
DSSSB TGT
PART(A+B)
MATHS
REAL ANALYSIS
(MCQ'S)
PART-02
LIVE
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1. If $\sum_{n=1}^{\infty} u_n$ be a convergent series of real numbers, then the sequence $\{u_n\}$ is

(a) bounded

(b) convergent

(c) a Cauchy sequence

(d) all of the above

2. If the sequence $\{|a_n|\}$ is convergent, then the sequence $\{a_n\}$ is

(a) convergent

(b) may not convergent

(c) oscillatory

(d) monotonic

3. The series $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \cdots$ is

(a) convergent for $x \geq 1$ and divergent for $x < 1$

(b) convergent for $x \leq 1$ and divergent for $x > 1$

(c) convergent for $x > 1$ and divergent for $x \leq 1$

(d) convergent for $x < 1$ and divergent for $x \geq 1$

$\Rightarrow \frac{x^0}{1} + \frac{x^1}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \cdots$

$\sum_{n=0}^{\infty} \frac{x^n}{n^2+1} = \sum_{n=0}^{\infty} u_n$

$\sum_{n=0}^{\infty} u_{n+1} \Rightarrow \frac{x^{(n+1)}}{(n+1)^2+1}$

$\frac{u_{n+1}}{u_n} \Rightarrow \frac{\cancel{x^n} \cdot x}{n^2+1+2n+1} \cdot \frac{n^2+1}{\cancel{x^n}} \Rightarrow \frac{n^2+1}{n^2+2n+2} \cdot x \Rightarrow x$

By Comparision Test.

$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n}$ (a) $u_n = \frac{1}{n}$

$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

(i) $u_{n+1} < u_n$

(ii) $\lim_{n\to\infty} u_n = 0$] Convergent

$\Rightarrow \lim_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = 0$

4. Which of the following is true?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ does not converge

(b) $\sum_{n=1}^{\infty} \frac{1}{n}$ converges

(c) $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{(n+m)^2}$ converges

(d) $\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{1}{(n+m)^2}$ diverges

(b) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ में $p>1$ तो Cgt

$p \le 1$ dgt

by p-series Test

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ does not converge

(b) $\sum_{n=1}^{\infty} \frac{1}{n}$ converges

(c) $\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{(n+m)^2}$ converges

(d) $\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{(n+m)^2}$ diverges

(c) $\sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \frac{1}{(n+3)^2} + \cdots$

$\Rightarrow \frac{1}{(1+1)^2} + \frac{1}{(1+2)^2} + \frac{1}{(1+3)^2} + \cdots + \left(\frac{1}{(2+1)^2} + \frac{1}{(2+2)^2} + \frac{1}{(2+3)^2} + \cdots\right)$

$\Rightarrow \left[\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots\right] + \left[\frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} \cdots\right]$

$\Rightarrow \frac{1}{2^2} + \frac{2}{3^2} + \frac{3}{4^2} + \cdots$

$u_n \Rightarrow \frac{n}{(n+1)^2}$ $\quad \frac{n}{n^2}$

$V_n = \frac{1}{n} \rightarrow$ dgt by

$\lim_{n\to\infty} \frac{u_n}{V_n} = \frac{\cancel{n} \times n}{\cancel{n^2}(1+\frac{1}{n})^2} \Rightarrow \frac{1}{(1+\frac{1}{n})^2} \Rightarrow$

5. The sequence $\left\{3 + \underbrace{(-1)^n \frac{1}{n}}\right\}$ is

(a) Oscillatory

(b) Monotone

(c) Convergent

(d) Bounded but not convergent

$3 + \langle -1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \frac{1}{6} \cdots \rangle$

$\Rightarrow -1 \leq u_n \leq \frac{1}{2}$

Bounded by not Convergent

$\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{3\sqrt{n}}$

$u_n = \frac{1}{3\sqrt{n}}$

6. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3\sqrt{n}}$

(a) converges but not absolutely

(b) converges absolutely

(c) diverges

(d) none of the above

Leibneitz's Test →

(i) $u_{n+1} < u_n$ ✓

(ii) $\lim_{n\to\infty} u_n = 0$ ✓

] Cgt

For absolute Convergent $= \lim_{n\to\infty} |u_n| \Rightarrow \lim_{n\to\infty} \frac{1}{n^{1/2}}$

by p-series Test it is divergent.

7. **The sequence $\{x_n\}$, where $x_n = (-1)^n \frac{2n-1}{n}$ is**

(a) convergent

(b) divergent

(c) finitely oscillatory

(d) infinitely oscillatory

$\langle -1, \frac{3}{4}, -\frac{5}{3}, \frac{7}{4}, \frac{-9}{5} \cdots \rangle$

$(-1)^1 \cdot 1 = -1$

$(-1)^2 \cdot 2 = 2$

$(-1)^3 \cdot 3 = -3$

$-1, 2, -3, 4, -5, 6, -7, 8$

8. The sequence $\{(-1)^n n\}$ is

(a) bounded above

(b) bounded below

(c) bounded sequence

(d) neither bounded above nor bounded bellow

9. **The series $\sum_{n=0}^{\infty}(\sin\ x)^n$ converges to the value $(4+2\sqrt{3})$ for some value of x in $\left(0,\frac{\pi}{2}\right)$, then the value of x is**

(a) $\boxed{\frac{\pi}{3}}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{5}$

(d) $\frac{\pi}{6}$

$0, \frac{\pi}{3}$

$\frac{\pi}{3} = 60°$

$\sum_{n=0}^{\infty}\left(\sin\frac{\pi}{3}\right)^n$

$\sum_{n=0}^{\infty}\left(\frac{\sqrt{3}}{2}\right)^n$

$\Rightarrow 1 + \frac{\sqrt{3}}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^3 + \dots \infty$

$\Rightarrow a = 1,\ r = \frac{\sqrt{3}}{2}$

$S_\infty \Rightarrow \frac{a}{1-r} \Rightarrow \frac{1}{1-\frac{\sqrt{3}}{2}} \Rightarrow \frac{2}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \Rightarrow (4+2\sqrt{3})$

$\left(\frac{1}{\sqrt{2}}\right)^n \quad \left(\frac{1}{2}\right)^n$

10. **Consider the statements** 0, +1

$S_1: 1-1+1-1+1-1+\cdots = \pm 1$

$S_2: \frac{1}{1+2} = 1-2+2^2-2^3+\cdots$ **Then**

(a) S_1 **is true but** S_2 **is false.**

(b) S_1 **is false but** S_2 **is true.**

(c) both S_1 **and** S_2 **is true.**

(d) both S_1 **and** S_2 **is false.**

$\frac{1}{1+2}$ $|2|<1$ ×

$\frac{1}{1+x} = 1-x+x^2-x^3+\cdots$

$(1+x)^{-1}$

$|x|<1$

तो Exist करता है।

वरना No Exist.