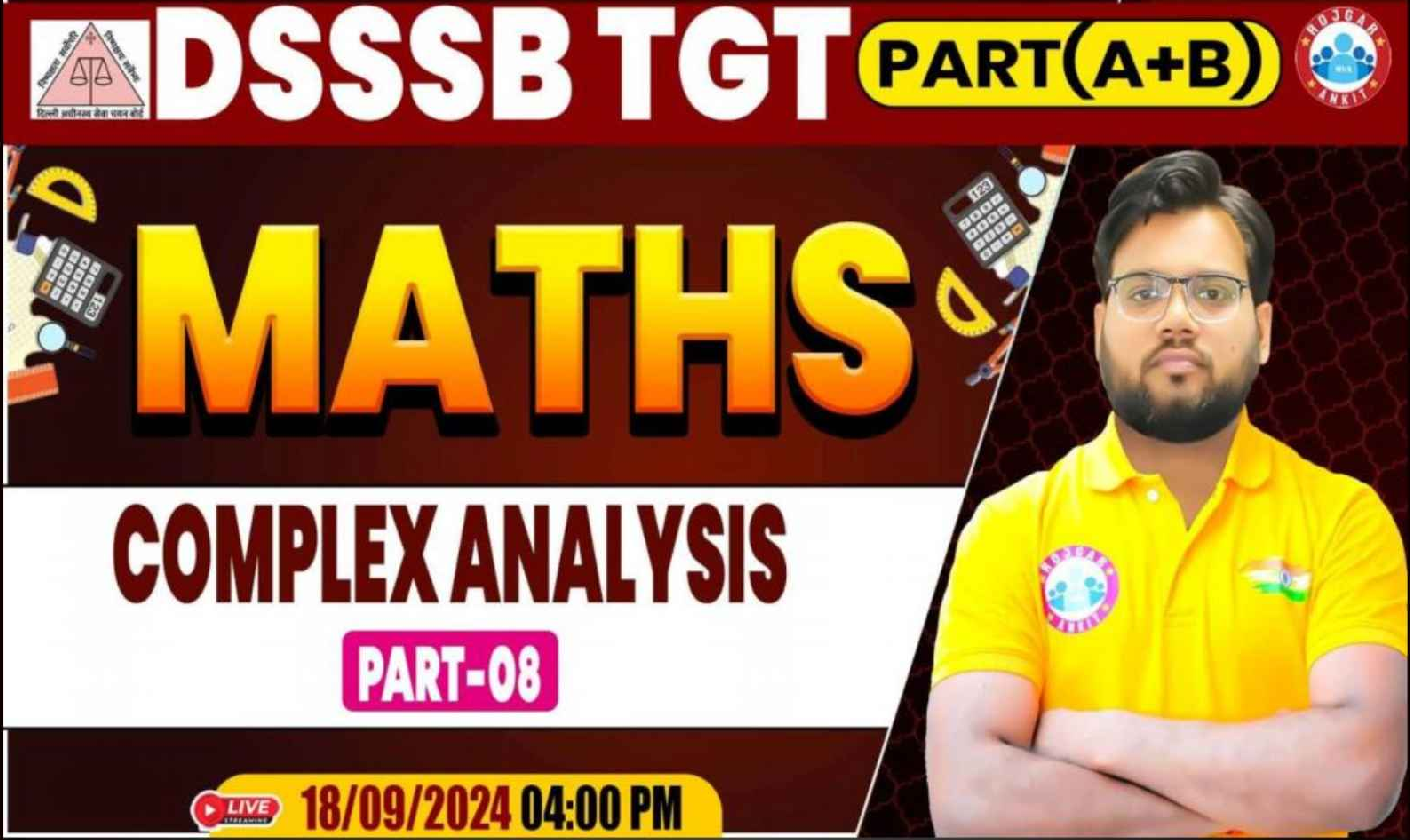


DSSSB TGT
PART(A+B)
MATHS
COMPLEX ANALYSIS
PART-08
LIVE
18/09/2024 04:00 PM

If ω is an imaginary cube root of unity, then $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ ---... 100 factors is equal to

