

TRIGONOMETRY

$$\sec^2 \theta - \tan^2 \theta = 1$$

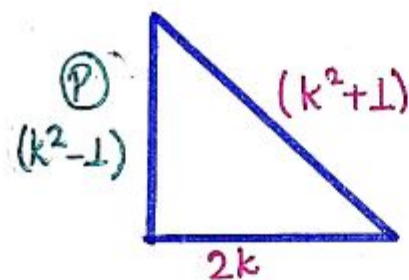
$$\sec \theta + \tan \theta = \frac{1}{\sec \theta - \tan \theta}$$

$\sec \theta + \tan \theta = k$

$$\sec \theta - \tan \theta = \frac{1}{k}$$

$$+ \quad \frac{2 \sec \theta = k + \frac{1}{k} = \frac{k^2 + 1}{k}}$$

$$\boxed{\sec \theta = \frac{k^2 + 1}{2k}}$$



$$P^2 = (k^2 + 1)^2 - (2k)^2$$

$$P^2 = k^4 + 1 + 2k^2 - 4k^2$$

$$k^4 + 1 - 2k^2$$

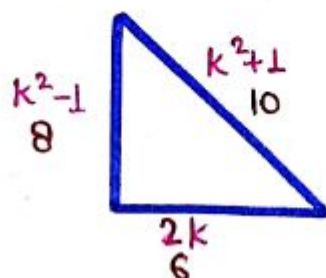
$$P^2 = (k^2 - 1)^2$$

$$P = \boxed{k^2 - 1}$$

Ex:-

$$\sec \theta + \tan \theta = 3 \rightarrow (k)$$

$$\sin \theta = ?$$

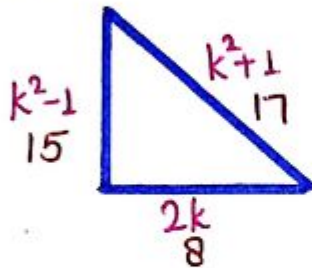


$$\sin \theta = \frac{8}{10} = \frac{4}{5}$$

Ex:-

$$\operatorname{Cosec} \theta + \cot \theta = 4 \rightarrow (k)$$

$$\cos \theta = ?$$



$$\cos \theta = \frac{8}{17}$$

SPECIAL CASE

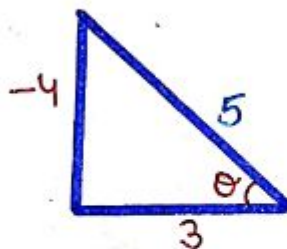
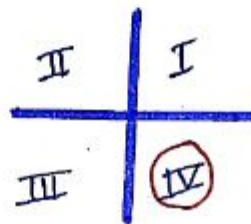
$$\overset{+}{\sec} \theta - \overset{-}{\tan} \theta = 3 \rightarrow (k)$$

$$\overset{+}{\sec} \theta + \overset{-}{\tan} \theta = \frac{1}{3}$$

$$2 \tan \theta = \frac{1}{3} - 3$$

$$2 \tan \theta = -\frac{8}{3}$$

$\tan \theta = -\frac{4}{3}$



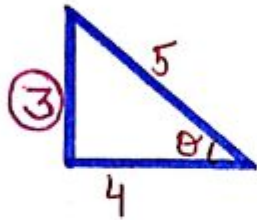
$$\sin \theta = -\frac{4}{5}$$

Q) If $\sec \theta = x + \frac{1}{4x}$ ($0^\circ < \theta < 90^\circ$), then what will be $\sec \theta + \tan \theta$ equal to?

यदि $\sec \theta = x + \frac{1}{4x}$ ($0^\circ < \theta < 90^\circ$) हों, तो $\sec \theta + \tan \theta$ किसके बराबर होगा?

