दिल्ली अधीनस्थ सेवा चयन बोर्ड

DSSSB TGT PART(A+B)

MATHS

REAL ANALYSIS
(LOGARITHMIC RATIO TEST)

LIVE STREAMING 02/09/2024 11:00 AM

Q. $\sum_{n=1}^{\infty} u_n = \frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \cdots$

$$\frac{u_{n+1}}{u_n} = r$$

$$\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \cdots \infty + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \cdots \infty$$

$r = \frac{1}{4}$ $\qquad$ $r = \frac{1}{9}$

$\boxed{-1 < r < 1} \rightarrow$ Convergent

Q. $\sum_{n=1}^{\infty}\left(\frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n}\right)$ is convergent or divergent?

$\lim_{n\to\infty}(U_n)^{\frac{1}{n}} \Rightarrow \lim_{n\to\infty}\frac{\left(\frac{n+1}{n}\right)^n}{3}$

$1^{\infty} = e \Rightarrow 2.718$

$\Rightarrow \frac{1}{3}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

$\Rightarrow \frac{1}{3}(1^{\infty}) \Rightarrow \frac{2.718}{3} < 1$ (cgt)

Q. $\frac{1^3}{3} + \frac{2^3}{3^2} + \frac{3^3}{3^3} + \cdots + \frac{n^3}{3^n}$ is convergent or divergent?

$$u_n = \frac{n^3}{3^n}$$

$$\lim_{n \to \infty} (u_n)^{\frac{1}{n}} = \frac{n^{\frac{3}{n}}}{3}$$

$$\Rightarrow \frac{1}{3} < 1 \text{ (cgt)}$$

LOGARITHMIC TEST – **If the series $\sum_{n=1}^{\infty} U_n$ is a positive term series such that $\lim_{n\to\infty} n\log\frac{u_n}{u_{n+1}} = l$, then series is.**

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(i) Convergent If $l > 1$ (अभिसारी)

(ii) Divergent If $\ell < 1$ (अपसारी)

(iii) Test fails If $l = 1$

• DE MORGAN's AND BERTRAND's TEST

If $\sum_{n=1}^{\infty} u_n$ is a series of positive terms such that $\lim_{n\to\infty}\left[n\cdot\left(\frac{u_n}{u_{n+1}}-1\right)-1\right]\log n = l$, then the series is.

(i) Convergent If $l > 1$

(ii) Divergent If $l < 1$

(iii) Test fails If $l = 1$

Q. The Sequence $\left\langle \log \frac{1}{n} \right\rangle$ is –

(a) Convergent

(b) Divergent to ∞

(c) Divergent to $-\infty$

(d) None of the above

$$\lim_{n\to\infty} \log\frac{1}{n}$$

$$\lim_{n\to\infty} [\overset{0}{\log 1} - \log n]$$

$$= \lim_{n\to\infty} [-\log n]$$

$$= -\log\infty$$

$$= -\infty$$

$$\log\frac{m}{n} = \log m - \log n$$

Q. The limit of the sequence $\left\{\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots+\frac{1}{3^n}\right\}_{n=1}^{\infty}$ is:-

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) 1

(d) 0

$u_n = \frac{1}{3^n}$

$\lim\limits_{n \to \infty} \frac{1}{3^n}$

$\Rightarrow \frac{1}{3^\infty} = \frac{1}{\infty} = 0$

Q. The given series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}$ is?

(a) Divergent

(b) Convergent

(c) Oscillatory

(d) Absolutely Convergent

$$1 + \frac{1}{3} + \frac{1}{5} - \frac{1}{2} - \frac{1}{4} - \frac{1}{6}$$

$$\Rightarrow \frac{60+20+12-30-15-10}{60}$$

$$\Rightarrow \frac{37}{60}$$

finite

unique

Q. $\sum \frac{1}{n^p}$, when $P > 1$ is –

(a) Convergent

(b) divergent

(c) oscillatory

(d) None of the above

$\frac{1}{n^p}$ में यदि $P>1$ Cgt
और यदि $P \leq 1$ Dgt

P-Series Test → $\sum \frac{1}{n^p}$ में $P>1$

Cgt by P-Test

Q. The sequence $\left\{\frac{2n+7}{3n+2}\right\}$ Summons which of the following value?

(a) $\frac{3}{2}$

(b) $-\frac{7}{2}$

(c) $\frac{2}{3}$

(d) $-\frac{2}{7}$

$$\lim_{n\to\infty}\left[\frac{\cancel{n}\left(2+\frac{7}{n}\right)}{\cancel{n}\left(3+\frac{2}{n}\right)}\right]$$

$\frac{2}{3}$

Q. Which of the following series is divergent?

(a) $\sum \frac{1}{n^2}$ $p = 2 > 1$ Cgt

(b) $\sum \frac{1}{n^3}$ $3 > 1$ Cgt

$\frac{1}{n^p}$ (c) $\sum \frac{1}{n}$ $p = 1$ Dgt

(d) $\sum \frac{1}{n^4}$ → $4 > 1$ (Cgt)

$$\boxed{\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=l}$$

Raabe's TEST (राबे परिक्षण):- यदि धनात्मक पदों की श्रेणी इस प्रकार है ताकि $\mathrm{Lim}_{n\to\infty}\, n\left(\frac{u_n}{u_{n+1}}-1\right)=l$ है तो

(i) Convergent If $l>1$

(ii) Divergent If $l<1$

(iii) Test fails if $l=1$

- **Note → राबे परिक्षण का उपयोग तब लिया जाता है जबकि** $\lim\limits_{n\to\infty}\frac{u_n}{u_{n+1}}=1$ **हो-**

Q. श्रेणी $\frac{1}{2}+\frac{1\cdot 3}{2\cdot 4}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}+\ldots\ldots\ldots$ के Test of ..... convergence जाचों?

$$u_n = \frac{1\times 3\times 5\times\cdots\times(2n-1)}{2\times 4\times 6\times\cdots\times(2n)}, \quad U_{n+1} = \frac{1\times 3\times 5\times\cdots\times(2n-1)(2n+1)}{2\times 4\times 6\times\cdots\times 2n(2n+2)}$$

$$\lim_{n\to\infty}\left(\frac{u_n}{U_{n+1}}\right) = \frac{2n+2}{2n+1}$$

$$\lim_{n\to\infty}\left[\frac{2+\frac{2}{n}}{2+\frac{1}{n}}\right]$$

$$\Rightarrow \frac{2}{2} = 1$$

$$\lim_{n\to\infty} n\left(\frac{U_n}{U_{n+1}} - 1\right)$$

$$\lim_{n\to\infty} n\left(\frac{2n+2}{2n+1} - 1\right)$$

$$\lim_{n\to\infty} \frac{n}{2n+1}$$

$$\Rightarrow \frac{\cancel{n}}{\cancel{n}\left(2+\frac{1}{n}\right)} = \frac{1}{2+\frac{1}{n}} = \frac{1}{2} < 1$$

Divergent By Raabe's Test