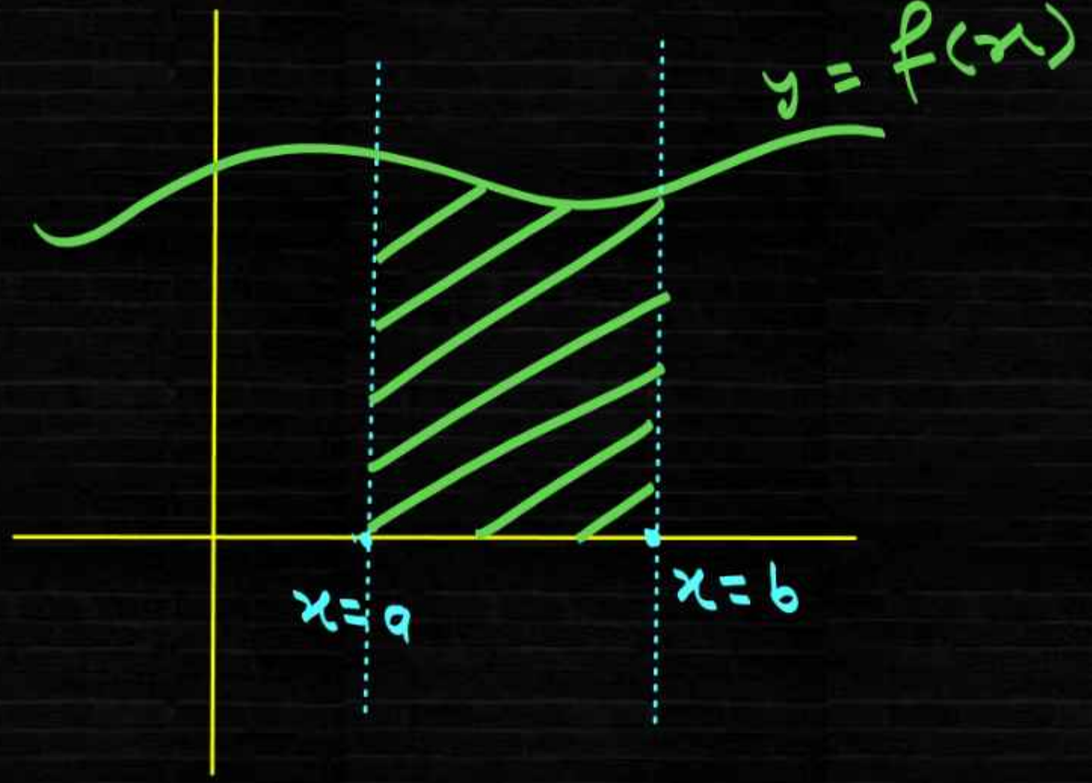
Application of integration.

(AOI).

