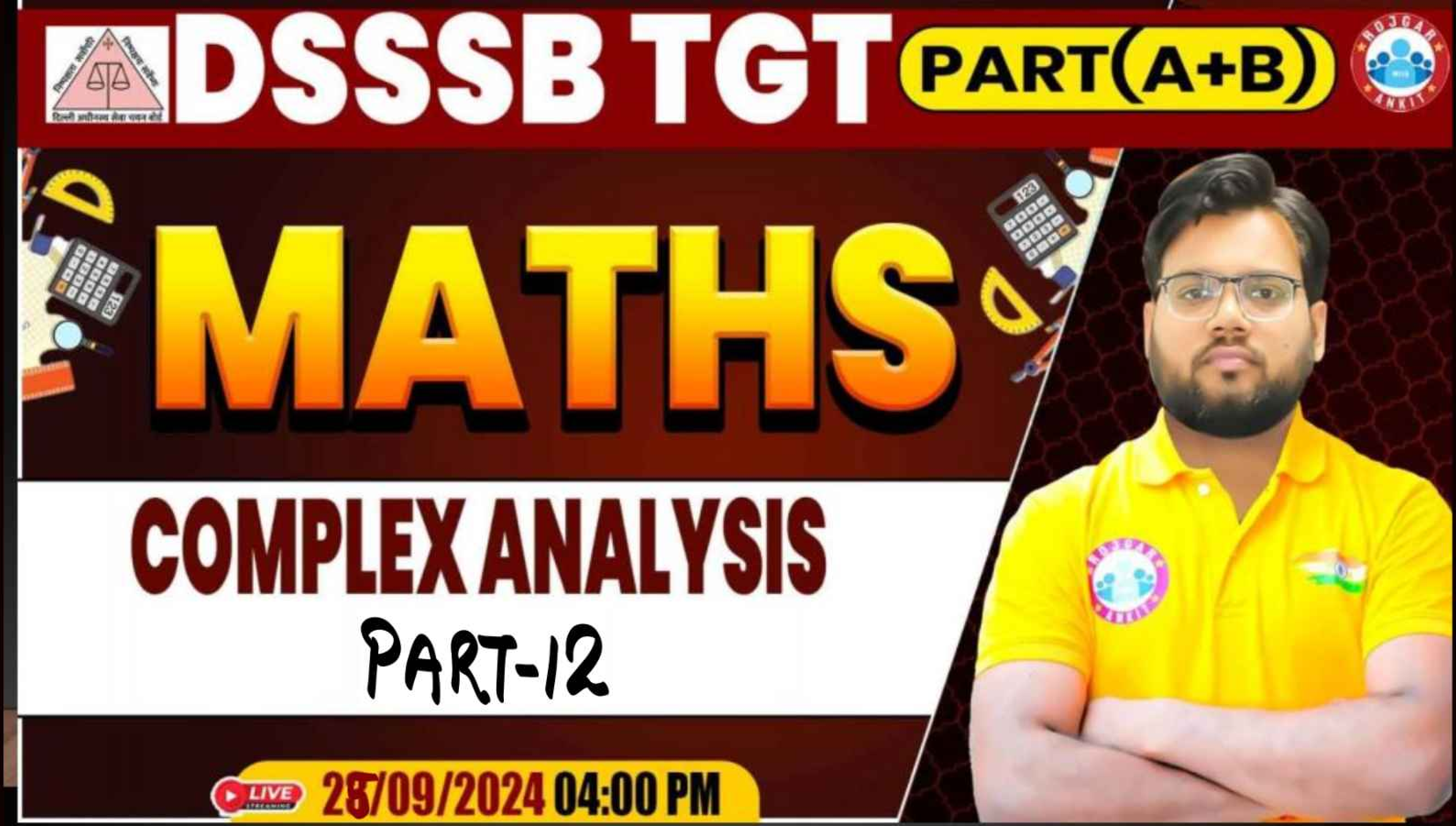
DSSSB TGT
PART(A+B)
MATHS
COMPLEX ANALYSIS
PART-12
LIVE
28/09/2024 04:00 PM

Entire Function: A function $f(z)$ is analytic in $\mathbb{C}$ is known as an/entire function

Theorem -1 If $f(z)\&(z)$ are entire function then $f(z)$ must be Constant Function& in this case $f'(z) = 0, \forall z \in \mathbb{C}$

Q. If p(z) & g(z) are non-zero polynomial s.t. p(z) g(z) is analytic in $\mathbb{C}$, then which of the following is/are true?

