



DSSSB TGT

PART(B)



MATHS

ORDINARY DIFFERENTIAL EQUATION

PART-06



23/09/2024 08:00 AM



Homogenous Linear Differential Equation

A differential Equation of the form $P_0x^n \frac{d^n y}{dx^n} + P_1x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + \dots + P_n y = X$

$P_0x^n \frac{d^n y}{dx^n} + P_1x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + \dots + P_n y = X$ से एक अंतर समीकरण

Where $P_0, P_1, P_2, \dots, P_n$ are constants and X is a function of x or constant, is known as Homogenous Linear differential Equation.

जहाँ $P_0, P_1, P_2, \dots, P_n$ स्थिरांक हैं और X, x का एक फलन या स्थिरांक है, इसे समरूप रैखिक अवकल समीकरण के रूप में जाना जाता है।

For Examples:-

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$$

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$$

$$\text{Let } x = e^z$$

$$z = \log x$$

$$\boxed{\frac{dz}{dx} = \frac{1}{x}}$$

$$\frac{dy}{dx} \Rightarrow \frac{dy}{dz} \times \frac{dz}{dx}$$

$$\frac{dy}{dx} \Rightarrow \frac{1}{x} \cdot \frac{dy}{dz}$$

$$\boxed{x \frac{dy}{dx} = \frac{dy}{dz}}$$

$$\frac{d^2 y}{dx^2} \Rightarrow \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

$$\Rightarrow \frac{d}{dx} \left(\underset{\text{I}}{\frac{1}{x}} \underset{\text{II}}{\frac{dy}{dz}} \right)$$

$$\Rightarrow \frac{1}{x} \cdot \frac{d^2 y}{dz^2} \times \frac{dz}{dx} + \frac{dy}{dz} \times \frac{-1}{x^2}$$

$$\frac{d^2 y}{dx^2} \Rightarrow \frac{1}{x^2} \cdot \frac{d^2 y}{dz^2} - \frac{1}{x^2} \cdot \frac{dy}{dz}$$

$$x^2 \cdot \frac{d^2 y}{dx^2} = \frac{d^2 y}{dz^2} - \frac{dy}{dz}$$

$$x^2 \cdot \frac{d^2 y}{dx^2} - 2x \cdot \frac{dy}{dx} - 4y = x^4$$

$$\frac{d^2 y}{dz^2} - \frac{dy}{dz} - 2 \cdot \frac{dy}{dz} - 4y = x^4$$

$$\frac{d^2 y}{dz^2} - 3 \frac{dy}{dz} - 4y = x^4$$

Homogeneous
differential
Equation

$$[D^2 - 3D - 4]y = x^4$$

Working Rule:-

1. Firstly put $x = e^z$ so that $z = \log x$ and $D = x \frac{d}{dx} = \frac{d}{dz}$ and

$$D(D-1) = x^2 \frac{d^2}{dx^2} \text{ etc..}$$

सबसे पहले $x = e^z$ रखें ताकि $z = \log x$ और $D = x \frac{d}{dx} = \frac{d}{dz}$ और $D(D-$

1) $= \frac{x^2 d^2}{dx^2}$ आदि हो।

$$\textcircled{\#} \quad x \cdot \frac{d}{dx} \Bigg| \frac{d}{dz} \rightarrow D$$

$$\textcircled{\#} \quad x^2 \cdot \frac{d^2 y}{dx^2} \rightarrow D^2 - D$$

2. Then given homogenous linear differential equation becomes Linear differential equation with constant coefficients in variable y and z .

फिर दिया गया समरूप रैखिक अंतर समीकरण चर y और z में स्थिर गुणांकों के साथ रैखिक अंतर समीकरण बन जाता है।

3. After getting solution replace z by x using $x = e^z$.

समाधान प्राप्त करने के बाद $x = e^z$ का उपयोग करके z को x से बदलें।

4. Example Solve the following Differential Equation

उदाहरण निम्नलिखित अंतर समीकरण हल करें

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$$

Ques $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$ Find the Homogeneous D. E.

↓

$$[D^2 - 2D - 4]y = x^4$$

$$D^2 - 2D - 4 = 0$$

$$(D-4)(D+1) = 0$$

$$D = -1, 4$$

different

$$C.F. \Rightarrow c_1 e^{4z} + c_2 e^{-z}$$

$$C.F. \Rightarrow c_1 x^4 + \frac{c_2}{x}$$

$$P.I. \rightarrow \frac{1}{f(D)} \cdot x^4$$

$$\Rightarrow \frac{1}{D^2 - 2D - 4} \times e^{4z}$$

$$\Rightarrow \frac{1}{16 - 12 - 4} \cdot e^{4z}$$

0

$$\frac{z \cdot e^{4z}}{2D - 3}$$

$$\frac{z \cdot e^{4z}}{5} \Rightarrow \frac{\log x \cdot x^4}{5}$$

$$\left[\begin{array}{l} x = e^z \rightarrow z = \log x \\ x^4 = e^{4z} \end{array} \right]$$

Soln $\rightarrow C.F. + P.I.$

$$\Rightarrow c_1 x^4 + \frac{c_2}{x} + \frac{\log x \cdot x^4}{5}$$

Q. 1 Solution of the differential equation $x^2 \frac{d^2 y}{dx^2} - 2y = x^2 + \frac{1}{x}$ is given by

अंतर समीकरण $\frac{x^2(d^2 y)}{dx^2} - 2y = x^2 + \frac{1}{x}$ का हल निम्न प्रकार दिया गया है

