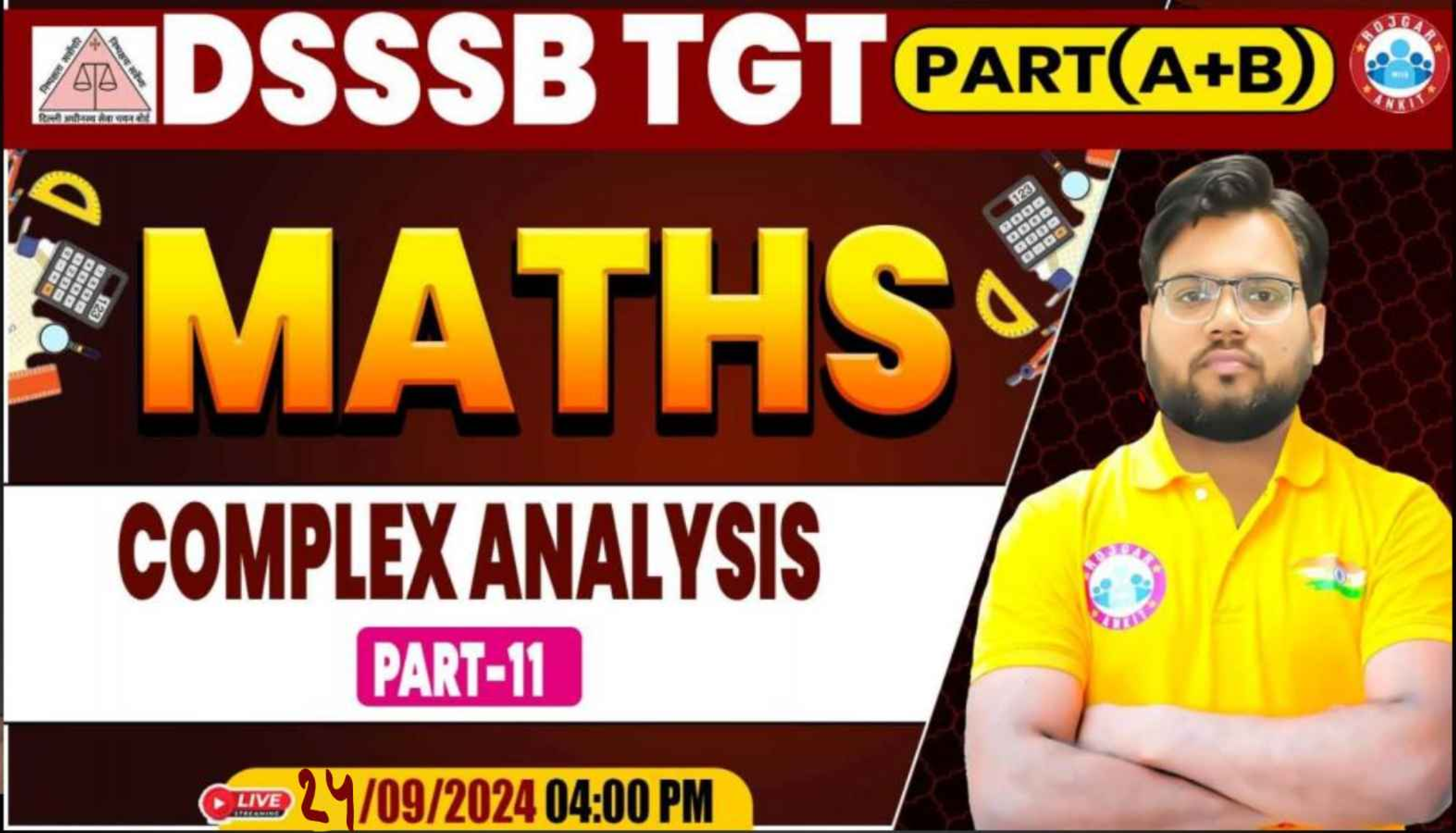
DSSSB TGT
PART(A+B)
MATHS
COMPLEX ANALYSIS
PART-11
LIVE
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Entire Function: A function $f(z)$ is analytic in $\mathbb{C}$ is known as an/entire function

(Complex field)

Analytic → Differentiable

Theorem -1 If $f(\mathrm{z})\&$[illegible](z) are entire function then $f(\mathrm{z})$ must be Constant Function& in this case $f'(\mathrm{z}) = 0, \forall \mathrm{z} \in \mathbb{C}$

Q. If p(z) & g(z) are non-zero polynomial s.t. p(z) g(z) is analytic in $\mathbb{C}$, then which of the following is/are true?

