DSSSB TGT PART(B)

MATHS

Partial Differential Equation

PART-02

LIVE 13/09/2024 08:00 AM

Total Differential

If $u = f(x, y)$ is differentiable function, then the differential or total differential of u is defined by $\boxed{du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy}$ where dx and dy are independent variables.

यदि u=f(x,y) अवकलनीय फलन है, तो u का अंतर या कुल अंतर $du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$ द्वारा परिभाषित किया जाता है, जहाँ dx और dy स्वतंत्र चर हैं।

Composite Functions

If u is a function of the variables x, y and these variables x, y are themselves functions of another variable t, then u is said to be a composite function of the variable t.

यदि u चर x,y का फलन है और ये चर x,y स्वयं किसी अन्य चर t के फलन हैं, तो u को चर t का संयुक्त फलन कहा जाता है।

In other words,

The relations $u = f(x, y)$ and $x = \phi(t)$ and $y = \psi(t)$ define u as composite function of t.

संबंध u=f(x,y) और x=ϕ(t) तथा y=ψ(t) u को t के संयुक्त फलन के रूप में परिभाषित करते हैं।

Note:-

If u is a composite function of t defined by the relations $u = f(x, y)$ and $x = \phi(t)$ and $y = \psi(t)$, then

यदि u, t का संयुक्त फलन है जिसे u=f(x,y) और x=ϕ(t) तथा y=ψ(t) संबंधों द्वारा परिभाषित किया जाता है, तो

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$$

Example:-

Find the total derivative of u with respect to t, when $u = e^x \sin y$, where $x = \log t, y = t^2$

t के सापेक्ष u का कुल अवकलज ज्ञात करें, जब $u = e^x \sin y$, जहाँ $x = \log t, y = t^2$

$u = e^x \sin y$

$y = t^2 \rightarrow \frac{dy}{dt} = 2t$

$x = \log t$

$e^x = t$

$\frac{dx}{dt} = \frac{1}{t}$

$$\frac{du}{dt} = \frac{du}{dx}\cdot\frac{dx}{dt} + \frac{du}{dy}\cdot\frac{dy}{dt}$$

$$\frac{du}{dt} \Rightarrow e^x \sin y \times \frac{1}{t} + e^x \cos y \cdot 2t \Rightarrow \boxed{\frac{du}{dt} = \sin t^2 + 2t^2 \cos t^2}$$

($e^x \rightarrow t$, $\sin y \rightarrow t^2$, $e^x \rightarrow t$, $\cos y \rightarrow t^2$)

Ans

Q. If $x^2 + y^2 + z^2 = \frac{1}{u^2}$ then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ is equal to

यदि $x^2 + y^2 + z^2 = \frac{1}{u^2}$ तो $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ बराबर है

(a) 1

(b) 0

(c) $2u$

(d) None of these

$$u^{-2} = x^2 + y^2 + z^2$$

$$\frac{-2}{u^3} \cdot \frac{du}{dx} = 2x + 0 + 0$$

$$-2u^{-3} \cdot \frac{du}{dx} = 2x$$

$$\frac{du}{dx} = -xu^3$$

$$\frac{d^2u}{dx^2} = -x \cdot 3u^2 \frac{du}{dx} - u^3$$

$$\frac{d^2u}{dx^2} = -x \cdot 3u^2 \cdot x - xu^3 - u^3$$

$$\boxed{\frac{d^2u}{dx^2} = 3x^2u^5 - u^3}$$

$$\frac{d^2u}{dy^2} = 3y^2u^5 - u^3$$

$$\frac{d^2u}{dz^2} = 3z^2u^5 - u^3$$

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = u^3\left[3x^2u^2 + 3y^2u^2 + 3z^2u^2 - 3\right]$$

$$= u^3\left[3u^2(x^2 + y^2 + z^2) - 3\right]$$

$$\Rightarrow u^3 \times 0$$

$$= 0$$

Q. If $z = e^{ax+by} f(ax - by)$, then $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y}$ is equal to

यदि $z = e^{ax+by} f(ax - by)$, तो $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y}$ बराबर है

(a) abz

(b) $2ab$

(c) ab

(d) $2abz$

$$b\frac{dz}{dx} = e^{ax+by} \cdot f'(ax-by) \cdot ab + f(ax-by) \cdot e^{ax+by} \times ab$$

$$b\frac{dz}{dx} = ab\, e^{ax+by} f'(ax-by) + ab\, f(ax-by) \cdot e^{ax+by} \quad -(I)$$

$$a\frac{dz}{dy} = e^{ax+by} \cdot f'(ax-by) \times -b \cdot a + f(ax-by) \cdot e^{ax+by} \cdot a.b \quad -(II)$$

(I) + (II)

$$b\frac{dz}{dx} + a\frac{dz}{dy} \Rightarrow 2ab \cdot e^{ax+by} \cdot f(ax-by) \Rightarrow 2abz$$ //