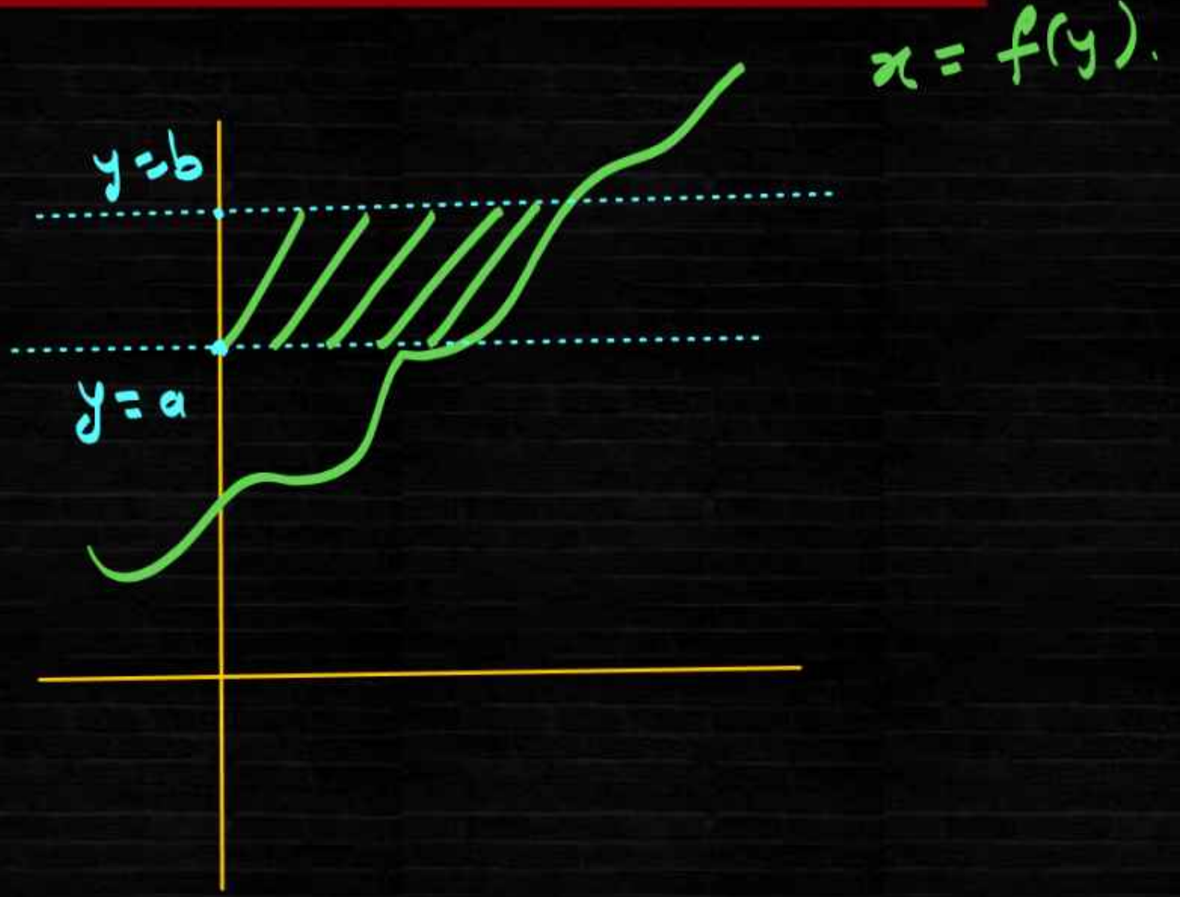
I. One function and two abscissa

$$\int_a^b f(x)\,dx.$$



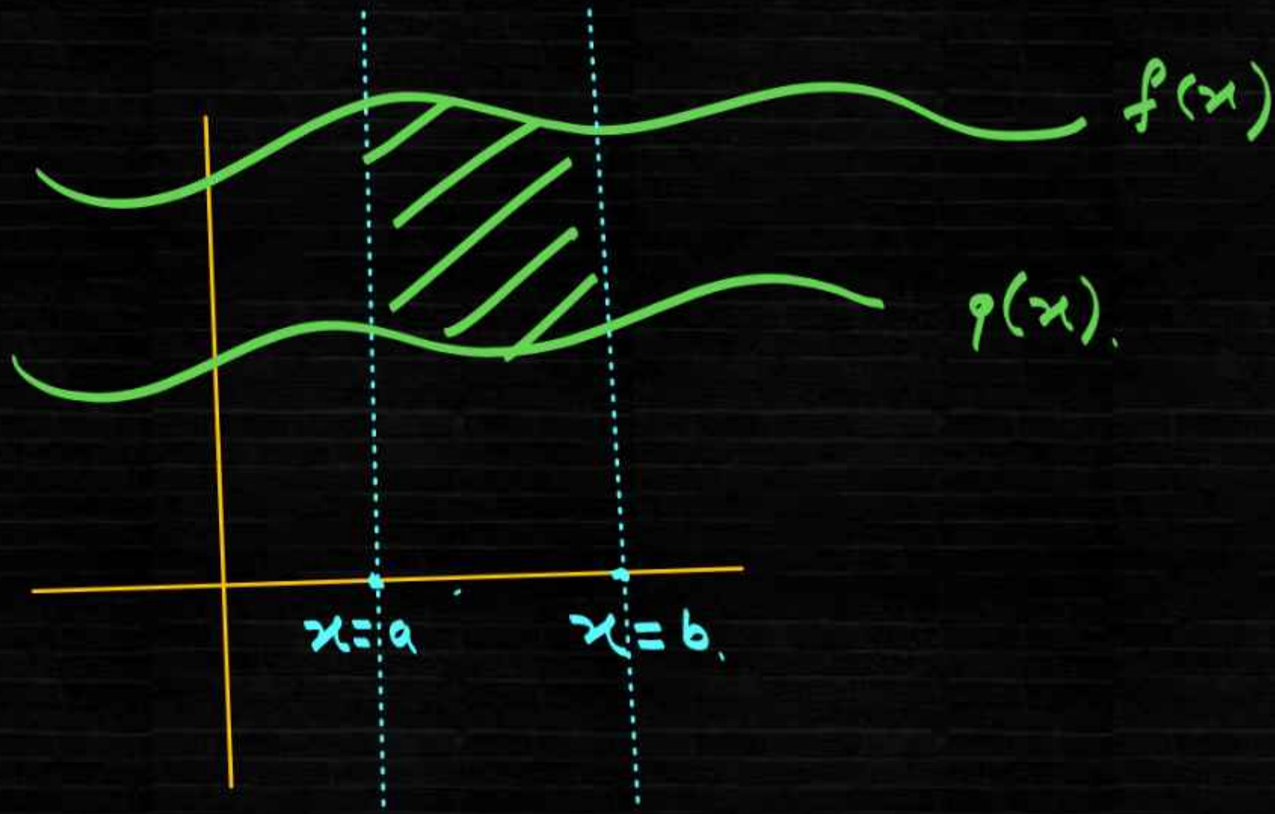
II. One function and two ordinates

$$\int_a^b f(y)\cdot dy.$$



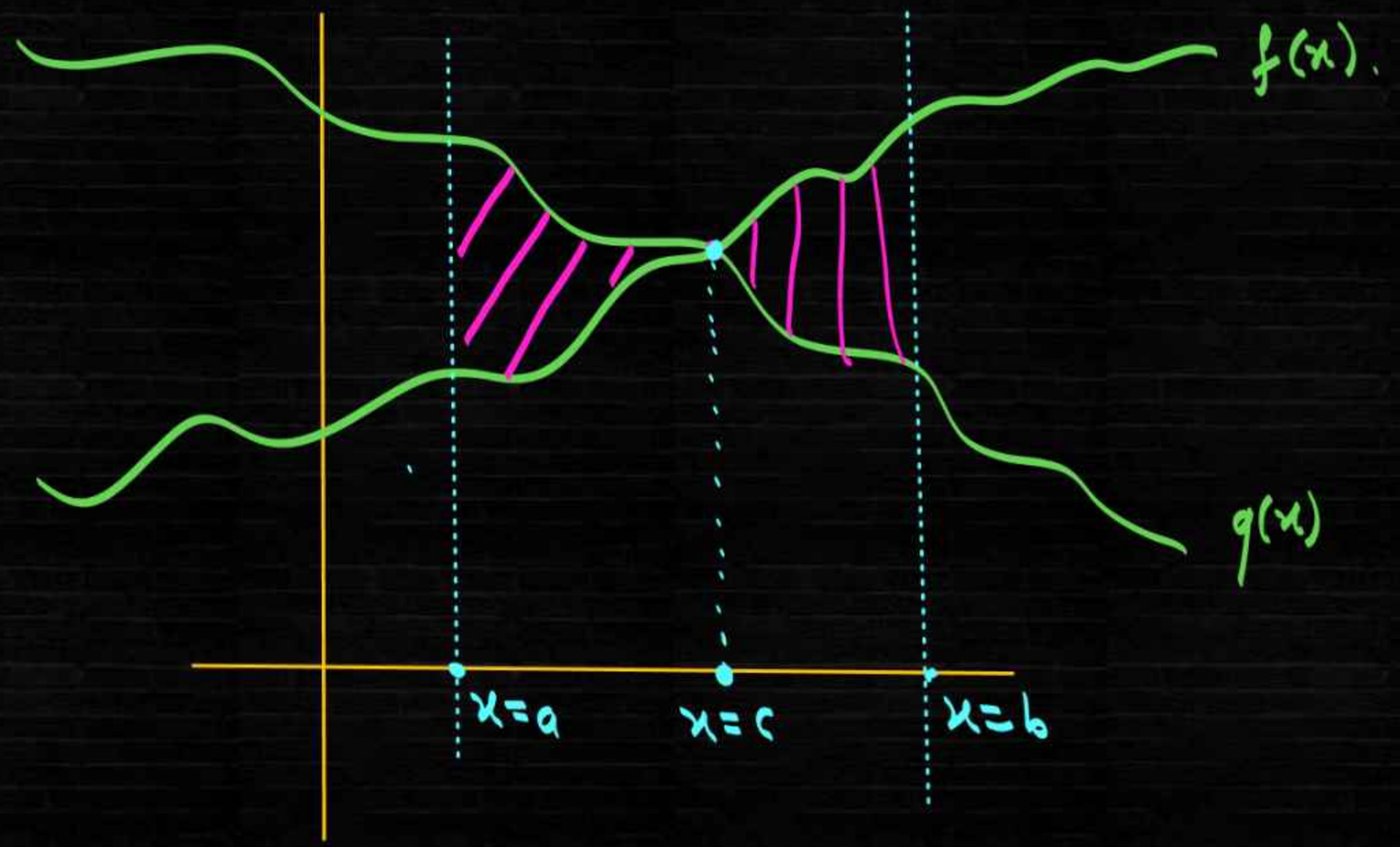
III. Two non-intersecting functions and two abscissa

$$\int_a^b [f(x) - g(x)]\,dx.$$



IV. Two intersecting functions and two abscissa

$$\int_a^c [g(x) - f(x)]\,dx + \int_c^b [f(x) - g(x)]\,dx.$$



Graph

- Line
- Circle
- Parabola
- Ellipse
- Hyperbola
- $\therefore$ $\sin x$ | $\cos x$.

Consider the curves $y = \sin x$ and $y = \cos x$.

Q. What is the area of the region bounded by the above two curves and the lines $x = 0$ and $x = \frac{\pi}{4}$?

उपरोक्त दो वक्रों और रेखाओं $x=0$ तथा $x = \frac{\pi}{4}$ से घिरे क्षेत्र का क्षेत्रफल क्या है?