Put, $x=1$

$$\sec \theta = 1 + \frac{1}{4} = \frac{5}{4}$$



$$\sec \theta + \tan \theta$$

$$\frac{5}{4} + \frac{3}{4} = \frac{8}{4} \quad (2)$$

$$2x = 2$$

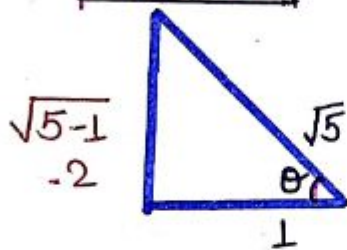
Q) If $\sec \theta + \tan \theta = 2 + \sqrt{5}$, then what will be the value of $\sin \theta + \cos \theta$?

यदि $\sec \theta + \tan \theta = 2 + \sqrt{5}$ हो, तो $\sin \theta + \cos \theta$ का मान क्या होगा?

$$\sec \theta + \tan \theta = \sqrt{5} + 2$$

$$\sec \theta - \tan \theta = \frac{1}{\sqrt{5} + 2} \times \frac{\sqrt{5} - 2}{\sqrt{5} - 2}$$

$$\frac{2\sec \theta = 2\sqrt{5}}{\boxed{\sec \theta = \sqrt{5}}} \quad \frac{\sqrt{5} - 2}{5 - 4} = \sqrt{5} - 2$$



$$\frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}}$$

$$\frac{3}{\sqrt{5}}$$

Q) The numerical value of $\frac{1}{1 + \cot^2 \theta} + \frac{3}{1 + \tan^2 \theta} + 2 \sin^2 \theta$ is :

$\frac{1}{1 + \cot^2 \theta} + \frac{3}{1 + \tan^2 \theta} + 2 \sin^2 \theta$ का संख्यात्मक मान होगा।

$$\frac{1}{\operatorname{cosec}^2 \theta} + \frac{3}{\sec^2 \theta} + 2 \sin^2 \theta$$

$$\sin^2 \theta + 3 \cos^2 \theta + 2 \sin^2 \theta$$

$$3 \sin^2 \theta + 3 \cos^2 \theta$$

$$3 (\sin^2 \theta + \cos^2 \theta)$$

$$3 \times 1 = 3$$

IInd Method

Value Putting

$$\theta = 45^\circ$$

$$\frac{1}{1+1} + \frac{3}{1+1} + 2 \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\frac{1}{2} + \frac{3}{2} + 2 \times \frac{1}{2}$$

$$\frac{2+4}{2} + 1 = 3$$

{ θ की कोई भी Value रख सकते हैं। }

{ $\left[\frac{1}{0}, \frac{0}{1} \right]$ की form नहीं बननी चाहिए }

Q) The value of $\frac{4}{1+\tan^2 \alpha} + \frac{1}{1+\cot^2 \alpha} + 3 \sin^2 \alpha$ is

$\frac{4}{1+\tan^2 \alpha} + \frac{1}{1+\cot^2 \alpha} + 3 \sin^2 \alpha$ का मान है।

$$\theta = 45^\circ$$

$$\frac{4}{1+1} + \frac{1}{1+1} + 3 \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\frac{4}{2} + \frac{1}{2} + 3 \times \frac{1}{2}$$

$$\frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$$

IInd Method

$$\frac{4}{\sec^2 \theta} + \frac{1}{\operatorname{cosec}^2 \theta} + 3 \sin^2 \theta$$

$$4 \cos^2 \theta + \sin^2 \theta + 3 \sin^2 \theta$$

$$4 (\cos^2 \theta + \sin^2 \theta)$$

$$4 \times 1 = 4$$

Q) If $4x = \sec \theta$ and If $\frac{4}{x} = \tan \theta$, then what will be $8(x^2 - \frac{1}{x^2})$?

