
DSSSB TGT
PART(A+B)
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ROJGAR WITH ANKIT
MATHS
COMPLEX ANALYSIS
LIVE STREAMING
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1. Comple~~x~~ Number

$\boxed{x, y \in R}$

A number of the form $z = x + iy$ where $x, y \in R$ and $i = \sqrt{-1}$. is called a complex number

$z = x + iy$ — Imaginary part

$i = \sqrt{-1} \Rightarrow \underline{i^2 = -1}$

Real part

iota

Also x, y are called respectively the real part of z and imaginary part of z and they are expressed

Re (z) and Im (z)

$z = 3 + 4i$

$Re(z) = 3 \quad Im(z) = 4$

Example -

$z = -5 + 1i \Rightarrow Re(z) \Rightarrow -5, Im(z) \neq 1$

$z = 8 + 0i \Rightarrow Re(z) \Rightarrow 8, Im(z) \neq 0$

$z = \sqrt{-9} \Rightarrow Re(z) \Rightarrow 0,$ $Im(z) = 3$

$z = 0 \Rightarrow Re(z) \neq 0, Im(z) \Rightarrow 3$

$z = 0 + 0i$

$Re(z) = 0$ $Im = 0$

$z = 8 + 0i$

$z = \sqrt{-9} \Rightarrow \sqrt{-1} \times \sqrt{9}$

$z \Rightarrow 3i$

$z = 0 + 3i$

2. Integral Powers of i

Since $i = \sqrt{-1}$

$i^{-1} \Rightarrow \frac{1}{i} = \frac{i^4}{i} \Rightarrow i^3 = -i$

$i^{-1} \Rightarrow \frac{1}{i} \times \frac{i}{i}$

$\Rightarrow \frac{i}{i^2} \Rightarrow \frac{i}{-1}$

$\Rightarrow -i$

$i = \sqrt{-1} = i$

$i^2 = -1$

$i^3 = i^2 \cdot i \Rightarrow -1 \cdot i = -i$

$i^4 = i^2 \times i^2 = (-1)(-1) = 1$

$i^5 = i^4 \times i \Rightarrow i$

$i^6 = i^4 \times i^2 \Rightarrow -1$

$i^7 = i^4 \times i^3 \Rightarrow -i$

$i^8 = i^4 \times i^4 \Rightarrow 1$

$i^9 = i$

$i^{10} \Rightarrow -1$

$i^{11} = -i$

$i^{12} = 1$

In General for any Integar k,

$i^{4k} = 1,$

$i^{4k+1} = i,$

$i^{4k+2} = -1,$

$i^{4k+3} = -i$

$i^{4k} = 1$

$i^{4k+1} = i$

$i^{4k+2} \Rightarrow -1$

$i^{4k+3} \Rightarrow -i$

$(i, 1, -i, -1)$

Example-

(i) $i^{123} \Rightarrow i^{4\times 30+3} \Rightarrow (i^4)^{30} \times i^3 \Rightarrow 1 \times -i = -i$

(ii) $i^{978} \Rightarrow (i^4)^{244} \cdot i^2 \Rightarrow 1 \times (-1) = -1$

(iii) $i^{-147} = i$

$\frac{1}{i^{147}}$

$\frac{1}{(i^4)^{36} \cdot i^3} \times \frac{i}{i}$

$\Rightarrow \frac{i}{i^4} \Rightarrow \frac{i}{1} = i$

(iv) $(-i)^{8n+3}, n \in N = i$

$i^4 = 1$

$(-i)^4 = 1$

(iv) $(-i)^8 \Rightarrow (-i)^{8n} \cdot (-i)^3$

$\Rightarrow ((-i)^8)^n \quad -i \times -i \times -i$

$\Rightarrow 1 \quad \Rightarrow +i^2 \times -i$

$\Rightarrow -1 \times -i \Rightarrow i$

Q. $i^n + i^{n+1} + i^{n+2} + i^{n+3}; \forall n \in \mathrm{N}$ is equal to.

(i) i

(ii) 0

(iii) $(i)^2$

(iv) $(i)^3$

$i^{n+1} = i^n \times i^1$

$i^{n+2} = i^n \times i^2$

$i^{n+3} = i^n \times i^3$

$i^n[1 + i + i^2 + i^3]$

$i^n[\cancel{1} + \cancel{i} - \cancel{1} - \cancel{i}]$

0

$\boxed{i^1 + i^2 + i^3 + i^4}$

$\boxed{i^8 + i^9 + i^{10} + i^{11}}$

Note-

The sum of four consecutive integral powers of i is always Zero.

