
DSSSB TGT
PART(A+B)
MATHS
REAL ANALYSIS
(CAUCHY'S & D'ALEMBERT'S RATIO TEST)
LIVE
31/08/2024 11:00 AM

Q. अनुक्रम $\left\{\frac{2n+5}{3n+2}\right\}$ की सीमा ज्ञात कीजिए?

$$\lim_{n\to\infty}\left\{\frac{2n+5}{3n+2}\right\}$$

Converges to $\frac{2}{3}$

$$\lim_{n\to\infty}\left\{\frac{\cancel{n}\left(2+\frac{5}{n}\right)}{\cancel{n}\left(3+\frac{2}{n}\right)}\right\} \Rightarrow \frac{2}{3}$$

Q. अनुक्रम $\left\{1+\frac{(-1)^n}{n}\right\}$ की सीमा ज्ञात करो?

$(-1)^{\infty \to odd} = -1$ $\quad$ $(-1)^{\infty \to Even} \Rightarrow +1$

$\frac{-1}{\infty} = 0$

$\frac{1}{\infty} = 0$

$\frac{2000}{\infty} = 0$

$$\lim_{n\to\infty} \left\{1 + \frac{(-1)^n}{n}\right\}$$

$$= 1 + 0$$

$$\Rightarrow 1$$

$\frac{-1}{\infty} = 0$

$\frac{1}{\infty} = 0$

Q. अनुक्रम $\left\{\frac{\sqrt{n}}{n+1}\right\}$ की प्रकृति ज्ञात करो।

(a) monotonically Increase

(b) Monotonically decrease

(c) convergent

(d) divergent

$0.50 > 0.47 > 0.43 > 0.40$

$\frac{1}{2} > \frac{\sqrt{2}}{3} > \frac{\sqrt{3}}{4} > \frac{2}{5}$

$u_n = \left\{\frac{\sqrt{n}}{n+1}\right\}$

$\lim_{n\to\infty} u_n = \left\{\frac{\sqrt{n}}{n+1}\right\}$

$= \left\{\frac{\cancel{\sqrt{n}}}{\cancel{\sqrt{n}}\left(\sqrt{n}+\frac{1}{\sqrt{n}}\right)}\right\}$

$= \frac{1}{\sqrt{\infty}+\frac{1}{\infty}} \Rightarrow 0$ {finite, unique}

$3\overline{)1.414}(0.47$
12
21

$4\overline{)1.732}(0.43$
16
13

Q. अनुक्रम $\left\{\frac{n^2+1}{n^2}\right\}$ की प्रकृति ज्ञात करो।

(a) monotonically Increase

(b) Monotonically decrease

(c) convergent अभिसारी

(d) divergent अपसारी

$u_n = \left\{\frac{n^2+1}{n^2}\right\}$

$= \left\langle \frac{2}{1}, \frac{5}{4}, \frac{10}{9}, \frac{17}{16}, \frac{26}{25}, \ldots \right\rangle$

$2 > 1.25 > 1.11 > 1.06 > 1.04$

$\lim_{n\to\infty} \left(\frac{\cancel{n^2}\left(1+\frac{1}{n^2}\right)}{\cancel{n^2}}\right)$

$1+\frac{1}{\infty} \Rightarrow 1+0 \Rightarrow 1$ finite, unique

Q. अनुक्रम $\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}$ की प्रकृति ज्ञात करो।

(a) Convergent

(b) divergent

(c) Oscillatory

(d) None of these

$u_n = \left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}$

$\lim_{n\to\infty}\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}$

$-1\cdot 2,\ \frac{3}{2}$

$-2,\ \frac{3}{2},\ -\frac{4}{3},\ \frac{5}{4}$

(Not a unique Limit)

