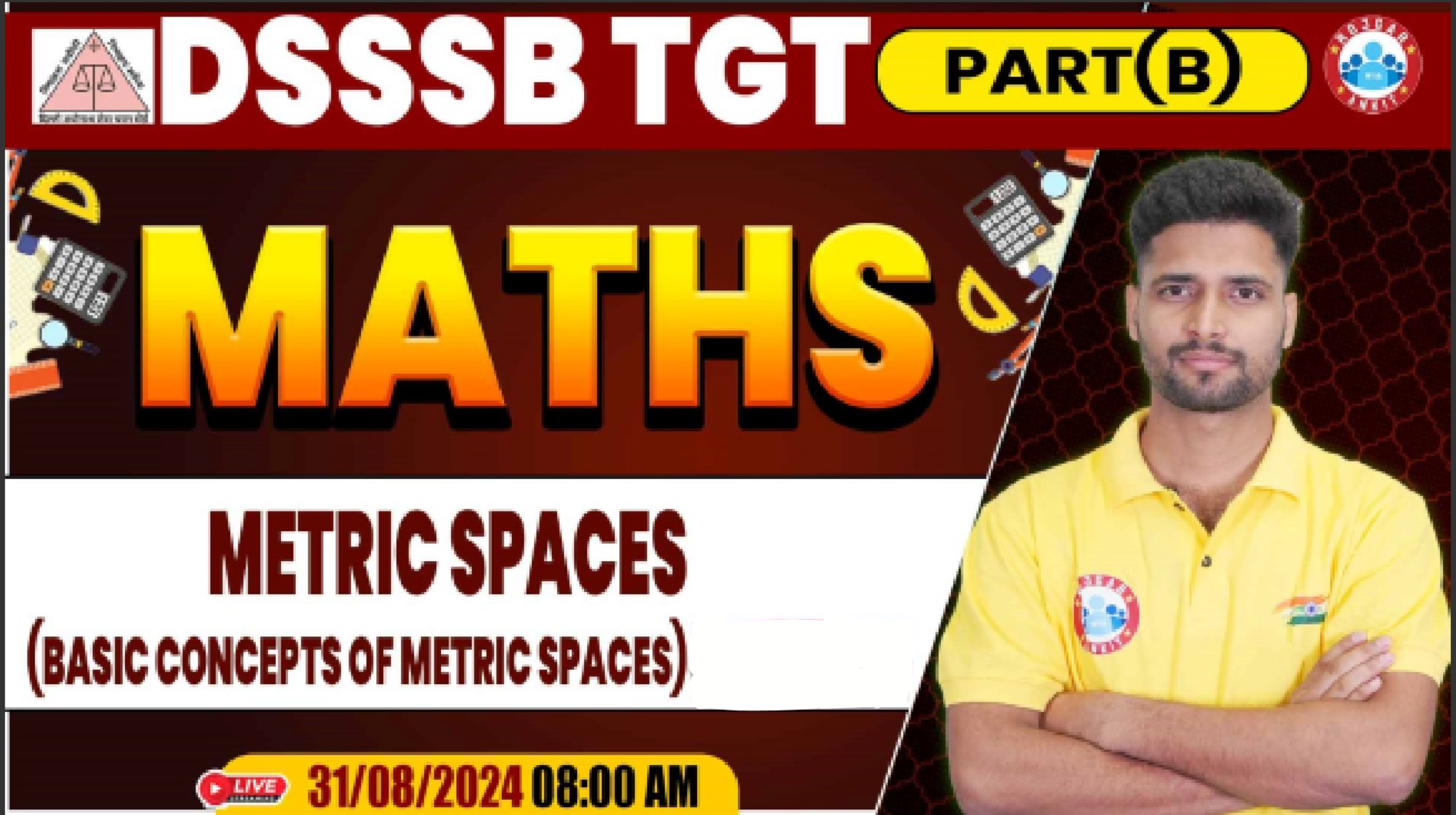
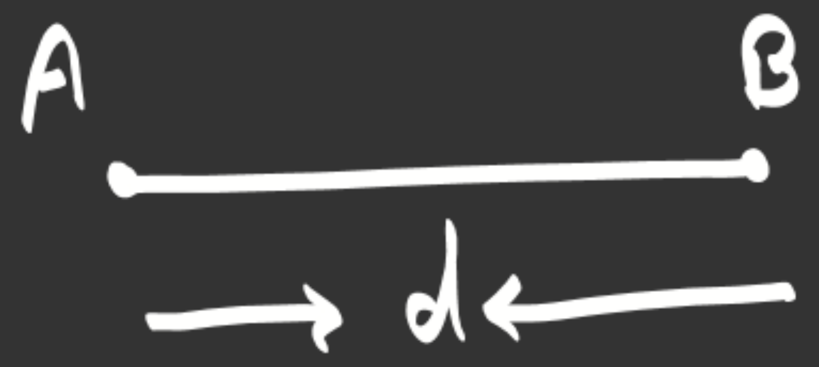