(a) $\sqrt{2} - 1$

(b) $\sqrt{2} + 1$

(c) $\sqrt{2}$

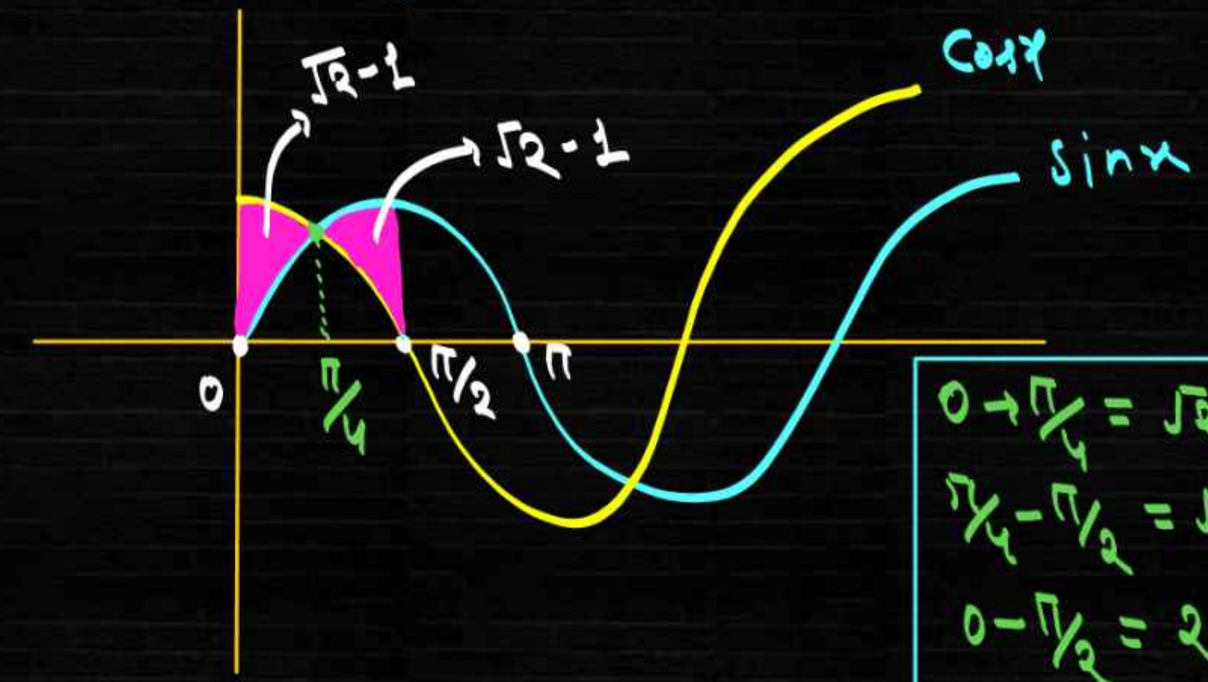
(d) 2

$$\int_0^{\pi/4} (\cos x - \sin x)\, dx$$

$$(\sin x + \cos x)_0^{\pi/4}$$

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (0+1)$$

$$\sqrt{2} - 1$$



$0 \to \pi/4 = \sqrt{2}-1$

$\pi/4 - \pi/2 = \sqrt{2}-1$

$0 - \pi/2 = 2\sqrt{2}-2$

Q. What is the area of the region bounded by the above two curves and the lines $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$?

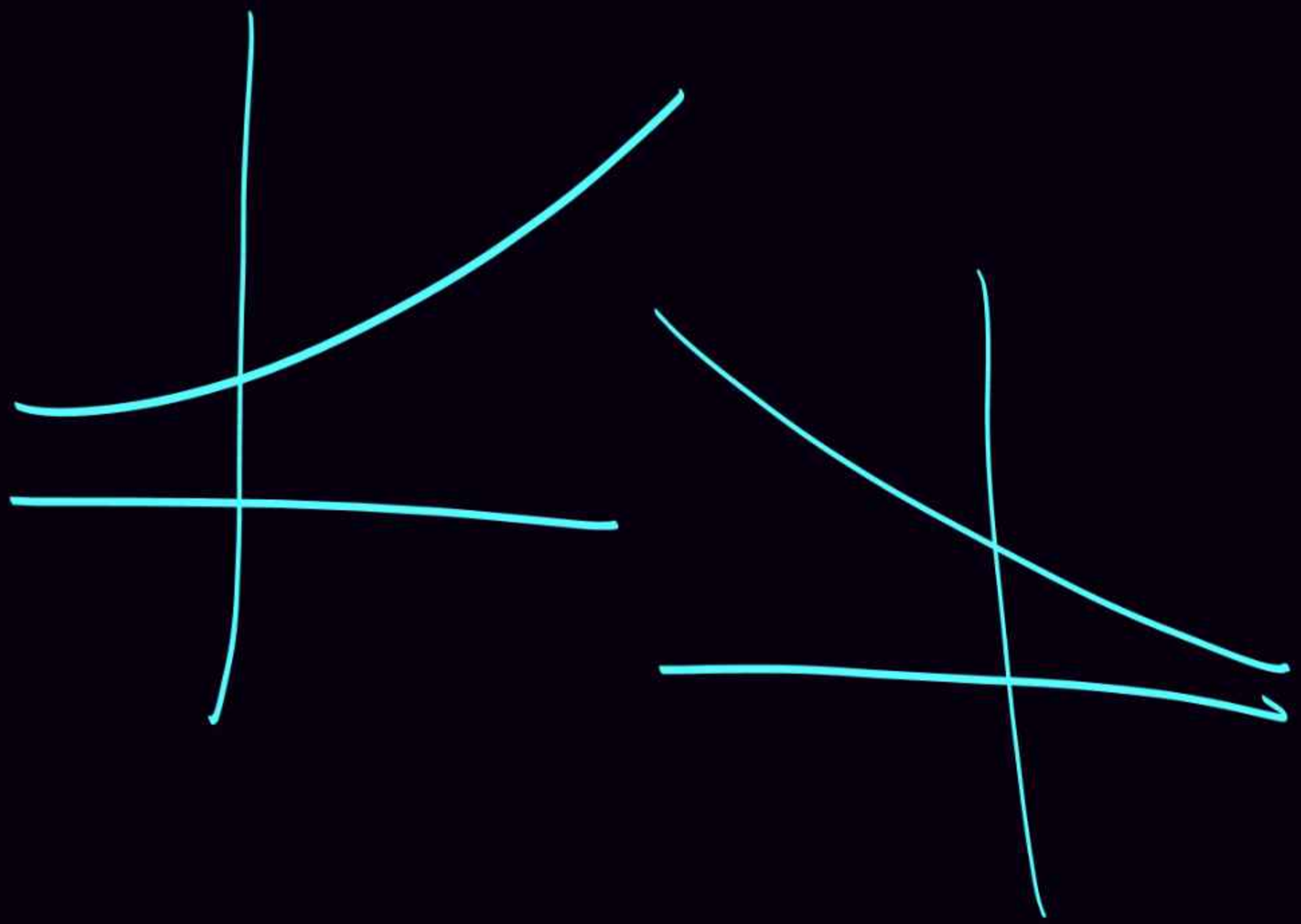
उपरोक्त दो वक्रों और रेखाओं $x = \frac{\pi}{4}$ तथा $x = \frac{\pi}{2}$ से घिरे क्षेत्र का क्षेत्रफल क्या है?

