Partial differential Equation

- Partial Differential Equation आंशिक अंतर समीकरण
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- Formation of Partial Differential Equation by Elimination of arbitrary functions मनमाने कार्यों के उन्मूलन द्वारा आंशिक अंतर समीकरण का गठन

Partial Differential Equation

If an equation contains a dependent variable, two or more independent variables and partial derivatives, is known as Partial Differential Equation

x, y, z → z dependent variable < x, y } Independent.

यदि किसी समीकरण में एक आश्रित चर, दो या अधिक स्वतंत्र चर और आंशिक व्युत्पन्न शामिल हों, तो उसे आंशिक अंतर समीकरण के रूप में जाना जाता है

$\frac{dz}{dx}, \frac{dz}{dy}$

Note:-

Generally x and y are taken as independent variables and z as a dependent variable.

सामान्यतः x और y को स्वतंत्र चर तथा z को आश्रित चर के रूप में लिया जाता है।

For Example:-

1. $z = x + y\frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$

2. $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial t}$

Formation of Partial Differential Equation by Elimination of arbitrary constants

Working Rule:-

1. Let $f(x,y,z,a,b)=0$...... (1) be the given equation, where z is regarded as a function of two independent variables x and y and a and b denote the arbitrary constants.

मान लीजिए f(x,y,z,a,b)=0....... (1) दिया गया समीकरण है, जहाँ z को दो स्वतंत्र चर x और y का एक फलन माना जाता है तथा a और b स्वेच्छ स्थिरांक दर्शाते हैं।

2. Differentiating (1) partially w.r.t. x and y, then we get $f\left(x,y,z,\frac{\partial z}{\partial x},\frac{\partial z}{\partial y},a,b\right)=0$.......... (2)

2. (1) को x और y के सापेक्ष आंशिक रूप से अवकलित करने पर, हम पाते हैं f(x,y,z,∂z/∂x,∂z/∂y,a,b)=0.......... (2)

3. Now Eliminating arbitrary constants a and b by using eq.(1) and eq.(2), we get the required Partial Differential Equation $f(x, y, z, p, q) = 0$ where $\frac{\partial z}{\partial x} = p$ and $\frac{\partial z}{\partial y} = q$

अब समीकरण (1) और समीकरण (2) का उपयोग करके मनमाने स्थिरांक a और b को हटाने पर, हमें आवश्यक आंशिक अंतर समीकरण f(x,y,z,p,q)=0 प्राप्त होता है जहाँ $\frac{\partial z}{\partial x} = p$ और $\frac{\partial z}{\partial y} = q$

Example

Form the partial differential equation by eliminating arbitrary constants a and b from the equation

समीकरण से मनमाने स्थिरांक a और b को हटाकर आंशिक अंतर समीकरण बनाएं

$$z = ax + by + a^2 + b^2$$

Qus $z = ax + by + a^2 + b^2$.

diff w.r to x and y.

$$\frac{dz}{dx} = a$$

$$\frac{dz}{dy} = b$$

P.D.E

$$z = x \cdot \frac{dz}{dx} + y \cdot \frac{dz}{dy} + \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2$$

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Formation-of partial Differential Equation by Elimination of arbitrary functions

Working Rule:-

1. Suppose u and v are two functions of x, y, z which are connected by the relation $f(u, v) = 0$ .... (1)

मान लीजिए u और v, x, y, z के दो फलन हैं जो संबंध f(u, v) = 0 से जुड़े हैं .... (1)

2. Differentiating (1) partially w.r.t. x, we get $\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial x}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z} \cdot \frac{\partial z}{\partial x}\right) = 0$ .... (2)

(1) को आंशिक रूप से w.r.t. में अवकलित करने पर, हम प्राप्त करते हैं $\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial x}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z} \cdot \frac{\partial z}{\partial x}\right) = 0$ .... (2)

3. Differentiating (1) partially w.r.t. y, we get $\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial y}\right)+\frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y}+\frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial y}\right)=0$ ……….. (3)

(1) को आंशिक रूप से w.r.t. में अवकलित करने पर y, हमें मिलता है $\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial y}\right)+\frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y}+\frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial y}\right)=0$ ……….. (3)

4. Now we shall eliminate $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ by using (2) and (3) then we get the required Partial differential equation of the form $Pp + Qq = R$

अब हम (2) और (3) का उपयोग करके $\frac{\partial f}{\partial u}$ और $\frac{\partial f}{\partial v}$ को हटा देंगे, फिर हमें *Pp+Qq=R* के रूप का आवश्यक आंशिक अंतर समीकरण मिलता है

Where $P = \frac{\partial u}{\partial y}\frac{\partial v}{\partial z} - \frac{\partial u}{\partial z}\frac{\partial v}{\partial y}$ $Q = \frac{\partial v}{\partial x}\frac{\partial u}{\partial z} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial z}$ **and** $R = \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}$