यदि $4x = \sec \theta$ तथा $\frac{4}{x} = \tan \theta$ हो, तो $8(x^2 - \frac{1}{x^2})$ क्या होगा?

$$x = \frac{\sec \theta}{4}, \quad \frac{1}{x} = \frac{\tan \theta}{4}$$

$$x^2 - \frac{1}{x^2} = \left(\frac{\sec^2 \theta}{16} - \frac{\tan^2 \theta}{16} \right)$$

$$\frac{1}{16} (\sec^2 \theta - \tan^2 \theta)$$

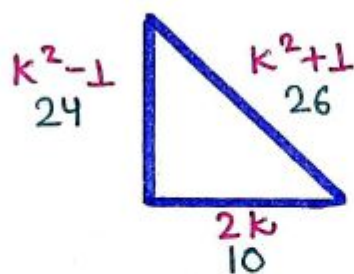
$$\frac{1}{16} \times 1 = \frac{1}{16}$$

$$8(x^2 - \frac{1}{x^2})$$

$$8 \times \frac{1}{16} = \frac{1}{2}$$

Q) If $\sec \theta + \tan \theta = 5$, then what will be the value of $\frac{\tan \theta + 1}{\tan \theta - 1}$?

यदि $\sec \theta + \tan \theta = 5$, तो $\frac{\tan \theta + 1}{\tan \theta - 1}$ का मान क्या होगा?



$$\frac{\frac{24}{10} + 1}{\frac{24}{10} - 1} \Rightarrow \frac{\frac{34}{10}}{\frac{14}{10}}$$

$$\frac{34}{14} = \frac{17}{7}$$

Q) What is the numerical value of $1 + \frac{1}{\cot^2 63^\circ} - \sec^2 27^\circ + \frac{1}{\sin^2 63^\circ} - \operatorname{cosec}^2 27^\circ$

$1 + \frac{1}{\cot^2 63^\circ} - \sec^2 27^\circ + \frac{1}{\sin^2 63^\circ} - \operatorname{cosec}^2 27^\circ$ का संख्यात्मक मान बताइए :

$$1 + \tan^2 63^\circ - \sec^2 27^\circ + \operatorname{cosec}^2 63^\circ - \operatorname{cosec}^2 27^\circ$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

$$\sec^2 (90 - 63) \qquad \qquad \operatorname{cosec}^2 (90 - 63)$$

$$\cancel{\sec^2 63^\circ} - \cancel{\operatorname{cosec}^2 63^\circ} + \cancel{\operatorname{cosec}^2 63^\circ} - \cancel{\sec^2 63^\circ}$$

$$= 0$$

Q) If θ be an acute angle and $7 \sin^2 \theta + 3 \cos^2 \theta = 4$, then the value of $\tan \theta$ is

यदि θ एक न्यूनकोण हो और $7 \sin^2 \theta + 3 \cos^2 \theta = 4$ हो, तो $\tan \theta$ का मान है ?

$$7 \sin^2 \theta + 3(1 - \sin^2 \theta) = 4$$

$$7 \sin^2 \theta + 3 - 3 \sin^2 \theta = 4$$

$$4 \sin^2 \theta = 1$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \sqrt{\frac{1}{4}} = +\frac{1}{2}$$

$$\boxed{\theta = 30^\circ}$$

$$\tan \theta$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

Q) If $\sec^2 \theta + \tan^2 \theta = \frac{7}{12}$, then $\sec^4 \theta - \tan^4 \theta = ?$

यदि $\sec^2 \theta + \tan^2 \theta = \frac{7}{12}$, तो $\sec^4 \theta - \tan^4 \theta = ?$

$$\sec^4 \theta - \tan^4 \theta$$

$$(\sec^2 \theta + \tan^2 \theta) (\sec^2 \theta - \tan^2 \theta)$$

$$\frac{7}{12} \times 1$$

$$\frac{7}{12}$$

Q) If $\sin \theta + \cos \theta = \sqrt{2} \cos \theta$, then $\sin \theta - \cos \theta = ?$