Q. अनुक्रम $\left\{\frac{n^2}{n+1}\right\}$ की प्रकृति ज्ञात करो।

$$\frac{1}{1.2} = \frac{1}{1\times 2} = \frac{1}{1} - \frac{1}{2}$$

$$\left\{\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \right\} \Rightarrow \frac{1}{1}\left\{\frac{1}{1} - \frac{1}{4}\right\}$$

अन्तर 1 1 1

$$\Rightarrow \frac{3}{4}$$

$$S_n \Rightarrow \frac{1}{(\text{अन्तर})}\left\{\text{Ist term} - \text{Last Term}\right\}$$

$$\Rightarrow \frac{1}{1}\left\{\frac{1}{1} - \frac{1}{100}\right\}$$

$$\frac{99}{100}$$

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12}$$

$$\frac{6+2+1}{12} \Rightarrow \frac{9}{12} \Rightarrow \frac{3}{4}$$

Q. अनुक्रम $\left\{\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n\cdot(n+1)}\right\}$ की प्रवृति ज्ञात करो।

(a) Converges to 0

(b) Converges to 1

(c) divergent

(d) None of these

$\left\{\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{n\times(n+1)}\right\}$

$S_n = \frac{1}{1}\left[\frac{1}{1} - \frac{1}{n+1}\right]$

$\lim_{n\to\infty} S_n = \left[\frac{n}{n+1}\right]$

$\lim_{n\to\infty} S_n = \frac{\cancel{n}}{\cancel{n}\left(1+\frac{1}{n}\right)}$

$= \frac{1}{1+\frac{1}{\infty}} \Rightarrow 1$

Q. अनुक्रम $\left\{\frac{1}{2\cdot 5}+\frac{1}{5\cdot 8}+\frac{1}{8\cdot 11}+\cdots+\frac{1}{n(n+3)}\right\}$ की प्रकृति ज्ञात करें।

$2 \xrightarrow{3} 5 \xrightarrow{3} 8 \xrightarrow{3} 11$

(a) Converges to 1

(b) Converges to $\frac{1}{3}$

(c) Converges to $\frac{4}{5}$

$\boxed{\text{(d) Converges to } \frac{1}{6}}$

$S_n \Rightarrow \frac{1}{\text{अन्तर}}\left[I_{st} - Last\right]$

$S_n = \frac{1}{3}\left[\frac{1}{2} - \frac{1}{n+3}\right]$

$S_n = \frac{1}{3}\left[\frac{n+1}{2(n+3)}\right]$

$\lim_{n\to\infty} S_n \Rightarrow \left[\frac{\cancel{n}\left(1+\frac{1}{n}\right)}{\cancel{n}\left(6+\frac{18}{n}\right)}\right]$

Converges to $\Rightarrow \frac{1}{6}$

$\frac{1}{4}$ $\frac{1}{8}$

Sequence $\left\{ \frac{1}{4\cdot 6} + \frac{1}{6\cdot 8} + \frac{1}{8\cdot 10} + \frac{1}{10\cdot 12} + \cdots + \frac{1}{n\cdot(n+2)} \right\}$

$$S_n = \frac{1}{2}\left[\frac{1}{4} - \frac{1}{n+2}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{n-2}{4(n+2)}\right]$$

$$\Rightarrow \frac{\cancel{n}\left(1-\frac{2}{n}\right)}{\cancel{n}\left(8+\frac{16}{n}\right)}$$

