Method to find Complementary Function from Auxiliary Equation

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Nature of roots	$C \cdot F$
Real and different Like m_1 and m_2	$C.F = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
Real and equal Like $m_1 = m_2 (= m)$	$C.F = c_1 e^{mx} + x c_2 e^{mx}$
Complex in pairs Like $\alpha \pm i\beta$	$\$C.F = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)\$$
Complex and repeated Like $\alpha \pm i\beta$ and $\alpha \pm i\beta$	$C.F = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x) + x e^{\alpha x}(c_3 \cos \beta x + c_4 \sin \beta x)$

Q. Solution of the differential Equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ **is given by**

a) $y = (c_1 + c_2x)e^{-2x}$

b) $y = (c_1 + c_2x)e^{x/2}$

c) $y = (c_1 + c_2x)e^{4x}$

d) $y = (c_1 + c_2x)e^{2x}$

$(D^2 - 4D + 4)y = 0$

$D^2 - 4D + 4 = 0$

$(D-2)^2 = 0$

$D = 2, 2$

$C.F \Rightarrow c_1e^{2x} + xc_2e^{2x}$

$P.I \Rightarrow \frac{1}{f(D)} \times x$

$P.I = 0$

Complete sol^n = C.F + P.I

$\Rightarrow e^{2x}(c_1 + xc_2)$

Q. Solution of the differential Equation $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = 0$ is given by

a) $y = c_1e^{-x} + c_2e^{-2x} + c_3e^{3x}$

b) $y = c_1e^{x} + c_2e^{-2x} + c_3e^{3x}$

c) $y = c_1e^{-x} + c_2e^{2x} + c_3e^{3x}$

d) $y = c_1e^{-x} + c_2e^{-2x} + c_3e^{-3x}$

$(D^3 - 7D - 6)\,y = 0$

$D^3 - 7D - 6 = 0$

$(D+1) \overline{)\, D^3 - 7D - 6 \,(\, D^2 - D - 6}$

$-D^3 + D^2$

$-D^2 - 7D - 6$

$-D^2 - D$

$-6D - 6$

$-6D - 6$

0

$(D+1)(D^2 - D - 6) = 0$

3×2

$(D+1)(D-3)(D+2) = 0$

$D = -1, -2, 3$

Q. Solution of the differential Equation $\frac{d^4y}{dx^4} + 4y = 0$ is given by

$(D^4 + 4)y = 0$

a) $y = e^{-x}(c_1 \cos x + c_2 \sin x) + e^{-x}(c_3 \cos x + c_4 \sin x)$

b) $y = e^{-x}(c_1 \cos x + c_2 \sin x) + e^{x}(c_3 \cos x + c_4 \sin x)$

c) $y = e^{x}(c_1 \cos x + c_2 \sin x) + e^{x}(c_3 \cos x + c_4 \sin x)$

d) $y = e^{x}(c_1 \cos x + c_2 \sin x) + e^{-x}(c_3 \cos x + c_4 \sin x)$

$(D^2)^2 + 2^2 = 0$

$(D^2+2)^2 - 4D^2 = 0$

$(D^2+2)^2 - (2D)^2 = 0$

$(D^2+2-2D)(D^2+2+2D) = 0$

$D = \frac{2 \pm \sqrt{4-8}}{2}$

$D = \frac{2 \pm 2i}{2} \Rightarrow 1 \pm i$

$D = \frac{-2 \pm \sqrt{4-8}}{2}$

$D = \frac{-2 \pm 2i}{2} \Rightarrow -1 \pm i$

Q. Solution of the differential Equation $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 3y = e^{2x}$ is given by

a) $y = c_1 e^{-\frac{x}{2}} + c_2 e^{-\frac{3}{2}x} + \frac{1}{21} e^{2x}$

b) $y = c_1 e^{-x/2} + c_2 e^{\frac{3}{2}x} + \frac{1}{21} e^{-2x}$

c) $y = c_1 e^{x/2} + c_2 e^{-\frac{3}{2}x} + \frac{1}{21} e^{2x}$

d) $y = c_1 e^{x/2} + c_2 e^{-\frac{3}{2}x} + \frac{1}{27} e^{2x}$

$(4D^2 + 4D - 3) = 0$

$(4D^2 + 6D - 2D - 3) = 0$

$2D(2D+3) - 1(2D+3) = 0$

$(2D-1)(2D+3) = 0$

$D = \frac{1}{2}, \quad D = -3/2$

$PI \Rightarrow \frac{1}{f(D)} \cdot x$

$\Rightarrow \frac{1}{4\times4 + 4\times2 - 3} \times e^{2x}$ (D → 2)

$= \frac{1}{24 - 3} = \frac{1}{21}.$

f(D)

