



DSSSB TGT

PART(B)



MATHS

ORDINARY DIFFERENTIAL EQUATION

PART-02



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Q. Solution of the differential equation $\frac{dy}{dx} + \frac{ax+hy+g}{hx+by+f} = 0$ is

अंतरसमीकरण $\frac{dy}{dx} + \frac{ax+hy+g}{hx+by+f} = 0$ का हल है

✓ (a) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

(b) $ax^2 + 2hxy + by^2 = 0$

(c) $2gx + 2fy + c = 0$

(d) None of these

$$\underbrace{(hx+by+f)}_N \cdot dy + \underbrace{(ax+hy+g)}_M \cdot dx = 0$$

$$\frac{dN}{dx} = \frac{dM}{dy}$$

$$h = h$$

Solⁿ $\int M \cdot dx + \int N \cdot dy = c$
y.constant x not contains.

$$\int (ax+hy+g) dx + \int (by+f) dy$$

$$\frac{ax^2}{2} + hy \cdot x + gx + \frac{by^2}{2} + fy = c$$

$$ax^2 + 2hxy + 2gx + by^2 + 2fy = c$$

Q. Solution of the differential equation $x(1+y^2)dx + y(1+x^2)dy = 0$ **is given by**

अंतर समीकरण $x(1+y^2)dx + y(1+x^2)dy = 0$ **का हल**

(a) $x^2 + (x^2 - 1)y^2 = 2c$

✓ (b) $x^2 + (x^2 + 1)y^2 = 2c$

(c) $x^2 - (x^2 + 1)y^2 = 2c$

(d) $x^2 - (x^2 - 1)y^2 = 2c$

$$\frac{dM}{dy} = \frac{dN}{dx}$$

$$2xy = 2xy$$

$$\int M \cdot dx + \int N \cdot dy = c$$

$y \text{ const}$ $x \text{ not const.}$

$$\int (x + xy^2) dx + \int y \cdot dy = c$$

$$\frac{x^2}{2} + \frac{x^2 y^2}{2} + \frac{y^2}{2} = c$$

$$x^2 + (x^2 + 1)y^2 = 2c //$$

Q. Solution of the differential equation $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$ is given by

अंतर समीकरण $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$ का हल "द्वारा दिया गया है

(a) $x^4 + 6x^2y^3 + y^4 = c$

✓ (b) $x^4 + 6x^2y^2 + y^4 = c$

(c) $x^4 + 6x^3y^2 + y^4 = c$

(d) $x^4 - 6x^2y^2 - y^4 = c$

$$\frac{dM}{dy} = \frac{dN}{dx}$$

$$6xy = 6xy$$

Solⁿ

$$\int M dx + \int N dy = c$$

$y \text{ const.} \quad x \text{ not const.}$

$$\int (x^3 + 3xy^2) dx + \int y^3 dy = c$$

$$\frac{x^4}{4} + 3y^2 \frac{x^2}{2} + \frac{y^4}{4} = c$$

$$x^4 + y^4 + 6x^2y^2 = 4c - c$$

Q. Solution of the differential equation $x^2 \left(\frac{dy}{dx}\right)^2 + xy \frac{dy}{dx} - 6y^2 = 0$ is given by

अंतर समीकरण $x^2 \left(\frac{dy}{dx}\right)^2 + xy \frac{dy}{dx} - 6y^2 = 0$ का हल "द्वारा दिया गया है

