DSSSB TGT PART(B)

MATHS

Partial Differential Equation

PART-03

LIVE STREAMING 14/09/2024 08:00 AM

Partial differential Equation

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Questions

Q. If $u = xyf\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

यदि $u = xyf\left(\frac{y}{x}\right)$ तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ बराबर है

(a) u

(b) $4u$

(c) 0

(d) $2u$

$u = x^2 \frac{y}{x} f\left(\frac{y}{x}\right)$

$f(u)$

$n \cdot \frac{f(u)}{f'(u)} \Rightarrow 2 \times \frac{y}{x}$

$= 2u$ //

Q. If $z = u^2 + v^2, u = r\cos\theta, v = r\sin\theta$, then $\frac{\partial z}{\partial r}$ is equal to

यदि $z = u^2 + v^2, u = r\cos\theta, v = r\sin\theta$, तो $\frac{\partial z}{\partial r}$ बराबर है

(a) r

(b) $2r$

(c) $\frac{r}{2}$

(d) 0

$z = r^2\cos^2\theta + r^2\sin^2\theta$

$z = r^2(\cos^2\theta + \sin^2\theta)$

$z = r^2$

$\frac{dz}{dr} = 2r$ //

Q. If $u = \phi(y + ax) + \psi(y - ax)$ then $\frac{\partial^2 u}{\partial x^2} - a^2 \frac{\partial^2 u}{\partial y^2}$ s equal to

यदि $u = \phi(y + ax) + \psi(y - ax)$ तो $\frac{\partial^2 u}{\partial x^2} - a^2 \frac{\partial^2 u}{\partial y^2}$ बराबर है

(a) u

(b) 0

(c) a

(d) None of these

$$\frac{dy}{dx} = \phi'(y+ax) \times a - \psi'(y-ax) \times a$$

$$\frac{d^2y}{dx^2} = \phi''(y+ax) \times a^2 + \psi'(y-ax) \times a^2 \quad \text{— (1)}$$

$$\frac{dy}{dy} \Rightarrow \phi'(y+ax) \cdot + \psi'(y-ax)$$

$$-a^2 \frac{d^2y}{dy^2} = -a^2 \phi''(y+ax) + \psi''(y-ax) \times -a^2 \quad \text{— (2)}$$

(1) + (2)

Q. If $u = \sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

यदि $u = \sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)$, **तो** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ **बराबर है**

Euler partial diff.

(a) 0

(b) 2

(c) 3

(d) None of these

$$\sin u = \frac{x^2\left(1-\frac{y^2}{x^2}\right)}{x^2\left(1+\frac{y^2}{x^2}\right)}$$

$$\sin u = x^0 \cdot f\left(\frac{y}{x}\right)$$

$\downarrow$

$f(u)$.

$$= n\cdot\frac{f(u)}{f'(u)}$$

$$\Rightarrow n\cdot\frac{f(u)}{f'(u)} \Rightarrow 0 \times \frac{\sin u}{\cos u} = 0//$$

Q. If $u = x^3 + y^3 + z^3 + 3xyz$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to

यदि $u = x^3 + y^3 + z^3 + 3xyz$, तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ बराबर है

(a) $2u$

(b) $3u$

(c) $4u$

(d) None of these

$$u = x^3\left(1 + \frac{y^3}{x^3} + \frac{z^3}{x^3} + 3\frac{y}{x}\cdot\frac{z}{x}\right)$$

$$u = x^3 f\left(\frac{y}{x}, \frac{z}{x}\right)$$

$f(u)$

$\Rightarrow nu$

$= 3 \times u$

$= 3u$

n.u.

Q. If $u = xy\tan^{-1}\left(e^{x/y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

यदि $u = xy\tan^{-1}\left(e^{x/y}\right)$, तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ बराबर है

(a) 0

(b) u

(c) $3u$

(d) $2u$

$u = x^2 \frac{y}{x} \tan^{-1}\left(e^{-y/x}\right)$

$\hookrightarrow n \cdot \frac{f(u)}{f'(u)}$

$u = x^n \cdot f\left(\frac{y}{x}\right)$

$f(u)$

$= 2x\frac{u}{x}$

$= 2u$ //

Result

if $u = x^n \cdot f\left(\frac{y}{x}\right)$.

$$x \cdot \frac{du}{dx} + y \frac{du}{dy} \Rightarrow n \cdot \frac{f(u)}{f'(u)}$$

$$x \frac{d^2u}{dx^2} + y \cdot \frac{d^2u}{dx \cdot dy} \Rightarrow (n-1) \cdot \frac{du}{dx}$$

$$y \cdot \frac{d^2u}{dx^2} + x \cdot \frac{d^2u}{dx \cdot dy} \Rightarrow (n-1) \frac{du}{dy}.$$