Ex $z = ax + by + a^2 + b^2$

$f(u, v) = 0$

(x, y, z)

diff w.r to x

$$\frac{df}{du}\left[\frac{du}{dx}\cdot\frac{dx}{dx} + \frac{du}{dy}\cdot\frac{dy}{dx} + \frac{du}{dz}\cdot\frac{dz}{dx}\right] \quad \text{— (I)}$$

diff w.r to y

$$\frac{df}{dv}\left[\frac{dv}{dx}\cdot\frac{dx}{dx} + \frac{dv}{dy}\times\frac{dy}{dx} + \frac{dv}{dz}\cdot\frac{dz}{dx}\right] \quad \text{— (II)}$$

(I) + (II)

$$\frac{df}{du}\left[\frac{du}{dx} + \frac{du}{dy}\times\frac{dy}{dx} + \frac{du}{dz}\cdot\frac{dz}{dx}\right] +$$

$$\frac{df}{dv}\left[\frac{dv}{dx} + \frac{dv}{dy}\cdot\frac{dy}{dx} + \frac{du}{dz}\cdot\frac{dz}{dx}\right] = 0$$

Example

Find the Partial Differential Equation by eliminating the function f from the relation $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$

संबंध $z = y^2 + 2f\left(\frac{1}{x} + lo\,g\,y\right)$ से फ़ंक्शन f को हटाकर आंशिक अंतर समीकरण ज्ञात करें

Q4 $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.

diff w.r.t x.

$$\frac{dz}{dx} = 2f'\left(\frac{1}{x} + \log y\right) \times \frac{-1}{x^2} \quad \text{— (I)}$$

diff w.r.t y.

$$\frac{dz}{dy} \Rightarrow 2y + 2f'\left(\frac{1}{x} + \log y\right) \times \frac{1}{y}$$

$$\frac{dz}{dy} - 2y = 2f'\left(\frac{1}{x} + \log y\right) \times \frac{1}{y} \quad \text{— (II)}$$

(I) ÷ (II)

$$\frac{\frac{dz}{dx}}{\frac{dz}{dy} - 2y} \Rightarrow \frac{\frac{-1}{x^2}}{\frac{1}{y}} \Rightarrow \frac{-y}{x^2}$$

$\frac{dz}{dx} \to p$, $\frac{dz}{dy} \to q$

$$\frac{p}{q - 2y} = \frac{-y}{x^2}$$

$$x^2 p = -yq + 2y^2$$

$$x^2 p + yq = 2y^2$$

$x^2 \to P$, $y \to Q$, $2y^2 \to R$

$$Pp + Qq = R$$

eliminate constant.

Q. Partial differential equation by eliminating h and k from the equation $(x-h)^2+(y-k)^2+z^2=\lambda^2$ is given by

समीकरण $(x-h)^2+(y-k)^2+z^2=\lambda^2$ से h और k को हटाकर आंशिक अंतर समीकरण इस प्रकार दिया गया है

a) $z^2\left(\frac{\partial z}{\partial x}\right)^2 - z^2\left(\frac{\partial z}{\partial y}\right)^2 - z^2 = \lambda^2$

b) $z^2\left(\frac{\partial z}{\partial x}\right)^2 + z^2\left(\frac{\partial z}{\partial y}\right)^2 + z^2 = \lambda^2$

c) $z^2\left(\frac{\partial z}{\partial x}\right)^2 - 2z^2\left(\frac{\partial z}{\partial y}\right)^2 + \frac{z^2}{2} = \lambda^2$

d) None of these

diff w.r to x.

$2(x-h) + 2z\cdot\frac{dz}{dx} = 0$

$x-h = -z\cdot\frac{dz}{dx}$ — (I)

diff w.r to y

$(y-k) = -z\cdot\frac{dz}{dy}$ — (II)

$z^2\left(\frac{dz}{dx}\right)^2 + z^2\left(\frac{dz}{dy}\right)^2 = (x-h)^2+(y-k)^2 \rightarrow \lambda^2 - z^2$

Q. The partial differential equation by eliminating the arbitrary constants λ and A from the equation $z = Ae^{-\lambda^2 t}\cos \lambda x$ is given by

समीकरण से मनमाने स्थिरांक λ और A को हटाकर आंशिक अंतर समीकरण $z = Ae^{-\lambda^2 t}\cos \lambda x$ द्वारा दिया गया है

a) $\frac{\partial^2 z}{\partial x^2} = 2\frac{\partial z}{\partial t}$

b) $\frac{\partial^2 z}{\partial x^2} = z\frac{\partial z}{\partial t}$

c) $z\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial t}$

d) $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial t}$

diff w.r.t t.

$\frac{dz}{dt} \Rightarrow A.e^{-\lambda^2 t}. \cos\lambda x$ — (i)

diff w.r.t x.

$\frac{dz}{dx} = -Ae^{-\lambda^2 t}\sin\lambda x \times \lambda$ — (ii)

$\frac{d^2z}{dx^2} \Rightarrow$

Home-work