(a) $(yx^3 + c) \left(\frac{y}{x} - c\right) = 0$

(b) $(yx^3 - c) \left(\frac{x}{y^2} + c\right) = 0$

(c) $(yx^2 + c) \left(\frac{y}{x^2} + c\right) = 0$

✓ (d) $(yx^3 - c) \left(\frac{y}{x^2} - c\right) = 0$

let $\frac{dy}{dx} = p$

$x^2 p^2 + xyp - 6y^2 = 0$

3×2

$x^2 p^2 + 3xyp - 2xyp - 6y^2 = 0$

$xp(xp + 3y) - 2y(px + 3y) = 0$

$(xp - 2y)(xp + 3y) = 0$

Case I

$xp = 2y$

$\frac{dy}{dx} = \frac{2y}{x}$

$\int \frac{dy}{2y} = \int \frac{dx}{x}$

$\frac{1}{2} \log y = \log x + c$

$\log y - \log x^2 = \log c$

$\frac{y}{x^2} = c$

$\frac{y}{x^2} - c = 0$

Case II

$$x^3 + 3y = 0$$

$$\frac{dy}{dx} = \frac{-3x}{x^3}$$

$$-\frac{1}{3} \int \frac{dx}{x} = \int \frac{dy}{y}$$

$$-\frac{1}{3} \log x = \log y + \log c$$

$$\log y = -3 \log x + \log c$$

$$\log y + \log x^3 = \log c$$

$$yx^3 - c = 0$$

Q. Solution of the differential equation $4p^2x - (3x - a)^2 = 0$ is given by

अंतर समीकरण $4p^2x - (3x - a)^2 = 0$ का हल निम्न प्रकार दिया गया है

(a) $(y - c)^2 = x(x + 3a)^2$

(b) $(y - c)^2 = x(x - 3a)^2$

(c) $(y - c)^2 = x(x - a)^2$

(d) None of these

$$4p^2x = (3x - a)^2$$

$$p^2 = \frac{(3x - a)^2}{4x}$$

$$p^2 - \frac{(3x - a)^2}{4x} = 0$$

$$\left(p - \frac{3x - a}{2\sqrt{x}}\right) \left(p + \frac{3x - a}{2\sqrt{x}}\right) = 0$$

Case I

$$\frac{dy}{dx} = \frac{3x - a}{2\sqrt{x}}$$

$$\int dy = \int \frac{3}{2}\sqrt{x} - \frac{a}{2}x^{-1/2} dx$$

$$y = \frac{3}{2} \times \frac{x^{3/2}}{3/2} - \frac{a}{2} \times \frac{x^{1/2}}{1/2} + c$$

$$y - c = x^{1/2} [x - a]$$

S.B.S

$$(y - c)^2 = x(x - a)^2 //$$

Q. The solution of the differential equation $xp^2 + (y - x)p - y = 0$ is given by

अंतर समीकरण $xp^2 + (y - x)p - y = 0$ का हल निम्न प्रकार दिया गया है

(a) $(y - 2x - c)(yx - c) = 0$

(b) $(y - x - c)(2yx - c) = 0$

(c) $(y + x - c)(yx - c) = 0$

~~(d)~~ $(y - x - c)(yx - c) = 0$

$$D = \frac{-(y-x) \pm \sqrt{(y-x)^2 + 4xy}}{2x}$$

$$D = \frac{-y+x \pm \sqrt{x^2+y^2}}{2x}$$

$$\frac{-y+x+x+y}{2x} \Rightarrow \frac{2x}{2x} = 1$$

$$\frac{-y+x-x-y}{2x} = -\frac{y}{x}$$

Case I $p = -\frac{y}{x}$

$$\frac{dy}{dx} = -\frac{y}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\log y = -\log x + \log c$$

$$\log x + \log y = \log c$$

$$\log xy = \log c$$

$$xy - c = 0$$

Case II $p = 1$

$$\frac{dy}{dx} = 1$$

$$\int dy = \int dx$$

$$y = x + c$$

$$y - x - c = 0$$

multiply

Q. The solution of the differential equation $p^2 - 2p \cosh x + 1 = 0$ is given by

अंतर समीकरण $p^2 - 2p \cosh x + 1 = 0$ का हल निम्न प्रकार दिया गया है

(a) $(y + e^x - c)(y + e^{-x} - c) = 0$

(b) $(y - e^x - c)(y - e^{-x} - c) = 0$

(c) $(y + e^x - c)(y - e^{-x} - c) = 0$

☒ (d) $(y - e^x - c)(y + e^{-x} - c) = 0$

$$p^2 - 2p \left(\frac{e^x + e^{-x}}{2} \right) + 1 = 0$$

$$p^2 - (e^x + e^{-x})p + 1 = 0$$

$$p = \frac{e^x + e^{-x} \pm \sqrt{(e^x + e^{-x})^2 - 4}}{2 \times 1}$$

$$p = \frac{e^x + e^{-x} \pm \sqrt{(e^x - e^{-x})^2}}{2}$$

$$p \Rightarrow \frac{e^x + e^{-x} + e^x - e^{-x}}{2}$$

$$p = \frac{e^x + e^{-x} - e^x + e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$p \Rightarrow e^x$ | $p = e^{-x}$

$$\int dy = \int e^x dx \quad \bigg| \quad \int dy = \int e^{-x} \cdot dx$$

$$y = e^x + c$$

$$\boxed{y - e^x - c = 0}$$

$$y = -e^{-x} + c$$

$$\boxed{y + e^x - c = 0}$$

Q. The solution of the differential equation $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$ is given by

अंतर समीकरण $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$ का हल निम्न प्रकार दिया गया है

(a) $(y - cx^2)(y^2 + 3x^2 - c) = 0$

(b) $(y - cx^2)(y^2 - 3x^2 - c) = 0$

(c) $(y - cx^2)(y^3 + 3x^2 - c) = 0$

(d) $(y - cx^2)(y^2 + 3x^3 - c) = 0$

Self Solved