# TRIGONOMETRY

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

* ⊕ $\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$

* ⊖ $\sin(A+B) - \sin(A-B) = 2 \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

* ⊕ $\cos(A+B) + \cos(A-B) = 2 \cos A \cos B$

* ⊖ $\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$

* $\pm[\cos(A-B) - \cos(A+B)] = \pm 2 \sin A \sin B$

* $\sin C + \sin D = 2 \sin\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

* $\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

* $\cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$

* $\cos C - \cos D = 2 \sin\left(\frac{C+D}{2}\right) \sin\left(\frac{D-C}{2}\right)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

(A=B)

* $\sin 2A = \sin A \cdot \cos A + \cos A \cdot \sin A$

↳ $2 \sin A \cdot \cos A$

↳ $= \frac{2 \tan A}{1 + \tan^2 A}$

* $\cos 2A = 1 - 2 \sin^2 A$

↳ $= 2 \cos^2 A - 1$

↳ $\cos^2 A - \sin^2 A$

↳ $= \frac{1 - \tan^2 A}{1 + \tan^2 A}$

* $\tan 2A = \left(\frac{2\tan A}{1-\tan^2 A}\right)$

* $\cot 2A = \left(\frac{\cot^2 A - 1}{2\cot A}\right)$

Q) Find $(\alpha+\beta)$, if $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$?

यदि $\tan\alpha = \frac{m}{m+1}$ और $\tan\beta = \frac{1}{2m+1}$ है, तो $(\alpha+\beta)$ ज्ञात कीजिये।

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\cdot\tan\beta}$$

$$\frac{\frac{m}{m+1}+\frac{1}{(2m+1)}}{1-\frac{m}{m+1}\times\frac{1}{2m+1}} = \frac{\frac{2m^2+m+m+1}{(m+1)(2m+1)}}{\frac{(m+1)(2m+1)-m}{(m+1)(2m+1)}}$$

$$\frac{2m^2+2m+1}{2m^2+m+2m+1-m}$$

$$\frac{2m^2+2m+1}{2m^2+2m+1} = 1$$

$$\tan(\alpha+\beta) = 1$$

$$\alpha+\beta = 45^\circ$$

$$= \frac{\pi}{4}$$

II nd Method

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}$$

Put = 1

$\tan\alpha = \frac{1}{2}$

$\tan\beta = \frac{1}{3}$

$$\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\tan(\alpha+\beta) = 1$$

$$\alpha+\beta = 45^\circ$$

$$= \frac{\pi}{4}$$

Q) $\cos 18° - \sin 18°$

$\cos 18° - \sin(90-72)$

$\cos 18 - \cos 72°$

$2 \sin \frac{90°}{2} \sin \frac{54}{2}$ (45°)

$2 \sin 45° \sin 27°$

$2 \times \frac{1}{\sqrt{2}} \times \sin 27$

$= \sqrt{2} \sin 27°$

$\left\{ \cos C - \cos D = 2 \sin \frac{C+D}{2} \cdot \sin \frac{D-C}{2} \right\}$

Q) $\sin 15° + \sin 75° = ?$

$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

$= 2 \sin 45° \times \cos 30°$

$2 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{\sqrt{2}}$

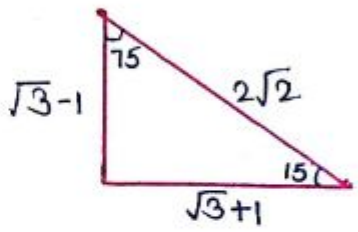
Q) $\sin 15° + \sin 165° = ?$

$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

$2 \times \sin 90° \cdot \cos 75°$

$2 \cos 75°$

$2 \times \frac{(\sqrt{3}-1)}{2\sqrt{2}}$




$\frac{\sqrt{3}-1}{\sqrt{2}} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{3-1}{\sqrt{2}(\sqrt{3}+1)}$

$\frac{2}{\sqrt{2}(\sqrt{3}+1)} = \frac{\sqrt{2}}{\sqrt{3}+1}$

Q) $\cot x (\sin 5x - \sin 3x) = ?$

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cot x (2\cos 4x \cdot \sin x)$

$\frac{\cos x}{\sin x} \times 2\cos 4x \cdot \sin x$

$= 2\cos x \cdot \cos 4x$

Q) $\frac{\sin 4\theta + \sin 2\theta}{\cos 4\theta + \cos 2\theta} = ?$

$\frac{2\sin 3\theta \cos\theta}{2\cos 3\theta \cos\theta}$

$= \tan 3\theta$

Q) $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ = ?$

$\frac{\sin 10 + \sin 50}{} + \frac{\sin 20 + \sin 40^\circ}{}$

$2\sin 30^\circ \times \cos 20^\circ + 2\sin 30 \cos 10^\circ$

$2\sin 30^\circ [\cos 20 + \cos 10]$

$2 \times \frac{1}{2}$

$= 2\cos 15^\circ \cdot \cos 5^\circ$

Q) $\frac{\cos 7A + \cos 11A}{\cos 7A - \cos 11A} = ?$

$\frac{2\cos 9A \cos 2A}{2\sin 9A \times \sin 2A}$

$\frac{\cot 9A}{\left(\frac{\sin 2A}{\cos 2A}\right)} = \frac{\cot 9A}{\tan 2A}$

Q) What is the value of $[(\sin 7x - \sin 5x) \div (\cos 7x + \cos 5x)] - [(\cos 6x - \cos 4x) \div (\sin 6x + \sin 4x)] = ?$