(a) $P(z)$ is constant polynomial

(b) $\overline{p(z)}$ is constant polynomial

(c) $g(z)$ is constant polynomial

(d) $\overline{g(z)}$ is constant polynomial

(e) $(c)\&(d)$ Both true

Complex Analysis

Theorem -2 If $f(\mathrm{z}) = u + iv$ is analytic in $\mathbb{C}$& if $\mathrm{Re}\,(f(\mathrm{z}))$ is Constant, then $f(z)$ is constant function.

$f(z) = u + iv$

$\mathrm{Re}(f(z)) = u$

Theorem -3 If $f(z) = u + iv$ is analytic in $\mathbb{C}$ & if Im($f(x)$) is Constant, then $f(z)$ is constant function.

$f(z) = u + iv$

$f(z)$ is analytic fu^n in $\mathbb{C}$

C-R Eq^n

$u_x = v_y = 0$

$u_y = -v_x = 0$

Q. Consider set $S = \{f \mid f(z) = 30 +$ iv & $f(z)$ is regular in $\mathbb{C}$ & $f: \mathbb{C} \to \mathbb{G}$, then

(a) If $f(5 + 7i) = 30 + i93$, then number of such function is unique

(b) S is singleton set

(c) S is uncountable set

(d) (a) & (c) both true

Theorem -4 If $f(z) = u + iv$ is analytic in $\mathbb{C}$ & if $|f(z)| = \sqrt{u^2 + v^2} = c$ (constant) then $f(z)$ is constant function.

Re ↓ Im ↑

$f(z) = 3 + 4i$

$|f(z)| = \sqrt{3^2 + 4^2} = 5 \rightarrow$ Constant

$z = x + iy$ ($x \to u$, $y \to v$)

$|z| = \sqrt{u^2 + v^2}$

Q. If $f: R^2 \to \{(x, y) \mid x^2 + y^2 = 100\}$ **is differentiable everywhere &** $f(15,25) = (10,0)$ **then** $f(10,0) =$

Theorem -5 If $f(\mathrm{z}) = u + iv$ is analytic in $\subseteq$ & if famp $f(z) = \tan^{-1}\left(\frac{v}{u}\right) = c$ then $f(\mathrm{z})$ is constant function.

Theorem - 6 If $f(z) = u + iv$ is analytic in $\mathbb{C}$ & if a, b, c are non-zero real constant s.t. $au + bv = c$, then $f(z)$ must be constant function.

$au + bv = c$ ⇐ $f(z) = 4 + 5i$

$5 \times 4 + 5 \times 5$

$= 45 =$ constant

$a = 5$

$b = 5$

Theorem -7 **If** $f(\mathrm{z}) = u + iv$ **is analytic in** $\mathbb{C}$ & $\mathrm{Re}\,(f(\mathrm{z})) = u(x, y) \geq 0; \forall \mathrm{z} \in \mathbb{C}$ **then** $f(z)$ **must be constant function**

Theorem -8 **If** $f(\mathrm{z}) = u + iv$ **is analytic in** $\mathbb{C}$ & $\operatorname{Im}(f(\mathrm{z})) = v(x, y) \geq 0; \forall \mathrm{z} \in \mathbb{C}$ **then** $f(\mathrm{z})$ **must be constant function**

Theorem -9 **If** $f(z) = u + iv$ **is analytic in** $\mathbb{C}\&\text{Re}\,(f(z)) = u(x, y) \leq 0; \forall z \in \mathbb{C}$. **then** $f(z)$ **must be constant function.**

Theorem -10 **If** $f(\mathrm{z}) = u + iv$ **is analytic in** $\mathbb{C}$&$\mathrm{Im}\,(f(\mathrm{z})) = v(x, y) \leq 0; \forall \mathrm{z} \in \mathbb{C}$. **then** $f(z)$ **must be constant function.**

Complex Analysis

Q. If one root of any equation is $(2 - 3i)$ (where $i = \sqrt{-1}$), then what is the equation?

(a) $\mathrm{x}^2 - 4\mathrm{x} + 12 = 0$

(b) $x^2 - 4x + 13 = 0$

(c) $x^2 + 4x + 12 = 0$

(d) $x^2 + 4x - 13 = 0$

Q. Roots of the quadratic equation $x^2 - 2ix + 3 = 0$ are:

(a) $3i, -i$

(b) $3i, i$

(c) $-3i, -i$

(d) $\pm 3i$

Q. Find the argument of this complex number $(1+i)(\sqrt{3}+i)$:

(a) $\frac{\pi}{12}$

(b) $\frac{5\pi}{12}$

(c) π

(d) $\frac{\pi}{4}$

$\tan 15^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\tan(60-45) \Rightarrow \frac{\tan 60 + \tan 45}{1 - \tan 60 \cdot \tan 45}$