Q. If $u = x^{n-1} y f\left(\frac{y}{x}\right)$, then $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x\,\partial y}$ is equal to

यदि $u = x^{n-1} y f\left(\frac{y}{x}\right)$, **तो** $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x\,\partial y}$ **बराबर है**

$u = x^n \cdot \frac{y}{x} f\left(\frac{y}{x}\right)$

$= (n-1) \cdot \frac{dy}{dx}$

(a) nu

(b) $n(n-1)u$

(c) $(n-1)\frac{\partial u}{\partial x}$

(d) $(n-1)\frac{\partial u}{\partial y}$

Q. If $x = r\cos\theta, y = r\sin\theta$, then the value of $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}$ is equal to

यदि $x = r\cos\theta, y = r\sin\theta$, तो $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}$ का मान बराबर है

(a) 1

(b) 0

(c) $\frac{\partial z}{\partial x}$

(d) $\frac{\partial x}{\partial y}$

$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta}$

$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\frac{y}{x}$

$\frac{d\theta}{dx} = \frac{1}{1+\frac{y^2}{x^2}} \times \frac{-y}{x^2} \Rightarrow \frac{-y}{x^2+y^2}$

$\frac{d^2\theta}{dx^2} = \frac{0 + y \times 2x}{(x^2+y^2)^2} \Rightarrow \frac{+2xy}{(x^2+y^2)^2}$ — (1)

$\frac{d\theta}{dy} \Rightarrow \frac{x^2}{x^2+y^2} \times \frac{1}{x} = \frac{x}{x^2+y^2}$

$\frac{d^2\theta}{dy^2} = \frac{-x \times 2y}{(x^2+y^2)^2}$ — (2)

(1) + (2)

Q. If $F(u) = f(x, y, z)$ is the homogenous function of degree n in x, y, z and $u_x = \frac{\partial u}{\partial x}$ etc, then $xu_x + yu_y + zu_z$ is equal to

यदि F(u)=f(x,y,z) x,y,z में डिग्री n का समरूप फलन है और $u_x = \frac{\partial u}{\partial x}$ आदि, तो $xu_x + yu_y + zu_z$ बराबर है

$x \cdot \frac{dy}{dx} + y \cdot \frac{dy}{dy} + z \cdot \frac{dy}{dz} \longrightarrow n \cdot \frac{f(y)}{f'(y)}$ //

(a) $nF(u)$

(b) $n\frac{F'(u)}{F(u)}$

(c) $n\frac{F(u)}{F'(u)}$

(d) $\frac{F(u)}{F'(u)}$

Q. If $u = f(x-y, y-z, z-x)$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ is equal to

यदि u=f(x-y,y-z,z-x) तो $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ बराबर है

(a) 3

(b) 0

(c) 1

(d) None of these

a b c

Self solve

Q. If $u = \sin\left(\frac{y}{x}\right) + \tan\left(\frac{x}{y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

यदि $u = \sin\left(\frac{y}{x}\right) + \tan\left(\frac{x}{y}\right)$ **तो** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ **बराबर है**

(a) $3u$

(b) $2u$

(c) u

(d) 0

$u = x^0\left[\sin\left(\frac{y}{x}\right) + \tan\left(\frac{\frac{1}{y}}{\frac{1}{x}}\right)\right]$

$\hookrightarrow n \cdot \frac{f(u)}{f'(u)}$

$u = x^0 \cdot f\left(\frac{y}{x}\right)$

$f(u)$

$= 0 \times \frac{f(u)}{f'(u)}$

$= 0$

Q. If $z = x^2 + 2y^2, x = r\cos\theta\ y = r\sin\theta$ then $\frac{\partial z}{\partial \theta}$ is equal to

यदि $z = x^2 + 2y^2, x = r\cos\theta\ y = r\sin\theta$ तो $\frac{\partial z}{\partial \theta}$ बराबर है

(a) $r^2 \sin 2\theta$

(b) $2r^2 \sin 2\theta$

(c) $r^2 \cos 2\theta$

(d) $2r^2 \cos 2\theta$

$z = r^2\cos^2\theta + 2r^2\sin^2\theta$

$z = r^2[\cos^2\theta + \sin^2\theta + \sin^2\theta]$

$z = r^2[1 + \sin^2\theta]$

$\frac{dz}{d\theta} \Rightarrow r^2[0 + 2\sin\theta.\cos\theta]$

$\Rightarrow r^2.\sin 2\theta$