(a) $\sqrt{2} - 1$

(b) $\sqrt{2} + 1$

(c) $2\sqrt{2}$

(d) 2



Q. What is the area bounded by the curves $y = e^x$, $y = e^{-x}$ and the straight line $x = 1$?

वक्र $y = e^x, y = e^{-x}$ और सीधी रेखा $x = 1$ से घिरा क्षेत्र क्या है?

(a) $\left(e + \frac{1}{e}\right)$ sq unit

(b) $\left(e - \frac{1}{e}\right)$ sq unit

(c) $\left(e + \frac{1}{e} - 2\right)$ sq unit

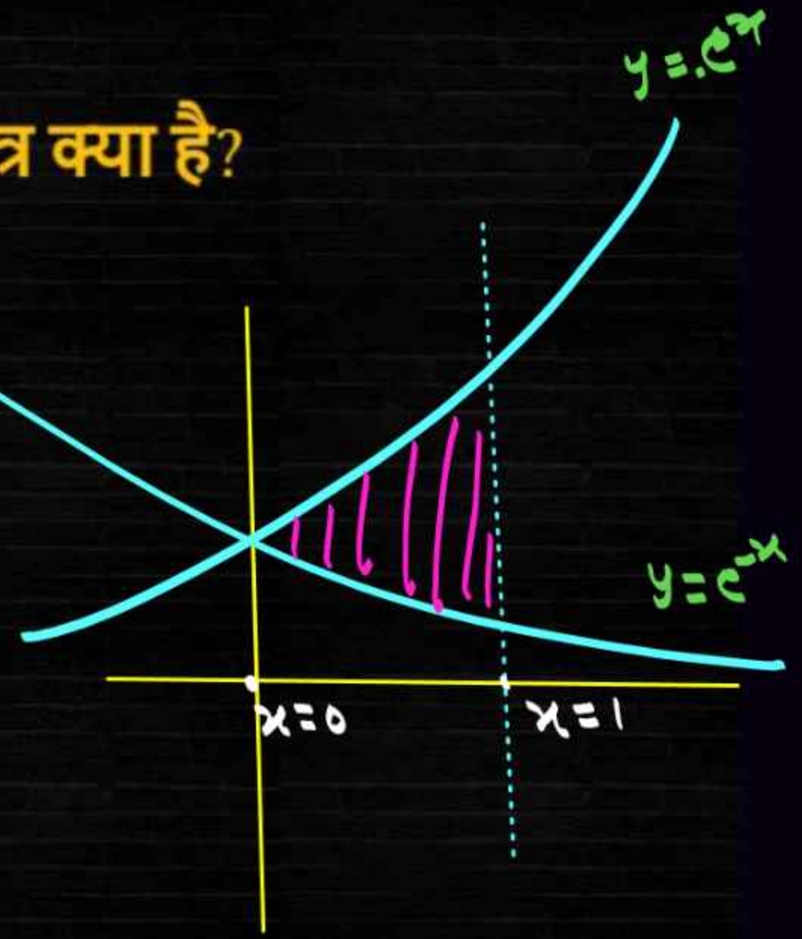
(d) $\left(e - \frac{1}{e} - 2\right)$ sq unit