~~a) $c_1 x^2 + \frac{c_2}{x} + \frac{1}{3} \left(x^2 + \frac{1}{x} \right) \log x$~~

b) $c_1 x^2 + \frac{c_2}{x^2} + \frac{1}{5} \left(x^2 - \frac{1}{x} \right) \log x$

c) $c_1 x^2 + \frac{c_2}{x} + \frac{1}{3} \left(x^2 - \frac{1}{x} \right) \log x$

d) None of these

let $x = e^z$
 $\rightarrow z = \log x$

$[n^2 - n - 2]y = e^{2z} + e^{-z}$

$n^2 - n - 2 = 0$

$n^2 - 2n + n - 2 = 0$

$(n-2)(n+1) = 0$

$n = -1, 2$

different

C.F $\rightarrow c_1 e^{2z} + c_2 e^{-z}$

$\Rightarrow c_1 x^2 + \frac{c_2}{x}$

P.I $\rightarrow \frac{1}{f(n)} x (e^{2z} + e^{-z})$

$\frac{1}{n^2 - n - 2} x \left(x^2 + \frac{1}{x} \right)$

$\frac{2 \cdot \left(x^2 + \frac{1}{x} \right)}{2n - 1} \Rightarrow \frac{\log x \left(x^2 + \frac{1}{x} \right)}{3}$

Q. 2 Solution of the differential equation $x \frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} = \frac{1}{x}$ **is given by**

Let $x = e^z \rightarrow z = \log x.$

अंतर समीकरण $x \frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} = \frac{1}{x}$ का हल निम्न प्रकार दिया गया है

- ✓ a) $c_1 + (c_2 + c_3 \log x)x + \frac{x}{2} (\log x)^2$
 b) $c_1 + (c_2 + c_3 \log x)x - \frac{x}{2} (\log x)^2$
 c) $c_1 + (c_2 + c_3 \log x)x + \frac{x^2}{2} (\log x)^2$
 d) $c_1 + (c_2 + c_3 \log x)x - \frac{x^2}{2} (\log x)^2$

C.F $\rightarrow c_1 e^{0z} + c_2 e^z + c_3 e^z$

C.P $\rightarrow c_1 + (c_2 + c_3 \log x) e^z$
 $\downarrow \log x \quad \downarrow x$

$n(n-1)(n-2) + n(n-1) = x$
 $(n^2 - n)(n-2) + n^2 - n$

$n^3 - 2n^2 - n^2 + 2n + n^2 - n = e^{-z}$

$n^3 - 2n^2 + n = e^{-z}$

$n^3 - 2n^2 + n = 0$

$n(n^2 - 2n + 1) = 0$

$n=0 \quad (n-1)^2 = 0$

$n = 1, 1$

$$P.I \rightarrow \frac{1}{f(n)} e^z$$

$$\Rightarrow \frac{1}{D^3 - 2D^2 + D} e^z$$

$$= \frac{1}{0} \cdot e^z$$

$$\Rightarrow \frac{z \cdot e^z}{3D^2 - 4D}$$

$$= \frac{z \cdot e^z}{0}$$

$$\frac{z^2 \cdot e^z}{6D - 4}$$

$$\frac{(\log x)^2}{2}$$

Q. 3 Solution of the differential equation $(x^2D^2 - 3xD + 4)y = x^m$ is given by

अंतर समीकरण $(x^2D^2 - 3xD + 4)y = x^m$ का हल निम्न प्रकार दिया गया है

a) $(c_1 + c_2 \log x)x^2 + \frac{1}{(m+2)^2} \cdot x^{2m}$

b) $(c_1 + c_2 \log x)x^2 + \frac{1}{(m-2)^2} \cdot x^{2m}$

c) $(c_1 + c_2 \log x)x^2 + \frac{1}{(m-2)^2} \cdot x^m$

d) $(c_1 + c_2 \log x)x^2 + \frac{1}{(m+2)^2} \cdot x^m$

Sol

Q. 4 Solution of the differential equation $x^2 \frac{d^2 y}{dx^2} + 8x \frac{dy}{dx} + 13y = \log x$ **is given by**

अंतर समीकरण $\frac{x^2(d^2y)}{dx^2} + 8x \frac{dy}{dx} + 13y = \log x$ **का हल निम्न प्रकार दिया गया है**

a) $x^{-7/2} \left[c_1 \cos \left(\frac{\sqrt{3}}{2} \log x \right) + c_2 \sin \left(\frac{\sqrt{3}}{2} \log x \right) \right] + \frac{1}{169} (\log x)$

b) $x^{-7/2} \left[c_1 \cos \left(\frac{\sqrt{3}}{2} \log x \right) + c_2 \sin \left(\frac{\sqrt{3}}{2} \log x \right) \right] + (\log x)$

c) $-x^{7/2} \left[c_1 \cos \left(\frac{\sqrt{3}}{2} x \right) + c_2 \sin \left(\frac{\sqrt{3}}{2} x \right) \right] + \frac{1}{169} (13 \log x - 7)$

d) None of these

Bel

Q. 5 Solution of the differential equation $(x + a)^2 \frac{d^2 y}{dx^2} - 4(x + a) \frac{dy}{dx} + 6y = x$

is given by

अंतर समीकरण $(x + a)^2 \frac{d^2 y}{dx^2} - 4(x + a) \frac{dy}{dx} + 6y = x$ **का हल निम्न द्वारा दिया गया है**

a) $y = c_1(x + a)^2 + c_2(x + a)^3 + \frac{1}{6}(3x - 2a)$

b) $y = c_1(x + a)^2 + c_2(x + a)^3 - \frac{1}{6}(3x + 2a)$

c) $y = c_1(x + a)^2 + c_2(x + a)^3 - \frac{1}{6}(3x - 2a)$

d) $y = c_1(x + a)^2 + c_2(x + a)^3 + \frac{1}{6}(3x + 2a)$

Self