
DSSSB TGT
PART (A+B)
MATHS
REAL ANALYSIS
(LEIBNITZ'S TEST & ABSOLUTE CONVERGENCE)
LIVE
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Convergence

For the ~~Convergence~~ of Alternating Series :

LEIBNITZ'S TEST- The alternating series $\sum_{n=1}^{\infty}(-1)^{n-1} \cdot u_n = u_1 - u_2 + u_3 - u_4 + \cdots (u_n > 0$ for all $n)$ is convergent if-

(i) $u_{n+1} < u_n \; \forall n \in N$

(ii) $\lim_{n\to\infty} u_n = 0$

$u_2 < u_1 \quad u_4 < u_3$

$u_3 < u_2$

$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot u_n$

$\Rightarrow u_1 - u_2 + u_3 - u_4$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot u_n$$

$$\lim_{n\to\infty} \frac{1}{n}$$

$$\Rightarrow \frac{1}{\infty} = 0$$

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

$$\Rightarrow \sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{n} \qquad \left(u_n = \frac{1}{n}\right)$$

(i) $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{n} + \cdots$

$u_{n+1} < u_n$ ✓

(ii) $\lim_{n\to\infty} u_n = 0$ ✓

The given series is Convergent by Leibnez test.

Q. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$.

$$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{2n-1}$$

$$\left\{ u_n = \frac{1}{2n-1} \right\}$$

$$\Rightarrow \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \cdots + \frac{1}{2n-1}$$

$$\lim_{n\to\infty} \left(\frac{1}{2n-1} \right)$$

$$\Rightarrow \frac{1}{2\times\infty-1}$$

$$\Rightarrow \frac{1}{\infty} = 0$$

(i) $u_{n+1} < u_n$ ✓

(ii) $\lim_{n\to\infty} u_n = 0$ ✓

The given series is Cgt.

Q. $1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots$

$0! = 1$

$1! = 1$

$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{(n-1)!}$ $\left(u_n = \frac{1}{(n-1)!} \right)$

$\lim_{n\to\infty} \frac{1}{(n-1)!}$

$\frac{1}{\infty} = 0$

$u_n = 1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \cdots \frac{1}{\infty}$

(i) $u_{n+1} < u_n$ ✓

(ii) $\lim_{n\to\infty} u_n = 0$

Q. $\frac{1}{\sqrt{2-1}} - \frac{1}{\sqrt{3-1}} + \frac{1}{\sqrt{4-1}} \cdots$

$$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \cdots -$$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{\sqrt{n}}$$

$$u_n = \frac{1}{\sqrt{n}}$$

$$\Rightarrow \frac{1}{1} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \cdots \infty$$

(i) $u_{n+1} < u_n$ ✓

(ii) $\lim_{n \to \infty} u_n = 0$ ✓ $\Big\}$ Cgt

Q. $\dfrac{1}{4^0 \cdot 1} - \dfrac{1}{4\cdot 3} + \dfrac{1}{4^2\cdot 5} - \dfrac{1}{4^3\cdot 7} + \cdots$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{4^{n-1}\cdot(2n-1)}$$

$$\frac{1}{4^0 \times 1} - \frac{1}{4^1 \times 3} + \frac{1}{4^2 \times 5} - \frac{1}{4^3 \times 7} + \cdots$$

$$\frac{100}{2} > \frac{100}{4} > \frac{100}{5} > \frac{100}{10} > \frac{100}{20} > \frac{100}{25}$$

$$u_n = \frac{1}{4^{n-1}\cdot(2n-1)}$$

$$u_{n+1} = \frac{1}{4^n \cdot (2n+1)}$$

(i) $u_{n+1} < u_n$

(ii) $\lim\limits_{n\to\infty} u_n = 0$

CGT

Absolute Convergence- A series $\sum_{n=1}^{\infty} u_n$ Which Contains the positive and Negative Terms is said to be absolute convergence if the series.

$\sum_{n=1}^{\infty} |u_n|$ is convergent

p-series

Cochy

D'Alembert's

and also

(i) $U_{n+1} < U_n$

(ii) $\lim_{n\to\infty} U_n = 0$

Q. $\sum u_n = 1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \cdots (-1)^{n-1} \frac{1}{2^{n-1}}$

$$\sum_{n=1}^{\infty} u_n = \sum_{n=}^{\infty} (-1)^{n-1} \cdot \frac{1}{2^{n-1}}$$

Leibnitz's $\left[\begin{array}{l} \text{(i) } u_{n+1} < u_n \checkmark \\ \text{(ii) } \lim_{n\to\infty} u_n = 0 \checkmark \end{array}\right]$ Cgt by Leibnitz's Test

(iii) $\sum_{n=1}^{\infty} |u_n| \Rightarrow \sum \frac{1}{2^{n-1}} \Rightarrow \frac{1}{1} + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots \infty$

$\Downarrow$ $\Rightarrow r = \frac{1}{2}$

Convergent by G.P. Test $\boxed{-1 < r < 1}$

So, the given series is absolutely Convergent series.

Q. $\sum_{n=1}^{\infty}(-1)^{n+1} \cdot \frac{1}{n^2} = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

By P-series Test

$\sum_{n=1}^{\infty} \frac{1}{n^p}$

यदि $P > 1$ Cgt

$P \leq 1$ Dgt

Leibnitz's

(i) $u_{n+1} < u_n$

(ii) $\lim_{n\to\infty} \frac{1}{n^2} = 0$

(iii) $\sum_{n=1}^{\infty} |u_n| = \sum_{n=1}^{\infty} \left|\frac{1}{n^2}\right| \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2}$ $P = 2 > 1$

$P > 1$

So, the given series is absolute convergent. Cgt

Conditional convergence → A series $\sum_{n=1}^{\infty} u_n$ is said to be conditionally convergent or semi convergent, If The series is convergent But not absolutely convergent.

Q. $\sum_{n=1}^{\infty} u_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$

$$\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{1}{n}$$

$$\left(u_n = \frac{1}{n} \right)$$

(i) $u_{n+1} < u_n$ ✓

(ii) $\lim_{n \to \infty} u_n = 0$ ✓

Convergent by Leibnitz's test

(iii) $\sum_{n=1}^{\infty} \left| \frac{1}{n} \right| \Rightarrow$ Divergent ($p = 1$)

$\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 1$ (Cgt) ✓

$p \leq 1$ (Dgt)

by p-series test