$$\int_0^1 (e^x - e^{-x})\, dx$$

$$(e^x + e^{-x})_0^1$$

$$e + e^{-1} - (1 + 1)$$

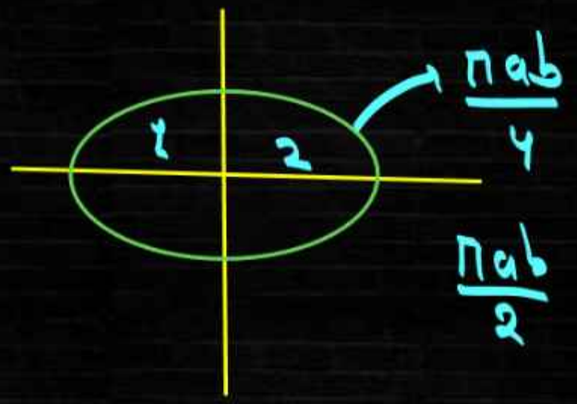
$$e + \frac{1}{e} - 2$$



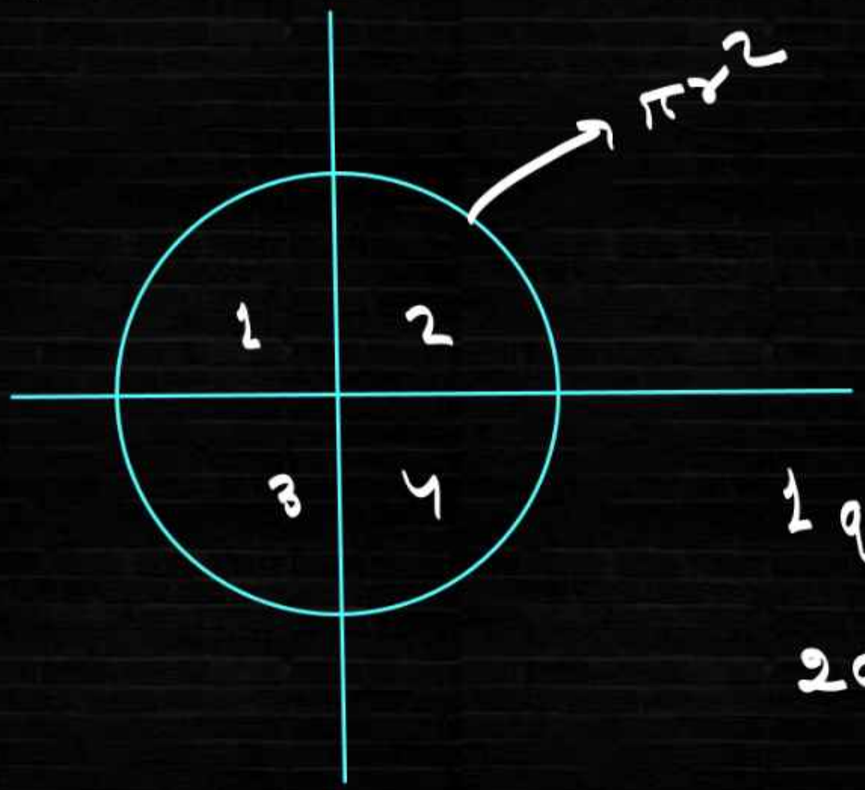
AREAS BOUNDED BY COMMON CURVES:-

(i) The entire area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab sq. units. As it is symmetric about both axes, clearly area in any one quadrant is $\frac{\pi ab}{4}$ sq. units.

दीर्घवृत्त $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ का संपूर्ण क्षेत्रफल πab वर्ग इकाई है। चूंकि यह दोनों अक्षों के प्रति सममित है, इसलिए स्पष्ट रूप से किसी एक चतुर्थांश में क्षेत्रफल $\frac{\pi ab}{4}$ वर्ग इकाई है।



$x^2 + y^2 = r^2$



πr^2

1 quad $\rightarrow \frac{\pi r^2}{4}$

2 quads $\rightarrow \frac{\pi r^2}{2}$

Q. What is the area enclosed by the equation $x^2 + y^2 = 2$?

समीकरण $x^2 + y^2 = 2$ द्वारा परिबद्ध क्षेत्र क्या है?

r^2

(a) 4π square units

(b) 2π square units

πr^2

$\pi 2$

(c) $4\pi^2$ square units

(d) 4 square units

2π

Q. What is the area bounded by $y = \sqrt{16 - x^2}, y \geq 0$ and the x-axis?

$y = \sqrt{16 - x^2}, y \geq 0$ और $x -$ अक्ष से घिरा क्षेत्र क्या है?

(a) 16π square units

(b) 8π square units

(c) 4π square units

(d) 2π square units

Half area

S.B.S

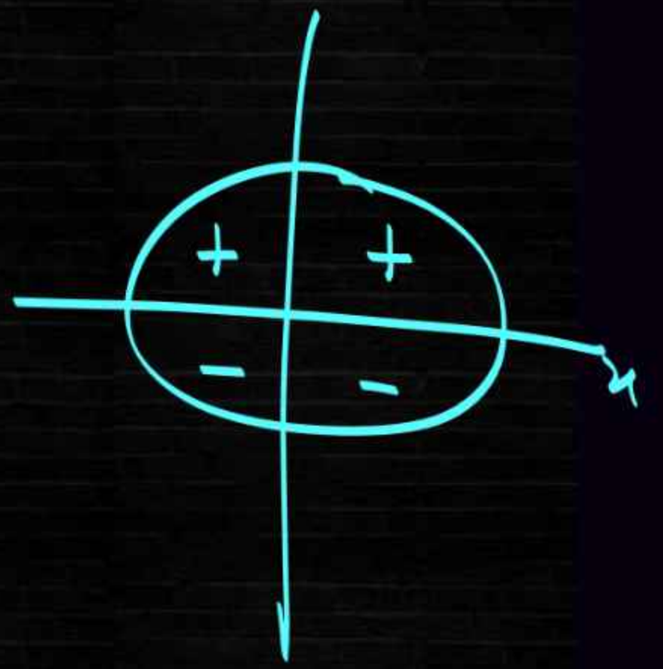
$y^2 = 16 - x^2$

$x^2 + y^2 = 16 \rightarrow$ circle

$\downarrow$

r^2

$\Rightarrow \frac{\pi \times 16}{2} = 8\pi$



(ii) Areas bounded by straight lines

- $\frac{x}{a}+\frac{y}{b}=1$ encloses an area $\frac{1}{2}|ab|$ sq. units with the co-ordinate axes.

