Partial differential equation

Part - 5

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Solution of Partial Differential Equation by direct Integration

For the solution of partial differential equation by the method of direct integration, keeping in view that in place of constant of integration, we will have the arbitrary function of variable kept constant in integration.

प्रत्यक्ष समाकलन विधि द्वारा आंशिक अवकल समीकरण के हल के लिए, यह ध्यान में रखते हुए कि समाकलन के स्थिरांक के स्थान पर, हमारे पास समाकलन में स्थिर रखे गए चर का स्वेच्छ फलन होगा।

Example

Solve $\dfrac{\partial^2 u}{\partial x\,\partial t} = e^{-t}\cos\, x$

$\frac{d}{dx}\left(\frac{du}{dt}\right) = e^{-t}\cos x \quad -①$

Integration w.r.to x, t is constant.

$\frac{du}{dt} = e^{-t}.\sin x + \phi(t) \quad -②$

Integration w.r.to t, x is constant

$u = -e^{-t}.\sin x + \int \phi(t).dt + \psi(x)$

$u = -e^{-t}.\sin x + \int \phi(t)\,dt + \psi(x)$

Lagrange's Partial Differential Equation

$p = \frac{dz}{dx}$ $q = \frac{dz}{dy}$

A linear partial differential equation of first order of the form $\boxed{Pp + Qq = R}$, where P, Q, R are functions of x, y, z is called Lagrange's partial differential equation.

Pp+Qq=R के रूप का प्रथम क्रम का एक रैखिक आंशिक अवकल समीकरण, जहाँ P,Q,R x,y,z के फलन हैं, लैग्रेंज आंशिक अवकल समीकरण कहलाता है।

For Example:-

1. $\overset{P}{yz}\,p + \overset{Q}{zx}\,q = \overset{R}{xy}$
2. $p\underbrace{\tan x}_{P} + q\underbrace{\tan y}_{Q} = \underbrace{\tan z}_{R}$

Solution of Lagrange's Linear Partial Differential Equation

Working Rule:-

1. Write the given equation in the from $Pp + Qq = R$ and find the value of P, Q, R.

दिए गए समीकरण को $\boxed{Pp+Qq=R}$ के रूप में लिखें तथा P,Q,R का मान ज्ञात करें।

2. Write the Lagrange's auxiliary equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

लैग्रेंज का सहायक समीकरण $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ लिखें

3. Solve the auxiliary equation obtained in setp 2 Let $u(x, y, z) = a$ and $v(x, y, z) = b$ be two independent solutions of it.

सेटप 2 में प्राप्त सहायक समीकरण को हल करें। मान लें कि u(x,y,z)=a और v(x,y,z)=b इसके दो स्वतंत्र समाधान हैं।

4. Finally general solution of the given equation can be written as $f(u, v) = 0$

अंत में दिए गए समीकरण का सामान्य समाधान f(u,v)=0 के रूप में लिखा जा सकता है

Example

Solve $\underbrace{yz}_{P}p + \underbrace{zx}_{Q}q = \underbrace{xy}_{R}$

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{yz} = \frac{dy}{zx}$$

$$\int x\,dx = \int y\,dy$$

$$x^2 - y^2 = 2c_1$$

$$\boxed{x^2 - y^2 = a}$$

$$\frac{dy}{zx} = \frac{dz}{xy}$$

$$\int y.dy = \int z\,dz$$

$$y^2 - z^2 = 2c_2$$

$$\boxed{y^2 - z^2 = b}$$

$$x^2 - y^2 = a \Rightarrow u(x, y, z)$$

$$y^2 - z^2 = b = v(x, y, z)$$

$$\rightarrow f(u, v) = 0$$

$$\boxed{f(x^2 - y^2,\ y^2 - z^2) = 0}$$

Q. The partial differential equation by eliminating the arbitrary constants a and b from the equation $z = (x+a)(y+b)$ is given by

समीकरण से मनमाने स्थिरांक a और b को हटाकर आंशिक अंतर समीकरण z=(x+a)(y+b) इस प्रकार दिया गया है

a) $z = \frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y}$

b) $z = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$

c) $z = \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}$

d) $z = 2\frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y}$

diff w.r.to x

$\frac{dz}{dx} = y+b$ -①

diff w.r.to y.

$\frac{dz}{dy} = (x+a)$.

$\frac{dz}{dx} \times \frac{dz}{dy} \Rightarrow (y+b)(x+a)$

$\boxed{z = \frac{dz}{dx} \times \frac{dz}{dy}}$

Q. The partial differential equation by eliminating the arbitrary functions from the equation $z = f(x - ay) + g(x + ay)$ is given by

समीकरण z=f(x-ay)+g(x+ay) से मनमाने कार्यों को हटाकर आंशिक अंतर समीकरण इस प्रकार दिया गया है

$$\boxed{\frac{d^2z}{dx^2} = a^2 \frac{d^2z}{dy^2}}$$

a) $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$

b) $\frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial x^2}$

c) $\frac{\partial^2 z}{\partial y^2} = -a^2 \frac{\partial^2 z}{\partial x^2}$

b) None of these

$\frac{dz}{dx} = f'(x-ay) + g'(x+ay)$

$\frac{d^2z}{dx^2} \Rightarrow f''(x-ay) + g''(x+ay)$ — 1

$\frac{dz}{dy} \Rightarrow f'(x-ay) \times -a + g'(x+ay) \times a$

$\frac{d^2z}{dy^2} \Rightarrow a^2 f''(x-ay) + a^2 g''(x+ay)$ — 2

Q. The partial differential equation by eliminating the arbitrary function from the equation $z = f(x^2 - y^2)$ is given by

समीकरण $z = f(x^2 - y^2)$ से मनमाना फ़ंक्शन को हटाकर आंशिक अंतर समीकरण इस प्रकार दिया गया है

a) $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$

$y\frac{dz}{dx} = f'(x^2-y^2) \times 2xy$

b) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$

$x\frac{dz}{dy} = f'(x^2-y^2) \times -2yx$

c) $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$

d) $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 0$

$y \cdot \frac{dz}{dx} + x\frac{dz}{dy} = 0$

Q. Solution of the partial differential equation $z(xy+z^2)(px-qy)=x^4$ is given by

आंशिक अंतर समीकरण $z(xy+z^2)(px-qy)=x^4$ का हल निम्न प्रकार दिया गया है

a) $f(xy, x^4-2xyz^2-z^4)=0$

b) $f(xy, x^4+2xyz^2+z^2)=0$

c) $f(yz, x^4-2xyz^2-z^2)=0$

d) $f(xz, x^4+2xyz^2+z^4)=0$

$\underbrace{zx(xy+z^2)}_{P}p-\underbrace{zy(xy+z^2)}_{Q}q=\underbrace{x^4}_{R}$

$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$

$\frac{dx}{\cancel{z}x(\cancel{xy+z^2})}=\frac{dy}{-\cancel{z}y(\cancel{xy+z^2})}$

$\int\frac{dx}{x}=-\int\frac{dy}{y}$

$\log x=-\log y+c_1$

$xy=a$

$\frac{dx}{z\cancel{x}(xy+z^2)}=\frac{dz}{x^{\cancel{4}3}}$

$\int x^3dx=\int 2xy+z^3$

$\frac{x^4}{4}=\frac{2z^2xy}{2\times2}+\frac{z^4}{4}$