यदि $\sin \theta + \cos \theta = \sqrt{2} \cos \theta$, तो $(\sin \theta - \cos \theta) = ?$

$$(\sin \theta + \cos \theta = \sqrt{2} \cos \theta)^2$$

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 2 \cos^2 \theta$$

$$1 + 2 \sin \theta \cos \theta = 2 \cos^2 \theta$$

$$\boxed{2 \sin \theta \cos \theta = 2 \cos^2 \theta - 1}$$

$$(\sin \theta - \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta$$

$$1 - (2 \cos^2 \theta - 1)$$

$$1 - 2 \cos^2 \theta + 1 = 2 - 2 \cos^2 \theta$$

$$2(1 - \cos^2 \theta)$$

$$2 \sin^2 \theta$$

$$\sin \theta - \cos \theta = \sqrt{2 \sin^2 \theta}$$

$$\pm 2 \sin \theta$$

Q) $\sqrt{\frac{1}{\sin^4(90+A)} + \frac{1}{\cos^2(90+A)-1}} = ?$

$$= \sqrt{\frac{1}{\cos^4 A} + \frac{1}{\sin^2 A - 1}}$$

$$= \sqrt{\frac{1}{\cos^4 A} + \frac{1}{-\cos^2 A}}$$

$$= \sqrt{\frac{1}{\cos^2 A} \left[\frac{1}{\cos^2 A} - 1 \right]}$$

$$= \frac{1}{\cos A} \sqrt{\frac{\sin^2 A}{\cos^2 A}}$$

$$= \frac{1}{\cos A} \sqrt{\tan^2 A}$$

$$\sec A \times \tan A$$

$$\begin{cases} 1 - \sin^2 \theta = \cos^2 \theta \\ \sin^2 \theta - 1 = -\cos^2 \theta \end{cases}$$

Q) $a^2 \sin^2 x + b^2 \cos^2 x = c^2$

$$2 \sin^2 x - \cos^2 x = ?$$

$$a^2 \sin^2 x + b^2 (1 - \sin^2 x) = c^2$$

$$a^2 \sin^2 x + b^2 - b^2 \sin^2 x = c^2$$

$$\sin^2 x (a^2 - b^2) = c^2 - b^2$$

$$\boxed{\sin^2 x = \frac{c^2 - b^2}{a^2 - b^2}}$$

$$\cos^2 x = 1 - \sin^2 x \Rightarrow 1 - \frac{c^2 - b^2}{a^2 - b^2}$$

$$\frac{a^2 - \cancel{b^2} - c^2 + \cancel{b^2}}{a^2 - b^2} = \frac{a^2 - c^2}{a^2 - b^2}$$

$$2 \sin^2 x - \cos^2 x$$

$$2 \left(\frac{c^2 - b^2}{a^2 - b^2} \right) - \frac{a^2 - c^2}{a^2 - b^2}$$

$$\frac{2c^2 - 2b^2 - a^2 + c^2}{a^2 - b^2}$$

$$\frac{3c^2 - 2b^2 - a^2}{a^2 - b^2}$$

1. What is the least value of $9\sin^2\theta + 16\cos^2\theta$?

$9\sin^2\theta + 16\cos^2\theta$ का न्यूनतम मान क्या है?

- (a) 0
- (b) 9
- (c) 16
- (d) 25

2. If $\operatorname{cosec}\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$, then what is

$m^{\frac{4}{3}}n^{\frac{2}{3}} + m^{\frac{2}{3}}n^{\frac{4}{3}}$ equal to?

यदि $\operatorname{cosec}\theta - \sin\theta = m$ तथा $\sec\theta - \cos\theta = n$, तो $m^{\frac{4}{3}}n^{\frac{2}{3}} + m^{\frac{2}{3}}n^{\frac{4}{3}}$ किसके बराबर है?