$[(\sin 7x - \sin 5x) \div (\cos 7x + \cos 5x)] - [(\cos 6x - \cos 4x) \div (\sin 6x + \sin 4x)$ का मान क्या है?

$$[(2\cos 6x \cdot \sin x) \div (2\cos 6x \cos x)] - [(2\sin 5x \cdot \sin(-x)) \div (2\sin 5x \cdot \cos x)]$$

$$\left[\frac{\cancel{2\cos 6x}\cdot \sin x}{\cancel{2\cos 6x}\cos x}\right] - \left[\frac{\cancel{2\sin 5x}(-\sin x)}{\cancel{2\sin 5x}\cos x}\right]$$

$$\tan - (-\tan x)$$

$$\tan x + \tan x$$

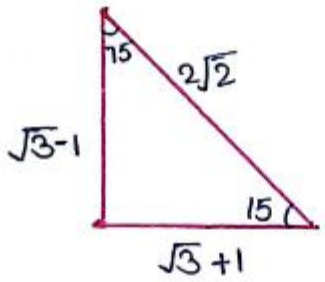
$$2\tan x$$

Q) $\tan 15^\circ - \cot 165^\circ = ?$

$$\tan 15^\circ - \cot(180 - 15)$$

$$\tan 15^\circ - (-\cot 15^\circ)$$

$$\boxed{\tan 15^\circ + \cot 15^\circ}$$




$$\frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{3-1}$$

$$\frac{4 - \cancel{2\sqrt{3}} + 4 + \cancel{2\sqrt{3}}}{2} = \frac{\cancel{8}}{\cancel{2}} 4$$

**1.** $\sin 0^\circ \sin 30^\circ \cdot \sin 45^\circ \sin 60^\circ \sin 90^\circ = ?$

(a) 1

(b) 0

(c) $\frac{\sqrt{6}}{8}$

(d) 4

**2. If** $\cos 2A = \sin 75^\circ$**, then the smallest positive value of** $A$ **is :**

**यदि** $\cos 2A = \sin 75^\circ$ **है, तो** $A$ **का सबसे छोटा धनात्मक मान क्या होगा?**

(a) $15^\circ$

(b) $7.5^\circ$

(c) $30^\circ$

(d) $37.5^\circ$

**3. If** $\theta$ **lies in the first quadrant and** $\cos^2\theta - \sin^2\theta = \frac{1}{2}$**, then find the value of** $\tan^2 2\theta + \sin^2 3\theta$

**यदि** $\theta$ **पहले चतुर्थांश में स्थित है और** $\cos^2\theta - \sin^2\theta = \frac{1}{2}$ **है, तो** $\tan^2 2\theta + \sin^2 3\theta$ **का मान ज्ञात करें।**

(a) $\frac{7}{2}$

(b) 3

(C) 4

(d) $\frac{4}{3}$

**4. The value of** $\sin 2\theta(\tan\theta + \cot\theta)$ **is :**

$\sin 2\theta(\tan\theta + \cot\theta)$ **का मान ज्ञात कीजिए।**

(a) $\frac{2}{3}$

(b) 1

(c) $\frac{1}{2}$

(d) 2

**5. What is the value of** $\text{cosec}\, 15^\circ \sec 15^\circ$ **?**

$\text{cosec}\, 15^\circ \sec 15^\circ$ **का मान क्या है?**

(a) 0.5

(b) 4

(c) 2

(d) 1

**6. The value of** $\frac{\sin 4\theta}{(1-\cos 4\theta)}$ **is :**

$\frac{\sin 4\theta}{(1-\cos 4\theta)}$ **का मान क्या होगा?**

(a) $\cot\theta$

(b) $\cot 2\theta$

(c) $\tan\theta$
(d) $\tan 2\theta$

7. $\frac{\sin 4\theta+\sin 2\theta}{\cos 4\theta+\cos 2\theta}=?$
(a) $\tan 3\theta$
(b) $\cot 3\theta$
(c) $\tan\theta$
(d) $\cot\theta$

8. What is the value of $(2\cos^2\theta-1)\left[\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}\right]$?
$(2\cos^2\theta-1)\left[\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}\right]$ का मान क्या है?
(a) 2
(b) 0
(c) $\frac{\sqrt{3}}{2}$
(d) 1

9. What is the value of $\sin 73^\circ+\cos 137^\circ$?
$\sin 73^\circ+\cos 137^\circ$ का मान क्या होगा?
(a) $\sin 13^\circ$
(b) $\cos 13^\circ$
(c) $\cos 18^\circ$
(d) $\sin 18^\circ$

