DSSSB TGT PART(A+B)

MATHS

REAL ANALYSIS
(MCQ'S)

LIVE 04/09/2024 04:00 PM

Numbers (Level-I)

n = odd = −1
n = Even = +1

1. The sequence $\{(-1)^n\}$ is

(a) bounded and convergent

(b) bounded but not convergent

(c) ~~convergent~~ but not bounded

(d) unbounded and divergent

$(-1)^n$

$\Rightarrow \langle -1, 1, -1, 1, -1, 1 \ldots \rangle$

$\{-1, 1\}$ bounded

2. The sequence $\left\{\frac{(-1)^n}{n}\right\}$ is

~~(a)~~ bounded
(b) increasing
(c) decreasing
(d) bounded above

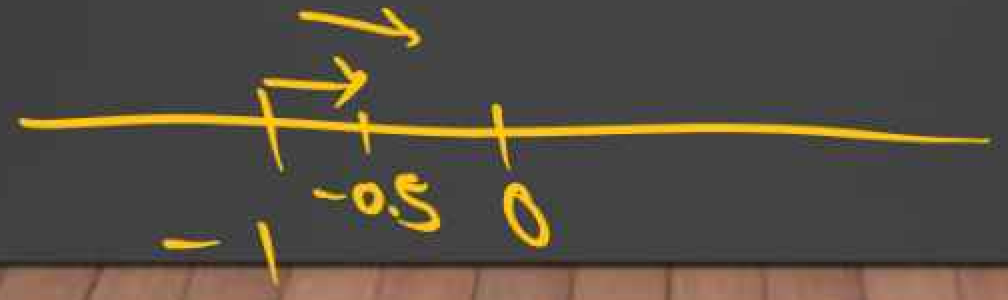
$$\frac{-1^n}{n}$$

$$\left\langle -1, \frac{1}{2}, \frac{-1}{3}, \frac{1}{4}, \frac{-1}{5}, \frac{1}{6} \dots \right\rangle$$

$$-1 \leq \frac{\{-1\}^n}{n} \leq \frac{1}{2}$$

$$\left\{-1, \frac{1}{2}\right\}$$

$$-1, \quad -\frac{1}{5} = -0.5$$



4. The sequence $\left\{\frac{(-1)^n}{n}\right\}$ (converges to

(a) zero

(b) 1

(c) -1

(d) ∞

$$\lim_{n\to\infty} \frac{(-1)^n}{n} \qquad -1^{\infty}$$

$$\Rightarrow \frac{(-1)^{(n)}}{\infty} = 0$$

$$(-1)^n \cdot \frac{1}{n} \qquad \left(u_n = \frac{1}{n}\right)$$

$$\lim_{n\to\infty} u_n$$

$$\lim_{n\to\infty} \frac{1}{n} \Rightarrow 0$$

5. The sequence $\{\sqrt{n+1}-\sqrt{n}\}$ converges to

(a) zero

(b) 1

(c) -1

(d) ∞

$$\frac{(\sqrt{n+1}-\sqrt{n})}{(\sqrt{n+1}+\sqrt{n})} \times (\sqrt{n+1}+\sqrt{n})$$

$$\Rightarrow \frac{(\sqrt{n+1})^2-(\sqrt{n})^2}{(\sqrt{n+1}+\sqrt{n})} \Rightarrow \frac{\cancel{n}+1-\cancel{n}}{\sqrt{n+1}+\sqrt{n}} = \frac{1}{\sqrt{n+1}+\sqrt{n}}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n+1}+\sqrt{n}}$$

$$\Rightarrow \frac{1}{\infty+\infty} = \frac{1}{\infty} = 0$$

6. The sequence $\left\{\frac{\sin n}{n}\right\}$ converges to

(a) 1

(b) 0

(c) -1

(d) 2

$$\lim_{n\to\infty}\left\{\frac{\sin n}{n}\right\}$$

$$\Rightarrow \frac{\sin n}{\infty} = \underline{\underline{0}}$$

$$\boxed{\lim_{n\to 0}\frac{\sin n}{n} = 1} \ *$$

$$\lim_{n\to\infty} \frac{-1}{n} \leq \frac{\sin n}{n} \leq \frac{1}{n}$$

$$-0 \qquad\qquad 0$$

$$0$$

7. The sequence $\{r^n\}$

(a) always convergent

(b) diverges to ∞ for $r > 1$

(c) does not converge

(d) none of the above

r^n

$|r| < 1 \rightarrow$ Convergent

8. The sequence $\{r^n\}$

(a) always converges for all r

(b) converges to zero for all r

(c) does not converge

(d) converges to zero for $|r| < 1$

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

$\left(\frac{1}{n}\right)^n$

0 —— 1

9. If the sequence $\{x_n\}$ converges to zero, then the sequence $\{\log(1 + x_n)\}$

(a) may converge to zero

(b) may not converge to zero

(c) must be converge to zero

(d) none of the above

log 1

0

10. The sequence $\left\{n^{\frac{1}{n}}\right\}$

(a) converges to 1

(b) converges to zero

(c) does not converge

(d) diverge to ∞

11. The sequence $\{x_n\}$, where $x_n = (2^n + 3^n)^{1/n}$ converges to

(a) zero

(b) 1

(c) 2

(d) 3

$$x_n = (2^n + 3^n)^{\frac{1}{n}}$$

$$x_n = \left[3^n\left(\frac{2^n}{3^n}+1\right)\right]^{\frac{1}{n}}$$

$$\lim_{n\to\infty} 3\left[\left(\frac{2}{3}\right)^n + 1\right]^{\frac{1}{n}}$$

$$\Rightarrow 3$$

$$x_n = (2^n + 3^n)^{\frac{1}{n}}$$

$$= \left[2^n\left(1+\frac{3^n}{2^n}\right)\right]^{\frac{1}{n}}$$

$$\lim_{n\to\infty} 2\left(1+\left(\frac{3}{2}\right)^n\right)^{\frac{1}{n}}$$

$$\Rightarrow 2$$

13. If a real sequence is not a Cauchy sequence, then it is a

~~(a)~~ divergent sequence

(b) bounded sequence

(c) convergent sequence

(d) none of the above

Remember

14. The set of all limit points of a bounded sequence is

(a) unbounded

(b) bounded

(c) not necessarily bounded

(d) none of the above

15. The sequence $\{x_n\}$, where $x_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$ is

(a) convergent

(b) monotonically decreasing

(c) not Cauchy

(d) oscillatory

*Note

$$\boxed{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e}$$

16. The sequence $\{x_n\}$, where $x_n = \left(1+\frac{1}{n}\right)^{2n}$ converges to

(a) e

(b) e^2

(c) $\sqrt{e}$

(d) none of these

$$x_n = \left(1+\frac{1}{n}\right)^{2n}$$

$$\lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)^{n}\right]^{2}$$

$$= e^2$$

17. **The sequence $\{x_n\}$, where $x_n = \left(1 + \frac{1}{2n}\right)^n$ converges to**

(a) e

(b) e^2

(c) $\sqrt{e}$

(d) **none of these**

$$x_n = \left(1 + \frac{1}{2n}\right)^n$$

$$\lim_{n \to \infty} \left[\left(1 + \frac{1}{2n}\right)^{2n}\right]^{\frac{1}{2}}$$

$$= e^{\frac{1}{2}} = \sqrt{e}$$

19. The sequence $\{x_n\}$, where $x_n = \left(1 + \frac{1}{2n}\right)^{3n}$ converges to

(a) $e^{\frac{2}{3}}$

(b) $e^{\frac{3}{2}}$

(c) e

(d) 0