- (a) 0
- (b) 1
- (c) mn
- (d) m^2n^2

3. $\frac{\cos^2\theta}{1-\tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta} = ?$

- (a) $1 - \sin\theta\cos\theta$
- (b) $1 + \sin\theta\cos\theta$
- (c) $1 + \sin\theta$
- (d) None of these

4. What will be the value of $(1 + \sin^4 A - \cos^4 A)\operatorname{cosec}^2 A$?

$(1 + \sin^4 A - \cos^4 A)\operatorname{cosec}^2 A$ का मान क्या होगा?

- (a) -2
- (o) 2
- (c) -1
- (d) 1

5. Find the value of $\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} - \tan^2\theta, 0^\circ < \theta < 90^\circ$.

$\frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} - \tan^2\theta, 0^\circ < \theta < 90^\circ$, का मान ज्ञात करें।

- (a) $2\sec^2\theta$
- (b) $\sec^2\theta + 1$
- (c) $\sec^2\theta$
- (d) $1 - \tan^2\theta$

6. If $x + y = 90^\circ$, then what is $\sqrt{\cos x \operatorname{cosec} y - \cos x \sin y}$ equal to?

यदि $x + y = 90^\circ$, तो $\sqrt{\cos x \operatorname{cosec} y - \cos x \sin y}$ किसके बराबर है?

- (a) $\cos x$ (b) $\sin x$
(c) $\sqrt{\cos x}$ (d) $\sqrt{\sin x}$

7. $\frac{[1-\tan(90-\theta)+\sec(90-\theta)]}{[\tan(90-\theta)+\sec(90-\theta)+1]}$ What is the value of "of"?/का मान क्या है ?

- (a) $\cot \frac{\theta}{2}$ (b) $\tan \frac{\theta}{2}$
(c) $\sin \theta$ (d) $\cos \theta$

8. If $p = \cot \theta + \tan \theta$ and $q = \sec \theta - \cos \theta$, then $(p^2 p)^{\frac{2}{3}}(q^2 p)^{\frac{2}{3}}$ is equal to

यदि $p = \cot \theta + \tan \theta$ तथा $q = \sec \theta - \cos \theta$, तो $((p^2 p)^{\frac{2}{3}}(q^2 p)^{\frac{2}{3}})$ बराबर है

- (a) 0 (b) 1
(c) 2 (d) 3

9. Consider the following :

निम्न पर विचार करें :

$$\text{I. } \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} = \cos^2 \theta (1 + \tan \theta)(1 - \tan \theta)$$

$$\text{II. } \frac{1+\sin \theta}{1-\sin \theta} = (\tan \theta + \sec \theta)^2$$

Which of the statements given above is/are correct?

उपर्युक्त में से कौन सा/से कथन सही है/हैं?

- (a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

10. If $\tan^2 \theta - 3\sec \theta + 3 = 0, 0^\circ < \theta < 90^\circ$, then the value of $(\sin \theta + \cot \theta)$ is:

यदि $\tan^2 \theta - 3\sec \theta + 3 = 0, 0^\circ < \theta < 90^\circ$, है, तो $(\sin \theta + \cot \theta)$ का मान है :

- (a)** $\frac{5\sqrt{3}}{6}$ **(b)** $2\sqrt{3}$
(c) $\frac{5\sqrt{3}}{3}$ **(d)** $3\sqrt{3}$

ANSWER SHEET

[illegible]

Worksheet solution

sol 1

$$\begin{aligned}y &= 9\sin^2\theta + 16\cos^2\theta \\&= 9(\sin^2\theta + \cos^2\theta) + 7\cos^2\theta \\&= 9 + 7(0) = 9\end{aligned}$$

sol 2

$$m = \operatorname{cosec}\theta - \sin\theta = \frac{1 - \sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$$

$$n = \sec\theta - \cos\theta = \frac{1 - \cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$$

$$m^{\frac{4}{3}} \cdot n^{\frac{2}{3}} + m^{\frac{2}{3}} \cdot n^{\frac{4}{3}} = (m \cdot n)^{\frac{2}{3}} \cdot \{m^{\frac{2}{3}} + n^{\frac{2}{3}}\}$$