Q. What is the area bounded by the lines $x = 0, y = 0$ and $x + y + 2 = 0$?

रेखाओं $x = 0, y = 0$ और $x + y + 2 = 0$ से घिरा क्षेत्र क्या है?

(a) $\frac{1}{2}$ square unit

(b) 1 square unit

(c) 2 square units

(d) 4 square units

$x + y = -2$

$\frac{x}{-2} + \frac{y}{-2} = 1$

a b

$\frac{1}{2} \times -2 \times -2 = 2$ sq. units

(iii) Area included betweent he parabola $y^2 = 4ax$ and the line y = mx is $\frac{8a^2}{3m^3}$ sq. units

$y^2 = 4ax$ and line $y = mx$.

Area $= \frac{8a^2}{3m^3}$

Q. What is the area of the region enclosed between the curve $y^2 = 2x$ and the straight line $y = x$?

वक्र $y^2 = 2x$ और सरल रेखा $y = x$ के बीच परिबद्ध क्षेत्र का क्षेत्रफल क्या है?

$4a = 2 \Rightarrow a = \frac{1}{2}$

(a) $\frac{2}{3}$ square units

(b) $\frac{4}{3}$ square units

(c) $\frac{1}{3}$ square units

(d) 1 square units

$$\Rightarrow \frac{8 \times \left(\frac{1}{2}\right)^2}{3 \times 1^2}$$

$$\Rightarrow \frac{2 \times \frac{1}{4}}{3} = \frac{1}{3}$$

(iv) Area included between the parabola $y^2 = 4ax$ and its latus rectum $x = a$ is

नाभिलम्ब

$\frac{8}{3}a^2$ **sq. units**

Q. What is the area of the parabola $y^2 = 4bx$ bounded by its latus rectum?

परवलय $y^2 = 4bx$ का उसके नाभि – रेक्टम से घिरा क्षेत्र क्या है?

↳ $x_b = x_a$

$b = 4$

(a) $2b^2/3$ **square unit**

(b) $4b^2/3$ **square unit**

(c) b^2 **square unit**

(d) $8b^2/3$ **square unit**

Q. What is the area of the parabola $y^2 = x$ bounded by its latus rectum?

परवलय $y^2 = x$ का उसके नाभि – रेक्टम से घिरा क्षेत्र क्या है?

$4a = 1$

$a = \frac{1}{4}$

(a) $\frac{1}{12}$ square unit

(b) $\frac{1}{6}$ square unit

(c) $\frac{1}{3}$ square unit

(d) None of the above

$\text{Area} \Rightarrow \frac{8}{3} \times a^2$

$= \frac{8}{3} \times \frac{1}{16}$

(v) Area enclosed between the parabolas $y^2 = 4ax$ and $x^2 = 4$ by is $\frac{16ab}{3}$ sq. units.

$y^2 = 4ax$, $x^2 = 4by$.