1 -1, w, w^2

$1+\omega+\omega^2=0$

$1+\omega=-\omega^2$

$1+\omega^2=-\omega$

$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ - - - 100 factors

$(1+\omega)(1+\omega^2)$

$(1+\omega^{16})(1+\omega^{32})$

$(1+\omega)(1+\omega^2)$

$+\omega^2\cdot\omega$

ω^3

$1 \times 1 \times 1 \times 1 \times 1$ - -

$1^{50} = 1$

Rational Number ($\mathbb{Q}$) → $\frac{p}{q}$ $(q \neq 0)$
$p, q \in R$

$1, 2, \frac{3}{4}, \frac{4}{3}, \frac{11}{17}, \frac{22}{7}$

$1.555555\ldots$

$3.79999\ldots$

$1.73\underline{2}9$

$$\boxed{R = \mathbb{Q} \cup \mathbb{Q}^c}$$

$$\boxed{\mathbb{Q} \cap \mathbb{Q}^c = \phi}$$

IRrational No. ($\mathbb{Q}^c$) →

$\sqrt{2}, \sqrt{3}, \pi, e, \log x,$

$\sqrt{10}, 4.71892174195\ldots$

Algebraic No →

$+, \times, -, (\ )^n, n \in R$

Complex Analysis

Algebraic Number: - A complex number is said to be Algebraic number if it satisfies a non-zero polynomial over Q.

→ All the rational No. are algebraic No.

→ Some irrationals No are also algebraic No.

Transcendental Number: - Non -algebraic numbers are said to be transcendental number.

$\rightarrow \underbrace{\pi, e, \log x}$

$\times, -, \times, (\ )^n$, $n \in R$

$(\sqrt{2})^2 - 2 = 0$

$\frac{22}{7} \times 7 - 22 = 0$

$= (\sqrt{2}+\sqrt{5})^2 - 7$

$\cancel{7} + 2\sqrt{10} - \cancel{7}$

$(2\sqrt{10})^2 - 40 = 0$

Q. Which of the following numbers is an algebraic number?

A. π ✗

B. e ✗

C. $\sqrt{2}$ → $(\sqrt{2})^2 - 2 = 0$ ✓

D. $\ln(2)$ log 2

Q. Which of the following statements is true?

$\propto$ A, All transcendental numbers are algebraic.

$\propto$ ~~B~~. All algebraic numbers are transcendental.

$\propto$ ~~C~~. Some algebraic numbers are transcendental.

~~D.~~ No algebraic number is transcendental.

$$\boxed{\mathbb{C} = A \cup T}$$

$$\boxed{A \cap T = \phi}$$

Complex Analysis

Eq^n of the circle in complex field.

(1) $|z-a| = r \Rightarrow |z-a| = r \to$ radius

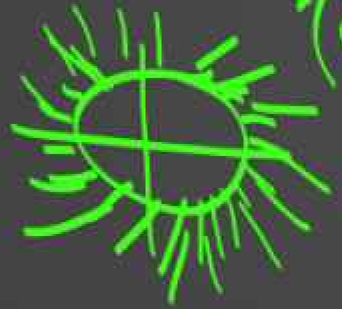
$a \to$ centre

(2) $|z-a| < r$ (open disc)

(3) $|z-a| \leq r$ (Closed disc)

(4) $|z-a| > r \to$ (Exterior without boundry disc)

(5) $|z-a| \geqslant r \to$ (Exterior with boundy disc)



Complex Analysis

Analytic Function or Regular Function or Holomorphic Function

Analytic Function:- **A complex valued function $f(z)$ is said to be analytic at point $z = z_0$ if $f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z) - f(z)}{\delta z}$ exists & is unique at z_0**

In Real Analysis → Lim, Continuity, differentiability

In Complex Analysis → Analytic fun.

Analytic Function: A single valued function $f(z)$ is said to be analytyic in a region R, if it is differentiableat each point of R.