$$\frac{1-\frac{2}{\infty}}{8+\frac{16}{\infty}}$$

$$\frac{1-0}{8+0} \Rightarrow \frac{1}{8}$$

Q. $\sum u_n = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots \lim_{n\to\infty} \sum u_n$ का मान कितना होगा?

(a) 0

(b) ∞

(c) $\sqrt{2}$

(d) None of these

$\lim_{n\to\infty} U_n$

$U_n = \frac{1}{\sqrt{n}}$

$\lim_{n\to\infty} \frac{1}{\sqrt{n}}$

$\Rightarrow \frac{1}{\sqrt{\infty}} \Rightarrow 0$

Q. $\frac{3}{5}+\frac{4}{5^2}+\frac{3}{5^3}+\frac{4}{5^4}+$ कैसी series है?

(a) Convergent

(b) divergent

(c) Oscillating

(d) None of these

$$\frac{3}{5}+\frac{4}{5^2}+\frac{3}{5^3}+\frac{4}{5^4}+\dots\dots\infty$$

GP GP

$$\boxed{\frac{3}{5}+\frac{3}{5^3}+\frac{3}{5^5}+\dots\infty}+\frac{4}{5^2}+\frac{4}{5^4}+\frac{4}{5^6}+\dots\infty$$

$r=\frac{1}{25} \leftarrow \boxed{-1<r<1} \rightarrow r=\frac{1}{25}$

$$a+ar+ar^2+ar^3+\dots\infty$$

$$S_n=\frac{a}{1-r}$$

$$\Rightarrow \frac{\frac{3}{5}}{1-\frac{1}{25}} \Rightarrow \frac{\cancel{3}\times \cancel{25}\,5}{\cancel{5}\times \cancel{24}\,8}$$

$$\Rightarrow \frac{\frac{4}{25}}{1-\frac{1}{25}} \Rightarrow \frac{4}{24}$$

$$\frac{5}{8}+\frac{1}{6} \Rightarrow \frac{15+4}{24} \Rightarrow \frac{19}{24}$$

Q. $(1+1)^1 + \left(1+\frac{1}{2}\right)^2 + \left(1+\frac{1}{3}\right)^3 + \cdots \infty$

कैसी series हैं?

~~(a)~~ Convergent

(b) divergent

(c) Oscillating

(d) None of these

$\lim_{n\to\infty} u_n = \left(1+\frac{1}{n}\right)^n$

$\left(1+\frac{1}{\infty}\right)^\infty$

$\Rightarrow (1+0)^\infty$

$\Rightarrow 1^\infty$

$\Rightarrow 1$

Q. $\frac{1}{2}+\frac{\sqrt{2}}{3}+\frac{\sqrt{3}}{8}+\cdots+\frac{\sqrt{n}}{n^2-1}+\cdots$ कैसी Series है?

(a) Convergent

(b) divergent

(c) Oscillating

(d) None of these

Q. श्रेणी $\frac{2}{1^p}+\frac{3}{2^p}+\frac{4}{3^p}+\ldots\ldots$

(a) Convergent If $P \geqslant 2$ और divergent If $P<2$

(b) Convergent If $P>2$ और divergent If $P \leq 2$

(c) Convergent If $P \geqslant 2$ और divergent If $P>2$

(d) Convergent If $P<2$ और divergent If $P \geqslant 2$

$\frac{2}{1^p}+\frac{3}{2^p}+\frac{4}{3^p}+\text{-----}\infty$

$U_n=\frac{(n+1)}{n^p} \Rightarrow \frac{n\left(1+\frac{1}{n}\right)}{n^p} \Rightarrow \frac{1+\frac{1}{n}}{n^{(p-1)}}$

$(p-1) \leqslant 1$

$\boxed{P \leqslant 2}$ dgt

$(p-1)>1$ cgt

$\boxed{P>2} \rightarrow$ cgt

By P-Series Test $\frac{1}{n^{(p)}}$ में

$P>1$ cgt

$P \leqslant 1$ dgt

Q. $\sum U_n = \sum \frac{n^p}{(n+1)^q}$

(a) Convergent If $q - p > 1$

(b) Divergent If $q - p \leqslant 1$

(c) both (a) & (b)

(d) None of these

$u_n = \frac{n^p}{(n+1)^q}$

$\Rightarrow \frac{n^p}{n^q\left(1+\frac{1}{n}\right)^q}$

$\lim_{n\to\infty} u_n \Rightarrow \frac{1}{n^{q-p}\underbrace{\left(1+\frac{1}{n}\right)^q}_{1}}$

