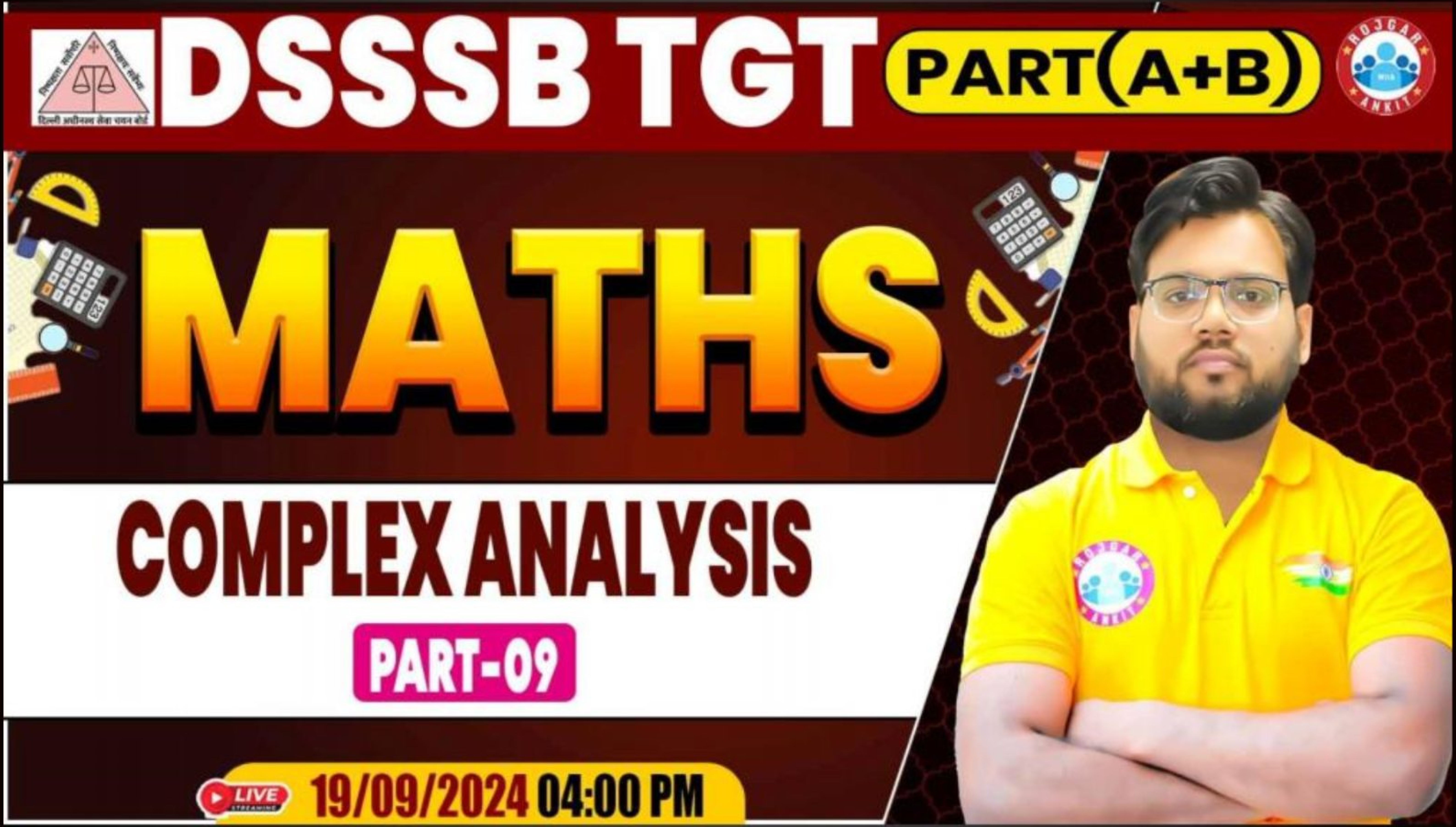
DSSSB TGT
PART(A+B)
MATHS
COMPLEX ANALYSIS
PART-09
LIVE
19/09/2024 04:00 PM

Complex Analysis

Algebraic Number: - A complex number is said to be Algebraic number if it satisfies a non-zero polynomial over Q.