$d = 3$

$x \leftarrow e^{3x}$

Q. Solution of the differential Equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y =$ is given by

$D^2 - 6D + 9 = 0$

$(D-3)^2 = 0$

$D = 3, 3$

a) $y = (c_1 + c_2x)e^x + \frac{x^2}{2}e^{3x}$

b) $y = (c_1 + c_2x)e^{3x} + \frac{x^2}{2}e^{3x}$

c) $y = (c_1 + c_2x)e^{3x} - \frac{x^2}{3}e^{3x}$

d) $y = (c_1 + c_2x)e^x - \frac{x^2}{2}e^{3x}$

P.I $\Rightarrow \frac{1}{f(D)} \times x$

$\frac{1}{D^2 - 6D + 9} \times e^{3x}$

$\frac{x \cdot e^{3x}}{2D - 6}$

$\frac{x^2 \cdot e^{3x}}{2}$

Case of failure $\leftarrow$ $f(D) = 0$

Q. Solution of the differential Equation $\frac{d^2y}{dx^2} - 4y = e^x +$ $\sin 2x$ **is given by**

a) $y = c_1e^{2x} + c_2e^{-2x} + \frac{1}{3}e^x + \frac{1}{8}\sin 2x$

✗ **b)** $y = c_1e^{2x} + xc_2e^{2x} - \frac{1}{3}e^x - \frac{1}{8}\sin 2x$

c) $y = c_1e^{2x} + c_2e^{-2x} - \frac{1}{3}e^x - \frac{1}{8}\sin 2x$

✗ **d)** $y = c_1e^{2x} + xc_2e^{2x} + \frac{1}{3}e^x + \frac{1}{8}\sin 2x$

$D^2 - 4 = 0$

$D = 2, -2$

$PI \Rightarrow \frac{1}{D^2-4} \times (e^x + \sin 2x)$

$D \leftarrow e^x$, $D^2 \Rightarrow -4$

$\frac{e^x}{D^2-4} + \frac{\sin 2x}{D^2-4}$

$\frac{e^x}{-3} - \frac{\sin 2x}{8}$

Q. Solution of the differential Equation $\frac{d^2y}{dx^2} - 4y = x^2$ is given by

a) $y = c_1e^{2x} + c_2e^{-2x} - \frac{1}{4}\left(x^2 + \frac{1}{2}\right)$

b) $y = c_1e^{2x} + c_2e^{-2x} - \frac{1}{4}\left(x^2 - \frac{1}{2}\right)$

c) $y = c_1e^{2x} + c_2e^{-2x} + \frac{1}{4}\left(x^2 - \frac{1}{2}\right)$

d) $y = c_1e^{2x} + c_2e^{-2x} + \frac{1}{4}\left(x^2 + \frac{1}{2}\right)$

$D^2 - 4 = 0$

$D = \pm 2$

$PI = \frac{1}{D^2-4} \times x^2$

$= \frac{1}{-4\left(1 - \frac{D^2}{4}\right)} \times x^2$

$= \frac{-1}{4}\left(1 - \frac{D^2}{4}\right)^{-1} \cdot x^2$

$= -\frac{1}{4}\left[1 + \frac{D^2}{4} + \frac{D^4}{16} + \cdots \infty\right] \cdot x^2$

$=$

Q. Solution of the differential Equation $\frac{d^2y}{dx^2} + y = x \cdot e^{2x}$ is given by

a) $c_1 \cos 2x + c_2 \sin 2x + \frac{1}{5} e^{2x} \left(x - \frac{4}{5}\right)$

b) $c_1 \cos x + c_2 \sin x + \frac{1}{5} e^{2x} \left(x + \frac{4}{5}\right)$

c) $c_1 \cos x + c_2 \sin x + \frac{1}{2} e^{5x} \left(x - \frac{4}{5}\right)$

d) None of these

self

Q. Solution of the differential Equation $\frac{d^2y}{dx^2} + 4y = x \cdot \sin x$ **is given by**

a) $y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{3} x \sin x - \frac{2}{9} \cos x$

b) $y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{3} x \sin x + \frac{2}{9} \cos x$

c) $y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{3} x \sin x - \frac{2}{9} \cos x$

d) None of these

Q. Solution of the differential Equation $\frac{d^2y}{dx^2} + y = \sin x \cdot \sin 2x$ **is given by**

a) $y = c_1 \cos x + c_2 \sin x - \frac{1}{4} x \sin x - \frac{1}{16} \cos 3x$

b) $y = c_1 \cos x + c_2 \sin x + \frac{1}{4} x \sin x + \frac{1}{16} \cos 3x$

c) $y = c_1 \cos x + c_2 \sin x - \frac{1}{16} x \sin x + \frac{1}{4} \cos 3x$

d) None of these