DSSSB TGT
PART(B)
MATHS
METRIC SPACES
(BASIC CONCEPTS OF METRIC SPACES)
LIVE
31/08/2024 08:00 AM

Metric Space

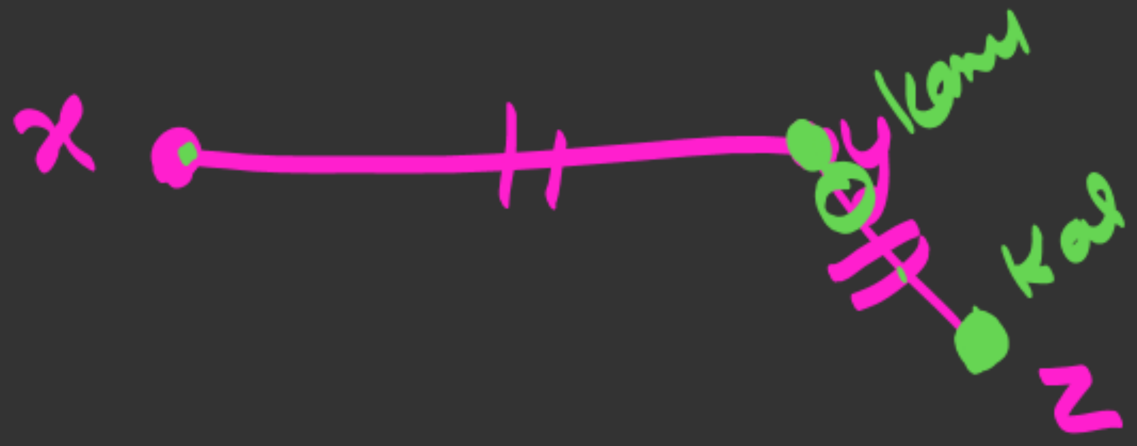


$|A-B|$

Let X be a non empty set

A function $d : X \times X \longrightarrow \mathbb{R}$

is said to be metric on X.



$|x-y| \geq 0$

Properties

(i) $d(x,y) \geq 0 \; ; \; \forall \, x,y \in R$

(ii) $d(x,y) = 0 \; ; \; x=y$

(iii) $d(x,y) = d(y,x) \; ; \; x,y \in R$

(iv) $d(x,z) \leq d(x,y) + d(y,z)$

$; \; x,y,z \in R.$

Que Prove that $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined as $d(x,y) = |x-y|$ is metric on $\mathbb{R}$.

①. $d(x,y) = |x-y| \geq 0 \; ; \; x,y \in \mathbb{R}$ (negativeness)

②. $d(x,y) = |x-y| = 0 \; ; \; x=y$ (definite)

③. $d(x,y) = d(y,x) \Rightarrow |x-y| = |y-x| \; ; \; x,y \in \mathbb{R}$ (Symmetric).

④. $d(x,y) = |x-y|$

$d(y,z) = |y-z|$

$d(x,z) = |x-z|$

(Transitive)

$A/X \quad B/Y$

<u>Que</u> $d : (0,\infty) \times (0,\infty) \to [0,\infty)$.

if $d(x,y) = \left|\frac{1}{x} - \frac{1}{y}\right|$ is a metric space?

✓ ①. $d(x,y) = \left|\frac{1}{x} - \frac{1}{y}\right| \geq 0$

✓ ②. $d(x,y) = \left|\frac{1}{x} - \frac{1}{y}\right| = 0$ $\quad$: $\frac{1}{x} - \frac{1}{y} = 0 \Rightarrow y - x = 0 \Rightarrow \boxed{x = y}$

✓ ③ $d(x,y) = \left|\frac{1}{x} - \frac{1}{y}\right| = \left|\frac{1}{y} - \frac{1}{x}\right| = d(y,x)$

$$\left|\frac{y-x}{\cancel{xy}}\right| = \left|\frac{x-y}{\cancel{xy}}\right|$$

④ $d(x,z) \leq d(x,y) + d(y,z)$.

$$\left|\frac{1}{x}-\frac{1}{z}\right| \leq \left|\frac{1}{x}-\frac{1}{y}\right| + \left|\frac{1}{y}-\frac{1}{z}\right|$$

$$\left|\frac{1}{x}-\frac{1}{z}\right| \leq \left|\frac{1}{x}-\frac{1}{z}\right|$$

#. Discrete metric space.

let X be a non empty set.

a map. $d : X \times X \longrightarrow \mathbb{R}$.
is defined as.

$$\left\{ \begin{aligned} d(x,y) &= 0 \quad \text{if } x = y \\ &= 1 \quad \text{if } x \neq y \end{aligned} \right\}$$

Ans By defined.

$d(x,y) \geq 0 \; \forall \; x,y \in R$

discrete metric space.

① $d(x,y) = 0 \; : \; x = y$

② $d(x,y) = 1 \; : \; x \neq y.$

$d(x,y) \neq 0 \; ;$