(a) $P(z)$ is constant polynomial

(b) $\overline{p(z)}$ is constant polynomial

(c) $g(z)$ is constant polynomial

(d) $\overline{g(z)}$ is constant polynomial

(e) $(c)\&(d)$ Both true

Complex Analysis

Theorem -2 **If** $f(\mathrm{z}) = u + iv$ **is analytic in** $\mathbb{C}$**& if** $\mathrm{Re}\,(f(\mathrm{z}))$ **is Constant, then** $f(z)$ **is constant function.**

Theorem -3 If $f(z) = u + iv$ is analytic inc & if Im($f(x)$) is Constant, then $f(\text{z}) is$ constant function.

Q. Consider set $S = \{f \mid f(z) = 30 +$ iv & $f(z)$ is regular in $\mathbb{C}$ & $f: \mathbb{C} \to \mathbb{G}$, then

(a) If $f(5 + 7i) = 30 + i93$, then number of such function is unique

(b) S is singleton set

(c) S is uncountable set

(d) (a) & (c) both true

$f(z) = 30 + iv$ (R, R)

$30 + 5i$

$30 + \frac{1}{3}i$

$x + iy$

Theorem -4 If $f(z) = u + iv$ is analytic in $\mathbb{C}$& if $|f(z)| = \sqrt{u^2 + v^2} = c$ (constant) then $f(z)$ is constant function.

Q. **If** $f: R^2 \to \{(x,y) \mid x^2 + y^2 = 100\}$ **is differentiable everywhere &** $f(15,25) = (10,0)$ **then** $f(10,0) =$

Theorem -5 **If** $f(\mathrm{z}) = u + iv$ **is analytic in** $\subseteq$ & **if famp** $f(z) = \tan^{-1}\left(\frac{v}{u}\right) = c$ **then** $f(\mathrm{z})$ **is constant function.**

Theorem - 6 If $f(z) = u + iv$ is analytic in $\mathbb{C}$ & if a, b, c are non-zero real constant s.t. $au + bv = c$, then $f(z)$ must be constant function.

Theorem -7 **If** $f(\mathrm{z}) = u + iv$ **is analytic in** $\mathbb{C}$ & $\mathrm{Re}\,(f(\mathrm{z})) = u(x, y) \geq 0; \forall z \in \mathbb{C}$ **then** $f(z)$ **must be constant function**

Theorem -8 **If** $f(z) = u + iv$ **is analytic in** $\mathbb{C}$ & $\operatorname{Im}(f(z)) = v(x, y) \geq 0; \forall z \in \mathbb{C}$ **then** $f(z)$ **must be constant function**

Theorem -9 **If** $f(z) = u + iv$ **is analytic in** $\mathbb{C}$ & $\mathrm{Re}\,(f(z)) = u(x, y) \le 0; \forall z \in \mathbb{C}$. **then** $f(z)$ **must be constant function.**

Theorem -10 **If** $f(\mathrm{z}) = u + iv$ **is analytic in** $\mathbb{C}$ & $\mathrm{Im}\,(f(\mathrm{z})) = v(x, y) \leq 0; \forall \mathrm{z} \in \mathbb{C}$. **then** $f(z)$ **must be constant function.**