$\left(\frac{\sqrt{3}+1}{1-\sqrt{3}}\right)$

$\sqrt{3} + i + i\sqrt{3} - 1$

$(\sqrt{3}-1) + i(1+\sqrt{3})$

$\theta = \tan^{-1}\left|\frac{\sqrt{3}+1}{\sqrt{3}-1}\right|$

$\theta = \frac{\pi}{12}$

$\arg(\theta) = \frac{\pi}{12}$

$x + iy$

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

Q. If ω be a cube root of 1 , then what is the value of $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^5)$?

(a) 8

(b) 7

(c) 9

(d) 1

Q. If $f(z) = |z|^2$, then

(a) f is differentiable everywhere except 0

(b) f is differentiable everywhere

(c) f is differentiable nowhere

(d) f is differentiable only at zero

Q. If $f(z) = |z|^2$, then:

(a) f is continuous but not differentiable at 0

(b) f is continuous and differentiable at 0

(c) f is neither continuous nor differentiable at 0

(d) f is discontinuous at 1

Complex Analysis

Algebraic Number: - A complex number is said to be Algebraic number if it satisfies a non-zero polynomial over Q.

Transcendental Number: - Non -algebraic numbers are said to be transcendental number.

Q. Which of the following numbers is an algebraic number?

A. π

B. e

C. $\sqrt{2}$

D. $\ln(2)$

Q. Which of the following statements is true?

A, All transcendental numbers are algebraic.

B. All algebraic numbers are transcendental.

C. Some algebraic numbers are transcendental.

D. No algebraic number is transcendental.

Complex Analysis

(1) $|z-a|=r$

(2) $|z-a|<r$

(3) $|z-a|\leq r$

(4) $|z-a|>r$

(5) $|z-a|\geqslant r$

Complex Analysis

Analytic Function or Regular Function or Holomorphic Function

Analytic Function:- A complex valued function $f(z)$ is said to be analytic at point $z = z_0$ if $f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z)-f(z)}{\delta z}$ exists & is unique at z_0

Analytic Function: A single valued function $f(z)$ is said to be andytyic in a region R, if it is differentiableat each point of R.

(C-R-Equation | Cauchy Riemann Equation)

For a function $f(z)$ to be analytic in a region R

Necessary: If $f(z)$ is analytic at a point, then u and v must satisfy the Cauchy-Riemann equations at that point.

Sufficient: If u and v satisfy the Cauchy-Riemann equations and the partial derivatives of u and v are continuous, then $f(z)$ is analytic in that region

Q. Let $f(z) = \frac{1}{z}, z \neq 0$, then

1. f does not satisfy Cauchy - Riemann equation for all $z \neq 0$
2. f is continuous only but nowhere differentiable
3. f satisfies Cauchy - Riemann equation, but not analytic for all $z \neq 0$
4. f is analytic for all $z \neq 0$,

Q. The Complex Function $f(z) = z^2$ is?

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None of these

Q. $f(z) = (|z|)^2$ is analytic or not?

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

Q. $f(z) = 2xy + i(x^2 - y^2)$

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

Q. $f(z) = z^3$

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

Q. $f(z) = e^{-x}(\cos\ y + i\sin\ y)$

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

Q. $f(z) = \sin z$

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

Harmonic Function

Q. $u = x^2 - y^2$

(a) Harmonic function

(b) Not Harmonic function

(c) May be Harmonic function

(d) None

Characteristics of Harmonic Functions in Complex Analysis

1. If $f(z) = u(x, y) + iv(x, y)$ is analytic in a region R, then both u and v are harmonic functions in R .
2. If $u(x, y)$ is harmonic in a connected region R, then u is the real part of an analytic function $f(z) = u(x, y) + iv(x, y)$.
3. If u and v are the real and imaginary parts of an analytic function, then u and v are considered as harmonic conjugates.

4.The sum of two harmonic functions results in another harmonic function.

5. An arbitrary pair of harmonic functions " u " and " v " may not necessarily be conjugates, unless $u+iv$ is an analytic function.