$(q-p) > 1$ तो Cgt

$(q-p) \leq 1$ तो Dgt

[By P-Series Test

$u_n \Rightarrow \frac{1}{n^p}$ में यदि

$p > 1$ तो Cgt

और $p \leq 1$ तो Dgt

$$\lim_{n\to\infty}(U_n)^{\frac{1}{n}} = l$$

- **Cauchy's Root Test (कोशी का मूल परीक्षण**

यदि $\sum u_n$ एकं positive पदों की श्रेणी इस प्रकार है।

ताकि $\lim_{n\to\infty}(u_n)^{\frac{1}{n}} = l$

(i) यदि $l < 1$ **तो** Σu_n **अभिसारी (convergent)**

(ii) यदि $l > 1$ **तो** $\sum u_n$ **अपसारी (divergent)**

(iii) यदि $l = 1$ **तो जाँच विफल**

Test fails

Test of Convergence

Q. निम्न श्रेणी का अभिसरण की जाँच करो।

$$\Rightarrow \frac{2}{3}x + \left(\frac{3}{4}x\right)^2 + \left(\frac{4}{5}x\right)^3 + \left(\frac{5}{6}x\right)^4 + \cdots \ldots$$

(a) **Convergent If** $x < 1$ **& divergent If** $x \leqslant 2$.

(b) **Cgt If** $x > 1$ & $\mathrm{Dg}\ t$ **If** $x < 1$

(c) **cgt If** $x < 1$ **& dgt If** $x > 1$

(d) **None of these**

$$U_n = \left(\frac{n+1}{n+2} \cdot x\right)^n$$

$$\lim_{n \to \infty} \left(\frac{(n+1)}{(n+2)} \cdot x\right)$$

$$\lim_{n \to \infty} \frac{\cancel{n}\left(1+\frac{1}{n}\right)}{\cancel{n}\left(1+\frac{2}{n}\right)} \cdot x$$

$$\boxed{l \Rightarrow x}$$

(i) $l < 1$ Cgt $\Rightarrow x < 1$ Cgt

(ii) $l > 1$ dgt $\Rightarrow x > 1$ dgt

(iii) $l = 1$ = Test fail

D' Alembert's Ratio Test (द- लम्बर अनुपात परीक्षण)

$$\lim_{n\to\infty} \frac{U_n}{u_{n+1}} = l$$

यदि धनात्मक पदों की श्रेणी $\sum u_n$ इस प्रकार है कि $\text{Lim}_{n\to\infty} \frac{u_n}{u_{n+1}} = l$

(i) यदि $l > 1$ तो $\sum u_n$ अभिसारी (cgt)

(ii) यदि $l < 1$ तो $\sum u_n$ अपसारी **(Dgt)**

(iii) यदि $l = 1$ तो **Test fails**

Q. श्रेणी $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\cdots+\frac{x^n}{n^2+1}$ के अभिसरण की जाँच करो।

$U_n = \frac{x^n}{n^2+1}$, $U_{n+1} = \frac{x^{n+1}}{(n+1)^2+1}$

$\lim_{n\to\infty} \frac{U_n}{U_{n+1}}$

$\Rightarrow \frac{x^n \times [(n+1)^2+1]}{(n^2+1)\times x^{n+1}}$

$l = \frac{1}{x}$

$\lim_{n\to\infty} \Rightarrow \frac{\cancel{n^2}[(1+\frac{1}{n})^2+\frac{1}{n^2}]}{\cancel{n^2}[1+\frac{1}{n^2}]} \cdot \frac{1}{x} \Rightarrow \frac{1}{x}$

$l > 1$ cgt

$\frac{1}{x} > 1$ cgt

$\boxed{1 > x}$ cgt

$l < 1$ dgt

$\frac{1}{x} < 1$ $\boxed{1 < x}$