10. Find the value of the following.
निम्न का मान ज्ञात कीजिए।
$\frac{\sin 8\theta\cos\theta-\sin 6\theta\cos 3\theta}{\cos 2\theta\cos-\sin 3\theta\sin 4\theta}$
(a) $\cot\theta$
(b) $\cot 2\theta$
(c) $\tan\theta$
(d) $\tan 2\theta$

## ANSWER SHEET

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| B | B | C | D | B | B | A | A | A | D |

# Worksheet solution

**sol 1**

$\sin 0° \cdot \sin 30° \sin 45° \sin 60° \cdot \sin 90°$

$\sin 0° = 0$

$\therefore$ All expression will become zero.

**sol 2**

$\cos 2A = \sin 75$

$\cos 2A = \cos 15$

$2A = 15$

$A = 7.5$

**sol 3**

$\cos^2\theta - \sin^2\theta = \frac{1}{2}$

$\cos 2\theta = \cos 60°$

$2\theta = 60°$

$\theta = 30°$

$\therefore \tan^2 2\theta + \sin^2 3\theta$

$\tan^2 60° + \sin^2 90°$

$(\sqrt{3})^2 + 1 = 4$

**sol 4**

$\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

$\therefore \sin 2\theta (\tan\theta + \cot\theta)$

$= 2\sin\theta\cos\theta (\sec\theta \operatorname{cosec}\theta)$

$= 2$

**sol 5**

$= \operatorname{cosec} 15° \cdot \sec 15°$

$= \frac{1}{\sin 15°} \cdot \frac{1}{\cos 15°}$

हर अंश और हर में गुणा करने पर,

$$= \frac{2}{2\sin 15^\circ \cos 15^\circ}$$

$$= \frac{2}{\sin 2\times 15^\circ} = \frac{2}{\sin 30^\circ} = 4$$

**sol 6**

$$= \frac{2\sin 2\theta \cdot \cos 2\theta}{1-(1-2\sin^2\theta)}$$

$$= \frac{2\sin 2\theta \cdot \cos 2\theta}{1-1+2\sin^2 2\theta}$$

$$= \frac{2\sin 2\theta \cdot \cos 2\theta}{2\sin^2 2\theta} = \cot 2\theta$$

**sol 7**

$$\frac{\sin 4\theta + \sin 2\theta}{\cos 4\theta + \cos 2\theta} = \frac{2\sin\frac{(4\theta+2\theta)}{2}\cdot\cos\frac{2\theta}{2}}{2\cos 3\theta \cdot \cos\theta}$$

$$= \frac{2\sin 3\theta \cdot \cos\theta}{2\cos 3\theta \cdot \cos\theta}$$

$$= \tan 3\theta$$

**sol 8**

$$= (2\cos^2\theta - 1)\left[\frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}\right]$$

$$= (2\cos^2\theta - 1)\left[\frac{(1+\tan\theta)^2 + (1-\tan\theta)^2}{(1-\tan\theta)\cdot(1+\tan\theta)}\right]$$

$$= (2\cos^2\theta - 1)\left[\frac{1+\tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta}{1-\tan^2\theta}\right]$$

$$= (2\cos^2\theta - 1) \times \frac{2(1+\tan^2\theta)}{1-\tan^2\theta}$$

$$= \cos 2\theta \times \frac{2}{\cos 2\theta} = 2$$

**Sol 9** $\sin 73^\circ + \cos 137^\circ$

$$= \sin 73^\circ + \cos(90+47^\circ)$$

$$= 2\cos\frac{73+47^\circ}{2}\sin\frac{73^\circ-47^\circ}{2}$$

$$= 2\cos 60^\circ \cdot \sin\frac{26^\circ}{2}$$

$$= 2\times\frac{1}{2}\cdot\sin 13^\circ = \sin 13^\circ$$

**Sol 10** $$\frac{\sin 8\theta\cos\theta - \sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta - \sin 3\theta\sin 4\theta}$$

$$= \frac{2\sin 8\theta\cos\theta - 2\sin 6\theta\cos 3\theta}{2\cos 2\theta\cdot\cos\theta - 2\sin 3\theta\cdot\sin 4\theta}$$

$$= \frac{\sin 9\theta + \sin 7\theta - \sin 9\theta - \sin 3\theta}{\cos 3\theta + \cos\theta - \cos\theta + \cos 7\theta}$$

$$= \frac{\sin 7\theta\cdot - \sin 3\theta}{\cos 3\theta + \cos 7\theta}$$

$$= \frac{2\cos\frac{7\theta+3\theta}{2}\sin\frac{7\theta-3\theta}{2}}{2\cos\frac{3\theta+7\theta}{2}\cos\frac{3\theta-7\theta}{2}} = \frac{\cos 5\theta\cdot\sin 2\theta}{\cos 5\theta\cdot\cos 2\theta}$$

$$= \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$$