Q. If $u = e^x(x\cos y - y\sin y)$ then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ is equal to

यदि $u = e^x(x\cos y - y\sin y)$ तो $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ बराबर है

(a) 0

(b) $e^x u$

(c) u

(d) 3

$$\frac{du}{dx} = e^x[\cos y - 0] + (x\cos y - y\sin y)e^x$$

$$\frac{d^2u}{dx^2} = e^x\cos y + \cos y[e^x + xe^x] - e^x y\sin y \quad \text{— (I)}$$

$$\frac{du}{dy} = -e^x \cdot x\sin y - e^x[y\cdot\cos y + \sin y] \quad \text{(I) + (II)}$$

$$\frac{d^2u}{dy^2} = -e^x x\cos y + e^x y\sin y - e^x\cos y - e^x\cos y \quad \text{— (II)}$$

Q. If $u = \log(\tan x + \tan y)$, then $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y}$ is equal to

यदि $u = \log(\tan x + \tan y)$, तो $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y}$ बराबर है

(a) u

(b) 0

(c) 2

(d) None of these

$$\sin 2x \frac{dy}{dx} = \frac{1 \times \sec^2 x}{\tan x + \tan y} \cdot 2 \sin x \cos x$$

$$\sin 2y \cdot \frac{dy}{dx} = \frac{2 \tan y}{\tan x + \tan y}$$

$$\sin 2x \cdot \frac{dy}{dx} = \frac{2 \tan x}{\tan x + \tan y} \rightarrow (+)$$

$$\frac{2(\tan x + \tan y)}{\tan x + \tan y}$$

$$= 2$$

Q. If $u = \sqrt{x^2 + y^2 + z^2}$ then $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2$ is equal to

यदि $u = \sqrt{x^2 + y^2 + z^2}$ तो $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2$ बराबर है

(a) 0

~~(b) 1~~

(c) $2u$

(d) u

$u^2 = x^2 + y^2 + z^2$

$2u \cdot \frac{dy}{dx} = 2x$

$\left(\frac{dy}{dx}\right)^2 = \left(\frac{x}{u}\right)^2$

✓ $\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{u^2}$

$\frac{x^2}{u^2} + \frac{y^2}{u^2} + \frac{z^2}{u^2} \Rightarrow \frac{x^2+y^2+z^2}{u^2} \Rightarrow \frac{u^2}{u^2} = 1$

Q. If $u = \sin^{-1}\frac{x^2+y^2}{x+y}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

यदि $u = \sin^{-1}\frac{x^2+y^2}{x+y}$ तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ बराबर है

(a) $\sin u$

(b) $\cos u$

(c) $\tan u$

(d) None of these

$\sin u = \frac{x^2+y^2}{x+y}$

$\sin u = \frac{x^2\left(1+\frac{y^2}{x^2}\right)}{x\left(1+\frac{y}{x}\right)}$

Euler

$\sin u = x^1 f\left(\frac{y}{x}\right)$

$f(u)$

$\Rightarrow 1.\frac{\sin u}{\cos u} = 1.\tan u$

Euler diHerenhia Equ^n.

$f(u) = x^n \cdot f\left(\frac{y}{x}\right)$

$x.\frac{du}{dx} + y.\frac{du}{dy} \Rightarrow n.\frac{f(u)}{f'(u)}$

Q. If $u = \sqrt{x^2 - y^2}\sin^{-1}\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

यदि $u = \sqrt{x^2 - y^2}\sin^{-1}\left(\frac{y}{x}\right)$ तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ बराबर है

(a) u

(b) $2u$

(c) $4u$

(d) **0**

$u = x\sqrt{1 - \frac{y^2}{x^2}} \cdot \sin^{-1}\left(\frac{y}{x}\right)$

$u = x^1 \cdot f\left(\frac{y}{x}\right)$

$1 \times \frac{u}{1}$

u //

Q. If $u = xyf\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

यदि $u = xyf\left(\frac{y}{x}\right)$ तो $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ बराबर है

(a) u

(b) $4u$

(c) **0**

(d) $2u$

Home-work

Q. If $z = u^2 + v^2, u = r\cos\theta, v = r\sin\theta$, then $\frac{\partial z}{\partial r}$ is equal to

यदि $z = u^2 + v^2, u = r\cos\theta, v = r\sin\theta$, तो $\frac{\partial z}{\partial r}$ बराबर है

(a) r

(b) $2r$

(c) $\frac{r}{2}$

(d) 0

Home - work

Q. If $u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$, then $\frac{\partial^2 u}{\partial x\,\partial y}$ is

यदि $u = x^2 tan^{-1}\left(\frac{y}{x}\right) - y^2 tan^{-1}\left(\frac{x}{y}\right)$, तो $\frac{\partial^2 u}{\partial x \partial y}$ है

(a) $\frac{x^2-y^2}{x^2+y^2}$

(b) $\frac{x^2+y^2}{x^2-y^2}$

(c) $\frac{2xy}{x^2+y^2}$

(d) $\frac{2xy}{x^2-y^2}$

Home-work