(C-R-Equation | Cauchy Riemann Equation)

For a function $f(z)$ to be analytic in a region R

Necessary: If $f(z)$ is analytic at a point, then u and v must satisfy the Cauchy-Riemann equations at that point.

$$f(z) = \underbrace{u}_{\text{Real Part}} + i\underbrace{v}_{\text{Imaginary Part}}$$

$$\left[\begin{aligned} u_x &= v_y \\ u_y &= -v_x \end{aligned}\right] \Rightarrow \left[\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} &= -\frac{\partial v}{\partial x} \end{aligned}\right]$$

Sufficient: If u and v satisfy the Cauchy-Riemann equations and the partial derivatives of u and v are continuous, then $f(z)$ is analytic in that region

$$u_x, \, u_y, \, v_x, \, v_y$$

$$\frac{\partial u}{\partial x}, \, \frac{\partial u}{\partial y}, \, \frac{\partial v}{\partial x}, \, \frac{\partial v}{\partial y}$$

$f(z) = \frac{1}{z}$

$z = x + iy$

$\Rightarrow \frac{1}{x+iy} \times \frac{x-iy}{x-iy}$

$f(z) = \frac{x-iy}{x^2+y^2}$

$f(z) = \underbrace{\frac{x}{x^2+y^2}}_{u} + i \underbrace{\frac{-y}{x^2+y^2}}_{v}$

Q. Let $f(z) = \frac{1}{z}, z \neq 0$, then

1. f does not satisfy Cauchy - Riemann equation for all $z \neq 0$

2. f is continuous only but nowhere differentiable

3. f satisfies Cauchy - Riemann equation, but not analytic for all $z \neq 0$

4. f is analytic for all $z \neq 0$,

$u_x = \frac{\partial u}{\partial x} = v_y = \frac{\partial v}{\partial y}$

$\Rightarrow \frac{1 \cdot (x^2+y^2) - x(2x)}{(x^2+y^2)^2}$

$\Rightarrow \frac{y^2-x^2}{(x^2+y^2)^2}$

$\Rightarrow -\left[\frac{1 \cdot (x^2+y^2) - y \cdot 2y}{(x^2+y^2)^2}\right]$

$= \frac{-(x^2-y^2)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$

(ii) $U_y = -V_x$

$\frac{\partial u}{\partial y}$

$\Rightarrow \frac{0 - x(2y)}{(x^2+y^2)^2}$

$\Rightarrow \frac{-2xy}{(x^2+y^2)^2}$

$\frac{\partial v}{\partial x}$

$\Rightarrow \frac{0 + y \cdot 2x}{(x^2+y^2)^2}$

$\Rightarrow \frac{2xy}{(x^2+y^2)^2}$

$U_y = -V_x$

Q. The Complex Function $f(z) = z^2$ is?

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None of these

$$f(x+iy) = (x+iy)^2$$

$$\Rightarrow x^2 - y^2 + 2xyi$$

$$\Rightarrow \underbrace{x^2 - y^2}_{u} + \underbrace{2xy}_{v}\, i$$

$$\frac{\partial u}{\partial x} = 2x = \frac{\partial v}{\partial y} = 2x$$

$$\frac{\partial u}{\partial y} = -2y = -\frac{\partial v}{\partial x} = -2y$$

Necessory Condition

$$\Rightarrow \left.\begin{array}{l} U_x = V_y \\ U_y = -V_x \end{array}\right\}$$

Q. $f(z) = (|z|)^2$ **is analytic or not?**

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

H.W.

Q. $f(z) = 2xy + i(x^2 - y^2)$

(a) Analytic

(b) Not analytic

(c) May be analytic

(d) None

$$f(z) = \underbrace{2xy}_{u} + i\underbrace{(x^2 - y^2)}_{v}$$

Necessary Condition $\Rightarrow$

$$U_x = V_y$$

$$U_y = -V_x$$

$$\frac{\partial u}{\partial x} = 2y \neq \frac{\partial v}{\partial y} = -2y$$