Complex Analysis

Q. If one root of any equation is $(2-3i)$ (where $i=\sqrt{-1}$), then what is the equation?

(a) $x^2-4x+12=0$

(b) $x^2-4x+13=0$

(c) $x^2+4x+12=0$

(d) $x^2+4x-13=0$

α, β

Eqⁿ →

$x^2-(\alpha+\beta)x+\alpha.\beta=0$

$x^2-(4)x+(4+9)=0$

$x^2-4x+13=0$

$\alpha=(2-3i)$

$\beta=(2+3i)$

Q. Roots of the quadratic equation $x^2 - 2ix + 3 = 0$ are:

(a) $3i, -i$

(b) $3i, i$

(c) $-3i, -i$

(d) $\pm 3i$

$\alpha = 3i$

$\beta = -i$

$\alpha + \beta = 3i - i = 2i$

$\alpha \cdot \beta = 3i \cdot (-i) = -3i^2 \Rightarrow -3(-1) \Rightarrow 3$

$x^2 - 2ix + 3 = 0$

$x^2 - (\alpha + \beta)x + \alpha \cdot \beta = 0$

$\alpha + \beta = 2i$

$\alpha\beta = 3$

Q. Find the argument of this complex number $(1+i)(\sqrt{3}+i)$:

(a) $\frac{\pi}{12}$

(b) $\frac{5\pi}{12}$

(c) π

(d) $\frac{\pi}{4}$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

$\theta = \tan^{-1}\left|\frac{\sqrt{3}+1}{\sqrt{3}-1}\right| \Rightarrow \theta = \frac{5\pi}{12}$

$\tan(45+30) = \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ}$

$\Rightarrow \frac{1+\frac{1}{\sqrt{3}}}{1-1\cdot\frac{1}{\sqrt{3}}} \Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$(1+i)(\sqrt{3}+i)$

$\sqrt{3}+i(1+\sqrt{3})-1$

$(\sqrt{3}-1)+i(1+\sqrt{3})$

$(\sqrt{3}-1)+i(\sqrt{3}+1)$

$x+iy$

$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\omega^3 = 1$

$1 + \omega + \omega^2 = 0$

$1 = -\omega - \omega^2$

$1 = -\omega^2 - \omega$

Q. If ω be a cube root of 1 , then what is the value of $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^5)$?

$(1-\omega)(1-\omega^2)(1-\omega)(1-\omega^2)$

$\left[(1-\omega)(1-\omega^2)\right]^2$

$\left[1-\omega^2-\omega+\omega^3\right]^2$

$\left[1+1+1\right]^2 \Rightarrow 9$

(a) 8

(b) 7

(c) 9

(d) 1

$$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8) + \cdots \text{ 200 factors}$$

$\omega^3 = 1$

$$\boxed{1+\omega+\omega^2=0}$$

$1+\omega = -\omega^2$

$1+\omega^2 = -\omega$

$$(1+\omega)(1+\omega^2)\ (1+\omega^3\cdot\omega)(1+\omega^3\cdot\omega^3\cdot\omega^2)$$

$$(1+\omega)(1+\omega^2)\ (1+\omega)(1+\omega^2)$$

$$\underbrace{(1+\omega)(1+\omega^2)}_{\omega^3}$$

$$\underbrace{-\omega^2 \times -\omega}_{\omega^3}$$

$1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

1^{100}

Q. If $f(z) = |z|^2$, then

(a) f is differentiable everywhere except

(b) f is differentiable everywhere

(c) f is differentiable nowhere

(d) f is differentiable only at zero

$f(z) = |x+iy|^2$

$(\sqrt{x^2+y^2})^2$

$f(z) \Rightarrow x^2+y^2 + 0i$

$u_x = 2x = 0 \Rightarrow x = 0$

$u_y = 2y = 0 \Rightarrow y = 0$

$-v_x = 0$

$v_y = 0$

$z = x+iy$

$|z| = \sqrt{x^2+y^2}$

$f(z) = |z|^2$

$f(z) = x^2 + y^2 + 0i$

C-R Eqn

$u_x = 2x = 0$

$x = 0$

$u_y = 2y = 0$

$y = 0$

Q. If $f(z) = |z|^2$, then:

(a) f is continuous but not differentiable at 0

(b) f is continuous and differentiable at 0

(c) f is neither continuous nor differentiable at 0

(d) f is discontinuous at 1