान है:

(a) 2
(b) 4
(c) 5
(d) 9

6. If $a\cot\theta + b\operatorname{cosec}\theta = x^2$ and $b\cot\theta + a\operatorname{cosec}\theta = y^2$, then the value of $(x^4 - y^4)$ is:
यदि $a\cot\theta + b\operatorname{cosec}\theta = x^2$ और $b\cot\theta + a\operatorname{cosec}\theta = y^2$ है, तो $(x^4 - y^4)$ का मान है:

(a) $a^2 + b^2$
(b) $a^2 - b^2$
(c) $b^2 - a^2$
(d) $(a - b)^2$

7. If a and b are real numbers such that $a\cos\theta + b\sin\theta = 4$ and $a\sin\theta - b\cos\theta = 3$, then $(a^2 + b^2)$ is:

यदि a और b वास्तविक संख्याएँ हैं जैसे कि $a\cos\theta + b\sin\theta = 4$ और $a\cos\theta + b\sin\theta = 4$, तो $(a^2 + b^2)$ है:

(a) 7
(b) 12
(c) 25
(d) $\sqrt{12}$

8. The value of $\frac{\sin\theta+\cos\theta-1}{\sin\theta-\cos\theta+1} \times \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is :

$\frac{\sin\theta+\cos\theta-1}{\sin\theta-\cos\theta+1} \times \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ का मान ज्ञात कीजिए।

(a) 1
(b) -1
(c) 2
(d) -2

9. Find the value $\frac{\sin\theta+\cos\theta-1}{\sin\theta-\cos\theta+1}\times\frac{\tan^2\theta(\operatorname{cosec}^2\theta-1)}{\sec\theta-\tan\theta}$

$\frac{\sin\theta+\cos\theta-1}{\sin\theta-\cos\theta+1}\times\frac{\tan^2\theta(\operatorname{cosec}^2\theta-1)}{\sec\theta-\tan\theta}$ का मान है।

(a) 1
(b) 3
(c) 2
(d) 4

10. What is $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}-\frac{\sin\theta+1}{\cos\theta}$ **equal to ?**

$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}-\frac{\sin\theta+1}{\cos\theta}$ किसके बराबर है?

(a) 0
(b) 1
(c) $2\sin q$
(d) $2\cos q$

ANSWER SHEET

1	2	3	4	5	6	7	8	9	10
D	A	A	A	D	C	C	A	A	A

Worksheet solution

Sol 1

$A+B+C=180^\circ$

$\tan A=\frac{1}{2}$, $\tan B=3$

$\tan(A+B)=-\tan C$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=-\tan C$$

$$\frac{\frac{1}{2}+3}{1-\frac{1}{2}\times 3}=-\tan C$$

$$-\tan C=\frac{7}{2}\times\frac{2}{-1}$$

$$\tan C=7$$

Sol 2

$A+B+C=90^\circ$

$\tan A=\frac{2}{3}$, $\tan B=\frac{3}{4}$, $\tan C=K$

$\tan(A+B)=\tan C$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=K$$

$$\frac{\frac{2}{3}+\frac{3}{4}}{1-\frac{2}{3}\times\frac{3}{4}}=K$$

$$=\frac{\frac{8+9}{12}}{\frac{6}{12}}=\frac{17}{6}$$

Sol 3

$x^2-y^2=(\cot\theta+\cos\theta)^2\cdot(\cot\theta-\cos\theta)^2$

$=4\cot\theta\cos\theta$

$=\frac{4\cos^2\theta}{\sin\theta}$

$xy=(\cot\theta+\cos\theta)(\cot\theta-\cos\theta)$

$=\cos^2\theta\cdot\left[\frac{1}{\sin^2\theta}-1\right]$

$=\cos^2\theta\cdot\frac{(1-\sin^2\theta)}{\sin^2\theta}$

$=\frac{\cos^4\theta}{\sin\theta}$

$\therefore\ \frac{\cos^2\theta}{\sin\theta}=\sqrt{xy}$

$(x^2-y^2)=4\sqrt{xy}$

Sol 4

$$m^2 - n^2 = (\tan x + \sin x)^2 - (\tan x - \sin x)^2$$

$$= 4 \tan x \sin x = \frac{4 \sin^2 x}{\cos x}$$

$$mn = (\tan x + \sin x)(\tan - \sin x) = (\tan^2 x - \sin^2 x)$$

$$= \sin^2 x \cdot \left[\frac{1}{\sin^2 x} - 1\right]$$

$$= \frac{\sin^2 x \cdot (1 - \cos^2 x)}{\cos^2 x} = \frac{\sin^4 x}{\cos^2 x}$$

$$\therefore \frac{\sin^2 x}{\cos x} = \sqrt{mn}$$

$$(m^2 - n^2) = 4\sqrt{mn}$$

Sol 5

$$3 \sin x + 5 \cos x = 5 \quad \text{---(i)}$$

$$3 \cos x - 5 \sin x = a \quad \text{---(ii)}$$

Squaring (i) and (ii) and adding

$$9(\sin^2 x + \cos^2 x) + 25(\cos^2 x + \sin x) = 25 + a^2$$

$$\therefore a^2 = 9$$

Alternative Method

$$a \sin x + b \cos x = m$$

$$a \cos x - b \sin x = n$$

$$a^2 + b^2 = m^2 + n^2$$

$\therefore \quad (3)^2 + (5)^2 = (5)^2 + a^2$

$a^2 = 9$

Sol 6 Squaring and subtracting the given equation-

$(x^4 - y^4) = a^2(\cot^2\theta - \operatorname{cosec}^2\theta) + b^2(\operatorname{cosec}^2\theta + \cot^2\theta)$

$= a^2[\operatorname{cosec}^2\theta - 1 - \operatorname{cosec}^2\theta] + b^2[1 + \cot^2\theta - \cot^2\theta]$

$\therefore \quad x^4 - y^4 = b^2 - a^2$

Sol 7 According to Property -

$a^2 + b^2 = 3^2 + 4^2$

$a^2 + b^2 = 25$

Sol 8

$$\frac{\sin\theta + \cos\theta - 1}{\sin\theta - \cos\theta + 1} = \frac{\cos\theta}{1 + \sin\theta}$$

eq^n में मान रखने पर

$$\frac{\cos\theta}{1+\sin\theta} \times \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \times \frac{1+\sin\theta}{1+\sin\theta}$$

$$\frac{\cos\theta}{1+\sin\theta} \times \frac{1+\sin\theta}{\cos\theta} = 1$$

Sol 9

$$\frac{\sin\theta + \cos\theta - 1}{\sin\theta - \cos\theta + 1} = \frac{\cos\theta}{1+\sin\theta}$$

equation में मान रखने पर –

$$= \frac{\cos\theta}{1+\sin\theta} \times \frac{\tan^2\theta(\operatorname{cosec}^2\theta - 1)}{\sec\theta - \tan\theta}$$

$$= \frac{\cos\theta}{1+\sin\theta} \times \frac{\tan^2\theta \times \cot^2\theta}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec\theta + \tan\theta} \times \frac{1}{\sec\theta - \tan\theta}$$

$$= \frac{1}{\sec^2\theta - \tan^2\theta} = 1$$

Sol 10

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1+\sin\theta}{\cos\theta}$$

equation में मान रखने पर

$$= \frac{1+\sin\theta}{\cos\theta} - \frac{\sin\theta + 1}{\cos\theta}$$

$$= \sec\theta + \tan\theta - (\tan\theta + \sec\theta)$$

$$= \sec\theta + \tan\theta - \sec\theta - \tan\theta)$$

$$= 0$$