$i^{-8}+i^{-7}+i^{-6}+i^{-5}$

$\frac{1}{i^8}+\frac{1}{i^7}+\frac{1}{i^6}+\frac{1}{i^5}$

$\frac{1}{i}\times\frac{i}{i} \Rightarrow \frac{i}{-1}$

$1+i-1-i=0$

$i^{79}+i^{80}+i^{81}+i^{82}=?=0$

$\underbrace{i^{103}+i^{104}+i^{105}+i^{106}}_{0}+i^{107}+i^{108}=?$

i^3+i^4

$-i+1 \Rightarrow 1-i$

$i^{103}+i^{104}+\underbrace{i^{105}+i^{106}+i^{107}+i^{108}}_{0}$

i^3+i^4

$\boxed{-i+1}$

$$i^{-1}+i^{-2}+i^{-3}+i^{-4}=0$$

$$\begin{array}{cccc}
\dfrac{1\times i}{i\times i} & \dfrac{1}{i^2} & \dfrac{1}{i^3} & \dfrac{1}{i^4} \\[2ex]
\dfrac{i}{-1} & \dfrac{1}{-1} & \dfrac{1}{-i\times i}\times i & \dfrac{1}{1} \\[2ex]
=-i & -1 & i & 1
\end{array}$$

3. Complex Number as an Ordered Pair of two Real Numbers

$Z = 3 + 4i \quad (3, 4)$
$z = 10 + 30i \quad (10, 30)$
$Z = 4i \quad (0, 4)$
$Z = 5 \quad (5, 0)$

Every complex number $z = x + iy$ may be considered as an ordered pair (x, y) of its two parts. The first number of the ordered pair is its real part, and the second number of the ordered pair is its imaginary part. Thus

$$\mathbb{Z} \equiv x + \mathrm{i}y \equiv (\mathrm{x}, y)$$

If $z = x + iy$, $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ are any complex numbers, then

(i) $z_1 = z_2 \Leftrightarrow x_1 = x_2$ **and** $y_1 = y_2$ **(equality)**

(ii) $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$ **(addition)**

(iii) $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$ **(subtraction)**

(iv) $z_1 z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$ **(multiplication)**

(v) $-z = (-x) + i(-y)$ **(negative)**

(vi) $z \neq 0, \frac{1}{z} = \left(\frac{x}{x^2+y^2}\right) + i\left(\frac{-y}{x^2+y^2}\right)$ **(reciprocal)**

(vii) $z \neq 0, \frac{z_1}{z_2} = \left(\frac{x_1 x_2 + y_1 y_2}{x_2^2+y_2^2}\right) + i\left(\frac{x_2 y_1 - x_1 y_2}{x_2^2+y_2^2}\right)$ **(division)**

$$z = x + iy$$

$$\frac{1}{z} = \frac{1}{(x+iy)} \times \frac{x-iy}{(x-iy)} \Rightarrow \frac{x-iy}{x^2-(iy)^2}$$

$$\Rightarrow \frac{x-iy}{x^2+y^2}$$

$$\frac{1}{z} = \frac{x}{x^2+y^2} + \frac{i(-y)}{x^2+y^2}$$

$$Z_1 = x_1 + iy_1 \qquad Z_2 = x_2 + iy_2$$

$$Z_1 \cdot Z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$\Rightarrow (x_1 \cdot x_2 + i\,\underbrace{x_1 y_2 + iy_1}\,x_2 + i^2 y_1 y_2)$$

$$\rightarrow x_1 \cdot x_2 + i(x_1 y_2 + x_2 y_1) - y_1 \cdot y_2$$

$$Z_1 Z_2 \Rightarrow (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

Geometrical Representation of a Complex Number

Geometrically every complex number $z = x + iy = (x, y)$ can be represented in xy-plane by a point whose cartesian coordinates are (x, y)

So, such a plane is named as complex plane or Argand plane or z-plane.