$$= \left(\frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta} \right)^{\frac{2}{3}} \cdot \left\{ \left(\frac{\cos^2\theta}{\sin\theta} \right)^{\frac{2}{3}} + \left(\frac{\sin^2\theta}{\cos\theta} \right)^{\frac{2}{3}} \right\}$$

$$= (\sin\theta \cdot \cos\theta)^{\frac{2}{3}} \cdot \left\{ \frac{(\cos^3\theta)^{\frac{2}{3}} + (\sin^3\theta)^{\frac{2}{3}}}{(\sin\theta \cdot \cos\theta)^{\frac{2}{3}}} \right\}$$

$$= (\sin\theta \cdot \cos\theta)^{\frac{2}{3}} \cdot \left\{ \frac{\cos^2\theta + \sin^2\theta}{(\sin\theta \cdot \cos\theta)^{\frac{2}{3}}} \right\}$$

$$= 1$$

sol 3

$$= \frac{\cos^2\theta}{1 - \tan\theta} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^2\theta}{1 - \frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^3\theta}{\cos\theta - \sin\theta} + \frac{\sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{[\cos \theta - \sin \theta] [\cos^2 \theta + \sin^2 \theta + \cos \theta \sin \theta]}{(\cos \theta - \sin \theta)}$$

$$= 1 + \cos \theta \sin \theta$$

sol 4

$$= (1 + \sin^4 A - \cos^4 A) \operatorname{cosec}^2 A$$

$$= [1 + (\sin^2 A)^2 - (\cos^2 A)^2] \operatorname{cosec}^2 A$$

$$= [1 + (\sin^2 A - \cos^2 A)(\sin^2 A + \cos^2 A)] \operatorname{cosec}^2 A$$

$$= [1 + \sin^2 A - \cos^2 A] \operatorname{cosec}^2 A$$

$$= 2 \sin^2 A \times \operatorname{cosec}^2 A = 2$$

sol 5

$$= \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - 1} + \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta + 1} - \tan^2 \theta$$

$$= \frac{\operatorname{cosec} \theta \theta (\operatorname{cosec} \theta + 1 + \operatorname{cosec} \theta - 1)}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)} - \tan^2 \theta$$

$$= \frac{\operatorname{cosec} \theta : 2 \operatorname{cosec} \theta}{\operatorname{cosec}^2 \theta - 1} - \tan^2 \theta$$

$$= \frac{2 \operatorname{cosec}^2 \theta}{\cot^2 \theta} - \tan^2 \theta$$

$$= 2 \times \frac{1}{\sin^2 \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta} - \tan^2 \theta$$

$$= 2 \sec^2 \theta - \tan^2 \theta$$

$$= \sec^2 \theta + (\sec^2 \theta - \tan^2 \theta)$$

$$= \sec^2 \theta + 1$$

sol 6

$$\begin{aligned} &= \sqrt{\cos x \operatorname{cosec} y - \cos x \sin y} \quad (\because x+y=90^\circ) \\ &= \sqrt{\cos x \cdot \operatorname{cosec}(90^\circ-x) - \cos x \cdot \sin(90^\circ-x)} \\ &= \sqrt{\cos x \cdot \sec x - \cos^2 x} \\ &= \sqrt{1 - \cos^2 x} = \sqrt{\sin^2 x} = \sin x \end{aligned}$$