$$\text{Area} = \frac{16ab}{3}$$

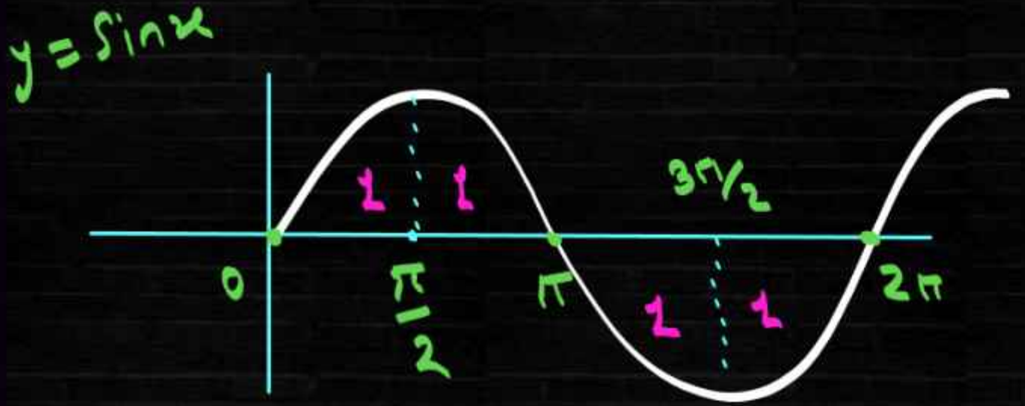
$y^2 = 2x$, $x^2 = 6y$

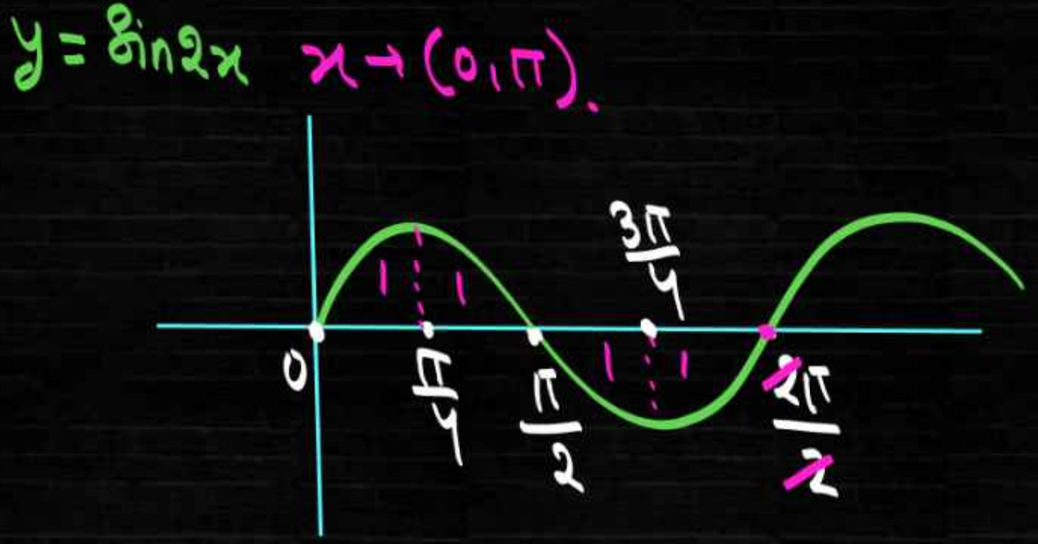
$4a = 2$ $\quad$ $4b = 6$

$a = 1/2$ $\quad$ $b = 3/2$

$$\text{Area} = \frac{4}{3} \times \frac{1}{2} \times \frac{3}{2} = 4 \text{ sq. units}$$

(vi) Area bounded by any one arc of $y = \sin ax$ or $y = \cos ax$ where its ordinate is zero, with the x-axis is $\frac{2}{a}$ sq. units.





Q. What is the area of one of the loops between the curve $y =$ csin x and x -axis?

वक्र y= sinx और x-अक्ष के बीच एक लूप का क्षेत्रफल क्या है?

(a) c

(b) 2 c

(c) 3 c

(d) 4 c

Q. What is the area of the portion of the curve $y = \sin x$, lying between $x = 0, y = 0$ and $x = 2\pi$?

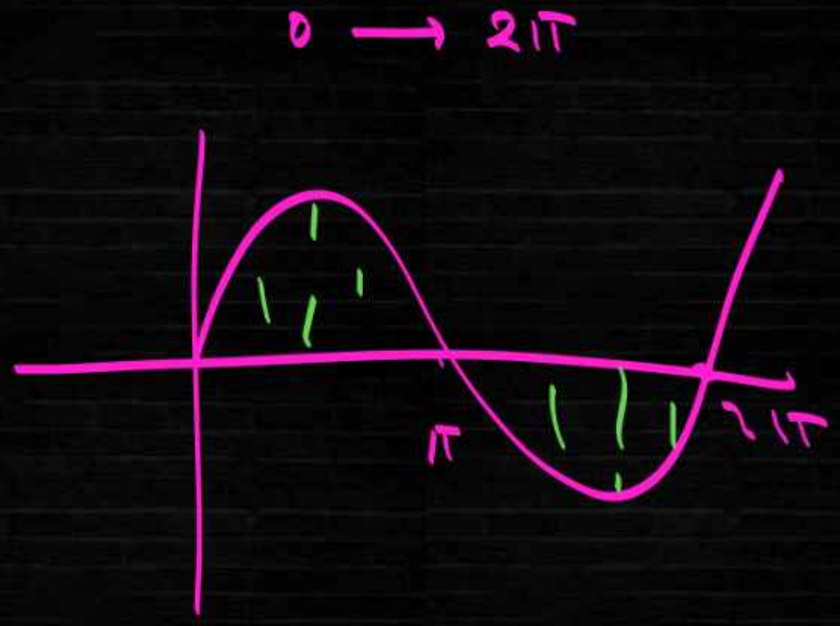
वक्र $y = sin\ x$ के उस भाग का क्षेत्रफल क्या है जो $x=0, y=0$ और $x=2\pi$ के बीच स्थित है?

(a) 1 square unit

(b) 2 square units

(c) 4 square units

(d) 8 square units



Q. The area bounded by the curve $|x| + |y| = 1$ is.

वक्र $|x|+|y|=1$ से घिरा क्षेत्र है।

(a) 1 square unit

(b) $2\sqrt{2}$ square units

(c) 2 square units

(d) $2\sqrt{3}$ square units

$|x| + |y| = a$

$Area = 2a^2$

2×1^2

$= 2$

Q. What is the area bounded by the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}\ (x,y\geq 0)$ and the coordinate axes?

वक्र $\sqrt{x}+\sqrt{y}=\sqrt{a}(x,y\geq 0)$ और निर्देशांक अक्षों से घिरा क्षेत्र क्या है?

(a) $\frac{5a^2}{6}$

(b) $\frac{a^2}{3}$

Area $= \frac{a^2}{6}$

(c) $\frac{a^2}{2}$

(d) $\frac{a^2}{6}$

Q. The area bounded by the coordinate axes and the curve $\sqrt{x} + \sqrt{y} = 1$, is

निर्देशांक अक्षों और वक्र $\sqrt{x} + \sqrt{y} = 1$ से घिरा क्षेत्र क्या है?

(a) 1 square unit

(b) $\frac{1}{2}$ square unik

(c) $\frac{1}{3}$ square unit

(d) $\frac{1}{6}$ square unit