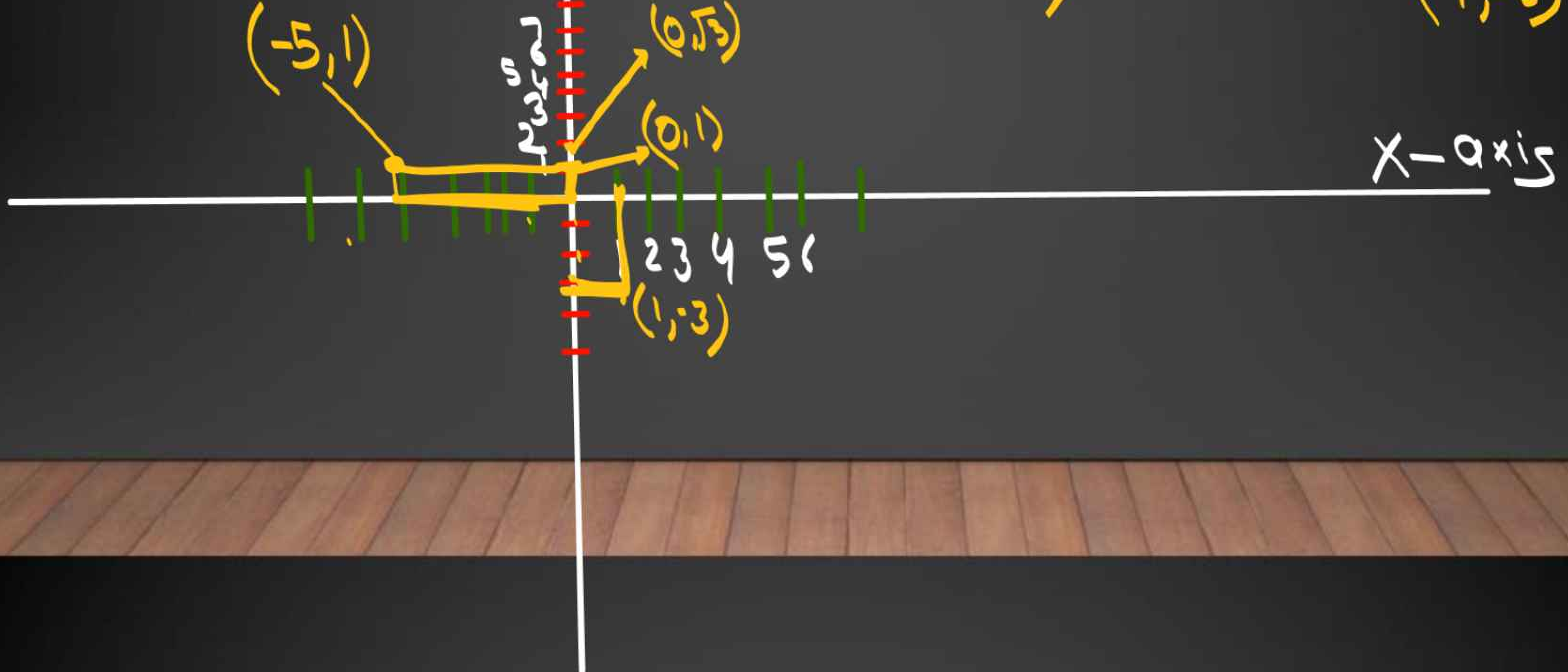
$i = (0+i) = (0,1)$

Example- Represent the following complex numbers in Argand diagram

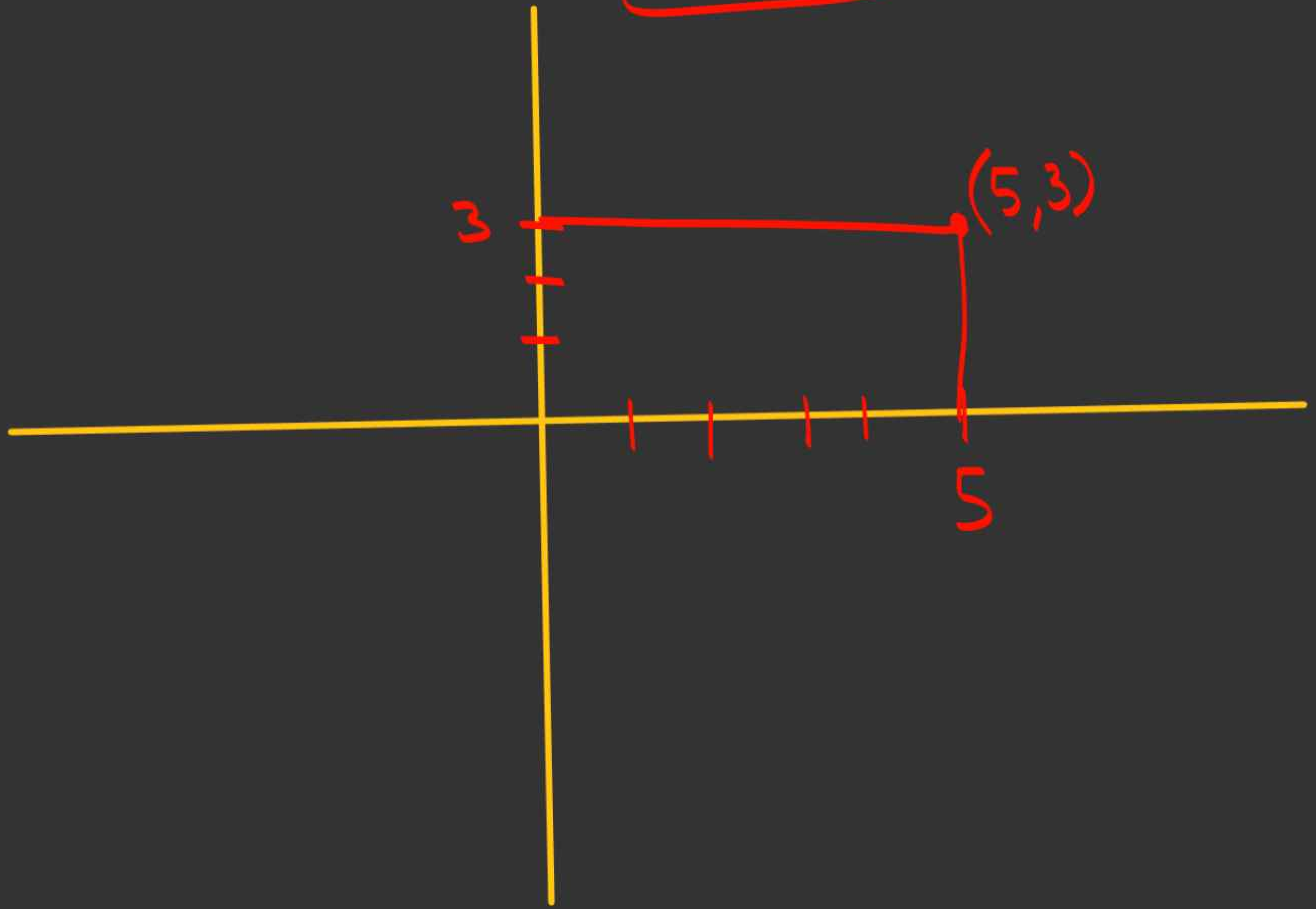
$$i, \sqrt{-3}, -5+i, 1-\sqrt{-9}$$

$(0+\sqrt{3}\,i)$ $\quad (-5,1) \quad$ $1-\sqrt{-9} \Rightarrow 1-3i$

$(1,-3)$



$\boxed{5+3i} = (5,3)$



Complex Conjugate

$$z = x + iy \Rightarrow \bar{z} = x - iy$$

$$z = (x, y) \Rightarrow \bar{z} = (x, -y)$$

(x, y)

$Z = x + iy$

$\bar{Z} = x - iy$

Example. (i) If $z = 5 + i$**, then** $\bar{z} = 5 - i$

(ii) If $z = 9$**, then** $\bar{z} = 9$

$Z = 3 + 4i \quad (3, 4)$

$\bar{Z} = 3 - 4i \quad (3, -4)$

$$\bar{Z} = \frac{1}{z} = \frac{1}{x+iy} \times \frac{x-iy}{x-iy} \Rightarrow \frac{x-iy}{x^2+y^2}$$

Propertics of Conjugate: **If** z, z_1, z_2 **are any complex numbers, then**

$z = 12,\ \bar{z} = 12$

$\boxed{z \in R}$

(i) $\bar{z} = z \Leftrightarrow z \in \mathrm{R}$