Transcendental Number: - Non -algebraic numbers are said to be transcendental number.

Q. Which of the following numbers is an algebraic number?

A. π

B. e

C. $\sqrt{2}$

D. $\ln(2)$

Q. Which of the following statements is true?

A, All transcendental numbers are algebraic.

B. All algebraic numbers are transcendental.

C. Some algebraic numbers are transcendental.

D. No algebraic number is transcendental.

Complex Analysis

(1) $|z-a|=r$

(2) $|z-a|<r$

(3) $|z-a|\leq r$

(4) $|z-a|>r$

(5) $|z-a|\geqslant r$

Complex Analysis

Analytic Function or Regular Function or Holomorphic Function

Analytic Function:- A complex valued function $f(z)$ is said to be analytic at point $z = z_0$ if $f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z)-f(z)}{\delta z}$ exists & is unique at z_0

Analytic Function: A single valued function $f(z)$ is said to be andytyic in a region R, if it is differentiableat each point of R.

(C-R-Equation | Cauchy Riemann Equation)

For a function $f(z)$ to be analytic in a region R

Necessary: If $f(z)$ is analytic at a point, then u and v must satisfy the Cauchy-Riemann equations at that point.

Sufficient: If u and v satisfy the Cauchy-Riemann equations and the partial derivatives of u and v are continuous, then $f(z)$ is analytic in that region

Q. Let $f(z) = \frac{1}{z}, z \neq 0$, then

1. f does not satisfy Cauchy - Riemann equation for all $z \neq 0$
2. f is continuous only but nowhere differentiable
3. f satisfies Cauchy - Riemann equation, but not analytic for all $z \neq 0$
4. f is analytic for all $z \neq 0$,

Q. The Complex Function $f(z) = z^2$ is?

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None of these

$f(z) = (x+iy)^2$

$\Rightarrow x^2 - y^2 + 2xyi$

$\frac{\partial u}{\partial x} = 2x$ $\qquad$ $\frac{\partial v}{\partial y} = 2x$

$\frac{\partial u}{\partial y} = -2y$ $\qquad$ $\frac{\partial v}{\partial x} = 2y$

$\frac{\partial^2 u}{\partial x^2} = 2$

$+$

$\frac{\partial^2 u}{\partial y^2} = -2$

0

Q. $f(z) = (|z|)^2$ **is analytic or not?**

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

$f(z) = |z|^2$

$f(z) = \underbrace{x^2+y^2}_{u} + i\cdot\underbrace{0}_{v}$

$z = x+iy$

$|z| = \sqrt{x^2+y^2}$

$|z|^2 = x^2+y^2$

$u_x \neq v_y$

$2x \neq 0$

$u_x = v_y$

$u_y = -v_x$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Q. $f(z) = 2xy + i(x^2 - y^2)$

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

$$f(z) = \underbrace{2xy}_{u} + i\underbrace{(x^2 - y^2)}_{v}$$

$$\frac{\partial u}{\partial x} = 2y \neq \frac{\partial v}{\partial y} = -2y$$

Q. $f(z) = z^3$ $\quad x+iy \quad a+ib$

(a) Analytic ✓

(b) Not analytic

(c) May be analytic

(d) None

$$(a+b)^3 = a^3+b^3+3ab(a+b)$$
$$\Rightarrow a^3+b^3+3a^2b+3ab^2$$

$z = x+iy$ $\qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$z^3 = (x+iy)^3$ $\qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$$= x^3 + i^3y^3 + 3x^2iy + 3xi^2y^2$$

$$\Rightarrow x^3 - iy^3 + 3x^2yi - 3xy^2$$

$f(z) = z^3$

$$\Rightarrow \underbrace{x^3-3xy^2}_{u} + i\,\underbrace{(3x^2y - y^3)}_{v}$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 = \frac{\partial v}{\partial y} = 3x^2-3y^2 \checkmark$$

$$\frac{\partial u}{\partial y} = -6xy \quad = \quad \frac{\partial v}{\partial x} = 6xy \checkmark$$