sol 7

$$\begin{aligned} &= \frac{[1 - \tan(90^\circ - \theta) + \sec(90^\circ - \theta)]}{[\tan(90^\circ - \theta) + \sec(90^\circ - \theta) + 1]} \\ &= \frac{1 - \cot \theta + \operatorname{cosec} \theta}{\cot \theta + \operatorname{cosec} \theta + 1} \\ &= \frac{(\operatorname{cosec}^2 \theta - \cot^2 \theta) + (\operatorname{cosec} \theta - \cot \theta)}{\cot \theta + \operatorname{cosec} \theta + 1} \\ &= \frac{(\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta) + (\operatorname{cosec} \theta - \cot \theta)}{\cot \theta + \operatorname{cosec} \theta + 1} \\ &= \frac{(\operatorname{cosec} \theta - \cot \theta)(\operatorname{cosec} \theta + \cot \theta + 1)}{\cot \theta + \operatorname{cosec} \theta + 1} \\ &= \operatorname{cosec} \theta - \cot \theta \\ &= \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} = \frac{1 - \cos \theta}{\sin \theta} = \frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2}} \\ &= \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \tan \frac{\theta}{2} \end{aligned}$$

Sol 8

$$p = \cot \theta + \tan \theta = \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = \frac{1}{\sin \theta \cos \theta}$$

$$q = \sec \theta - \cos \theta = \frac{1}{\cos \theta} - \cos \theta = \frac{\sin^2 \theta}{\cos \theta}$$

$$= (p^2 q)^{\frac{2}{3}} - (q^2 p)^{\frac{2}{3}}$$

$$= \left(\frac{1}{\sin^2 \theta \cdot \cos^2 \theta} \times \frac{\sin^2 \theta}{\cos \theta} \right)^{\frac{2}{3}} - \left(\frac{\sin^4 \theta}{\cos^2 \theta} \times \frac{1}{\sin \theta \cos \theta} \right)^{\frac{2}{3}}$$

$$= \left(\frac{1}{\cos^3 \theta} \right)^{\frac{2}{3}} - \left(\frac{\sin^3 \theta}{\cos^3 \theta} \right)^{\frac{2}{3}}$$

$$= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1 - \sin^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta}{\cos^2 \theta} = 1$$

Sol 9

$$(I) \text{ R.H.S } \cos^2 \theta (1 + \tan \theta)(1 - \tan \theta)$$

$$= \cos^2 \theta (1 - \tan^2 \theta)$$

$$= \cos^2 \theta \left(\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta} \right)$$

$$= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} = \text{L.H.S}$$

(II)

$$\text{L.H.S } \frac{1 + \sin \theta}{1 - \sin \theta} \times \frac{(1 + \sin \theta)}{(1 - \sin \theta)}$$

$$= \frac{(1 + \sin \theta)^2}{1 - \sin^2 \theta} = \left(\frac{1 + \sin \theta}{\cos \theta} \right)^2$$

$$= (\sec \theta + \tan \theta)^2$$

option (C) Both (i) and (ii)

Sol 10

$$\Rightarrow \tan^2 \theta - 3 \sec \theta + 3 = 0$$

$$\Rightarrow \sec^2 \theta - 1 - 3 \sec \theta + 3 = 0$$

$$\Rightarrow \sec^2 \theta - 3 \sec \theta + 2 = 0$$

$$\Rightarrow \sec^2 \theta - 2 \sec \theta - \sec \theta + 2 = 0$$

$$\Rightarrow \sec \theta (\sec \theta - 2) - 1 (\sec \theta - 2) = 0$$

$$\Rightarrow (\sec \theta - 1) (\sec \theta - 2) = 0$$

$$\sec \theta = 2 \quad \sec \theta \neq 1 \quad (0^\circ < \theta < 90^\circ)$$

$$\sec \theta = 2 \Rightarrow \sec \theta = \sec 60^\circ$$

$$\theta = 60^\circ$$

$$= (\sin \theta + \cot \theta)$$

$$= \sin 60^\circ + \cot 60^\circ$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} = \frac{3+2}{2\sqrt{3}} = \frac{5}{2\sqrt{3}}$$

$$= \frac{5\sqrt{3}}{2\sqrt{3} \times \sqrt{3}} = \frac{5\sqrt{3}}{6}$$