(ii) $\bar{z} = -z \Leftrightarrow z = 0$ **or purely imaginary**

(iii) $\overline{(\bar{z})} = z$

(iv) $z + \bar{z} = 2\mathrm{Re}\,(z)$

(v) $z - \bar{z} = 2i\mathrm{lm}\,(z)$

(vi) $z\bar{z} = x^2 + y^2$

(vii) $\frac{z}{\bar{z}} = \frac{z^2}{|z|^2}$

(viii) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

(ix) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

(x) $\overline{z_1 Z_2} = \bar{z}_1 \bar{z}_2$

(xi) $\overline{(z^n)} = (\bar{z})^n, n \in N$

(xii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}, z_2 \neq 0$

(xiii) $((\bar{z})^{-1}) = \left(\overline{z^{-1}}\right)$

$z = 3i$

$\bar{z} = -3i$

$\boxed{\bar{z} = -z}$

$z = 3 + \cancel{4i}$

$\bar{z} = 3 - \cancel{4i}$

$z + \bar{z} = 2 \times 3$

Modulus of a Complex Number:

$$z = x + iy \Rightarrow |z| = \sqrt{x^2 + y^2}$$

$$||z\ |= \sqrt{\{\mathrm{Re}\ ((z)\}^2 + \{\mathrm{lom}\ (z)\}^2}$$

Unimodular: **A complex number z is said to be unimodular if $|z| = 1$**