Q. $v = 3x^2y - y^3$

(a) Harmonic function

(b) Not Harmonic function

(c) May be Harmonic function

(d) None

Q. $u(x,y) = x^2 - y^2$, **find** $v(x,y)$ **?**

(a) $3x + 2y + k$

(b) $2xy + k$

(c) $3xy^2 + k$

(d) None

Q. $v(x, y) = 3x^2y - y^3$, **Find** $u(x, y)$

(a) $x^2 - 3xy^3 + c$

(b) $x^3 - 2xy + c$

(c) $x^3 - 3xy^2 + c$

(d) None

Q. For analytic function $f(z) = u + iv$, if $u(x, y) = 3x^2 - 3y^2$, find $v(x, y)$

(a) $6xy + 3x^2 + c$

(b) $6y - 6x + c$

(c) $6x - 6y + c$

(a) $6xy + c$

Q. An analytic Function $f(z) = u(x, y) + iv(x, y)$ if $u = xy$, find v

(a) $\frac{x^2 - y}{2} + k$

(b) $\frac{x - y^2}{2} + k$

(c) $\frac{(\mathrm{x}+\mathrm{y})^2}{2} + \mathrm{k}$

(d) $\frac{y^2 - x^2}{2} + k$

Milne -Thomson Method

Construct Analytic function By Milne -Thomson Method

(1) $f(z) = \int (u_x)_{(z,0)} dz - i \int (u_y)_{(z,0)} dz + c$; **if** $u(x, y)$ **is given**

(2) $f(z) = \int (v_y)_{(z,0)} dz + i \int (v_x)_{(z,0)} dz + c$; **if** $v(x, y)$ **is given**

Q. If $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2$, **Find Analytic** $f(z)$ **?**

(a) $z^3 - z^2 + c$

(b) $z^3 + 3z^2 + c$

(c) $3z^3 + z^2 + c$

(d) None of these.

Q. $u(x, y) = x^3 - 3xy^2$. **Find Analytic** $f(z)$ **?**

(a) $z^2 + c$

(b) $z^3 + z^2 + c$

(c) $z^3 + c$

(d) None of these.

Q. $(x, y) = 3x^2y - y^3$. **Find Analytic** $f(z)$ **?**

(a) $z^2 + c$

(b) $z^3 + z^2 + c$

(c) $z^3 + c$

(d) None of these.

Q. Every holomorphic function is :-

1. commutative
2. analytic
3. complex vector space
4. complex

Q. What is the analytic region of $f(z) = (x - y)^2 + 2i(x + y)$ **?**

A. $x - y = 1$

B. $x + y = 2$

C. $x + y = -2$

D. $x - y = -1$

$f(z) = (x-y)^2 + i\,2(x+y)$

$u = (x-y)^2$

$V = 2(x+y)$

$U_x = V_y$

$2(x-y) = 2$

$x - y = 1$ ✓

$U_y = -V_x$

$+2(x-y) = +2$

$x - y = 1$

Q → $(\cos\theta + i\sin\theta).(\cos\phi + i\sin\phi) =$

A. $\cos(\theta - \phi) + i\sin(\phi - \theta)$

B. $\cos(\theta + \phi) + i\sin(\theta + \phi)$

C. $\sin(\theta - \phi) + i\cos(\theta - \phi)$

D. $\sin(\phi - \theta)) + i\cos(\theta - \phi)$

$\cos(A+B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

$(\cos\theta + i\sin\theta)(\cos\phi + i\sin\phi)$

$\cos\theta \cdot \cos\phi + i\cos\theta \cdot \sin\phi + i\sin\theta \cdot \cos\phi - \sin\theta \cdot \sin\phi$

$(\cos\theta \cdot \cos\phi - \sin\theta \cdot \sin\phi) + i(\sin\phi\cos\theta + \cos\phi\sin\theta)$

$\cos(\theta+\phi) + i\sin(\theta+\phi)$

Q. If $f\,z$ is analytic function whose real part is constant then $f\,z$ is....... .

A. Function of x and y both

B. constant

C. Function of y only

D. Function of x only

U → Real Part

V → Imaginary part

$f(z) = u + iv$

$u_x = v_y$

$\Rightarrow u_y = -v_x$

$0 = 0$

If u is constant then

$u_x = 0 = v_y$

Q. Does C-R equations are necessary and sufficient for a function to be analytic?

A. depend on range of functions

B. FALSE

C. TRUE

D. can't say about nature of equations