$f(x+iy) = (x+iy)^3$

$= \underbrace{x^3 - 3xy^2}_{u} + i(3x^2y - y^3)$

$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$

$\frac{\partial^2 u}{\partial x^2} = 6x$

$+$

$\frac{\partial u}{\partial y} = -6xy$

$\frac{\partial^2 u}{\partial y^2} = -6x = 0$

Q. $f(z) = z^3$

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\Rightarrow a^3 + b^3 + 3a^2b + 3ab^2$

Harmonic

fun

$f(x+iy) = (x+iy)^3$

$= \underbrace{x^3 - 3xy^2}_{u} + i(3x^2y - y^3)$

$v = 3x^2y - y^3$

$\frac{\partial v}{\partial x} = 6xy$ $\frac{\partial v}{\partial y} = 3x^2 - 3y^2$

$\frac{\partial^2 v}{\partial x^2} = 6y + \frac{\partial^2 v}{\partial y^2} = -6y = 0$ Harmonic fun

Q. $f(z) = z^3$

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$\Rightarrow a^3 + b^3 + 3a^2b + 3ab^2$

$$f(z) = \underbrace{e^{-x}\cos y}_{u} + i\,\underbrace{e^{-x}\sin y}_{v}$$

$$\frac{\partial u}{\partial x} = -e^{-x}\cos y \neq \frac{\partial v}{\partial y} = e^{-x}\cos y$$

Q. $f(z) = e^{-x}(\cos y + i\sin y)$

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

$f(z) = \sin z$

$f(x+iy) = \sin(x+iy)$

$= \sin x \cdot \cos iy + \cos x \cdot \sin iy$

$= \underbrace{\sin x \cdot \cosh y}_{u} + i\,\underbrace{\cos x \cdot \sinh y}_{v}$

$\frac{\partial u}{\partial x} = \cos x \cdot \cosh y = \frac{\partial v}{\partial y} = \cos x \cdot \cosh y$

$\frac{\partial u}{\partial y} = \sin x \cdot \sinh y = -\frac{\partial v}{\partial x} = -\sin x \cdot \sinh y$

Q. $f(z) = \sin z$

~~(a) Analytic~~

(b) Not analytic

(c) May be analytic

(d) None

* $\cos i\theta = \cosh \theta$

$\sin i\theta = i \sinh \theta$

Harmonic function:→

C-R Eqⁿ→

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$U_x = V_y \Rightarrow U_{xx} = V_{xy}$$

$$U_y = -V_x \Rightarrow U_{yy} = -V_{xy}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x \partial y}$$

$$\frac{\partial^2 u}{\partial y^2} = -\frac{\partial^2 v}{\partial x \partial v}$$

$$U_{xx} + U_{yy} = V_{xy} - V_{xy}$$

$$\boxed{U_{xx} + U_{yy} = 0}$$

Laplace Eqⁿ

Harmonic Function

Q. $u = x^2 - y^2$

(a) Harmonic function

(b) Not Harmonic function

(c) May be Harmonic function

(d) None

$\frac{\partial u}{\partial x} = 2x$ $\qquad$ $\frac{\partial u}{\partial y} = -2y$

$\frac{\partial^2 u}{\partial x^2} = 2$ $\qquad$ $\frac{\partial^2 u}{\partial y^2} = -2$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2 - 2 = 0$$

Characteristics of Harmonic Functions in Complex Analysis

1. If $f(z) = u(x, y) + iv(x, y)$ is analytic in a region R, then both u and v are harmonic functions in R .
2. If $u(x, y)$ is harmonic in a connected region R, then u is the real part of an analytic function $f(z) = u(x, y) + iv(x, y)$.
3. If u and v are the real and imaginary parts of an analytic function, then u and v are considered as harmonic conjugates.

4.The sum of two harmonic functions results in another harmonic function.

5. An arbitrary pair of harmonic functions " u " and " v " may not necessarily be conjugates, unless $u+iv$ is an analytic function.

Q. $v = 3x^2y - y^3$

(a) Harmonic function

(b) Not Harmonic function

(c) May be Harmonic function

(d) None

H.W.