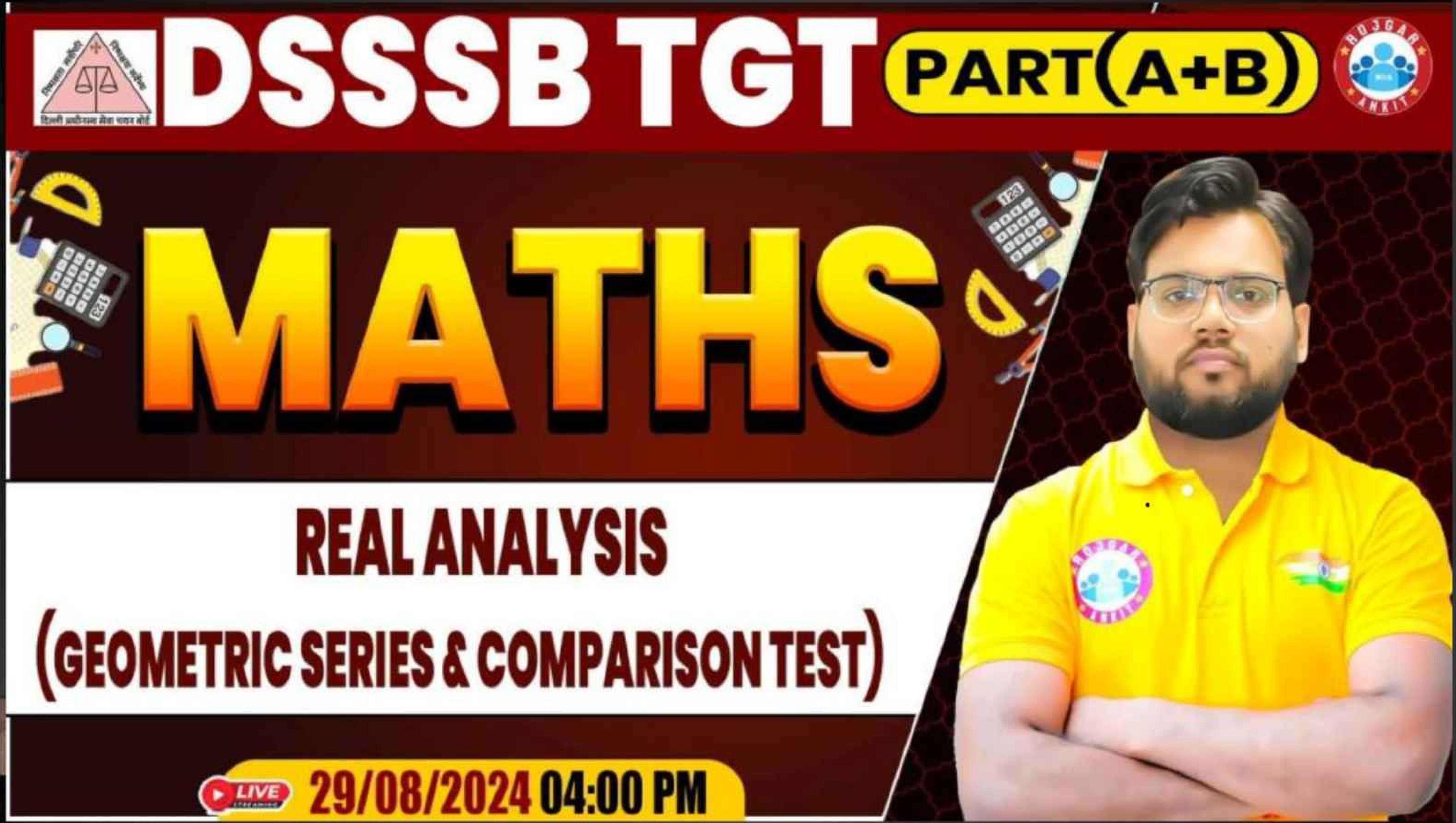

DSSSB TGT
PART(A+B)
MATHS
REAL ANALYSIS
(GEOMETRIC SERIES & COMPARISON TEST)
LIVE
29/08/2024 04:00 PM

आज 5:00 pm से

Vishal

Matic space

⇒ Convergent Sequence →

$\langle u_n \rangle \rightarrow$ **sequence;** $n \in N$

Q. $\langle u_n \rangle = \left\langle \frac{1}{n} \right\rangle$, जहाँ $n \in N$, कैसी sequence होगी?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

Q. $\langle u_n \rangle = \left\langle \frac{1}{2n-1} \right\rangle, n \in N$, कैसी sequence होगी?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

Q. $\langle u_n \rangle = \left\langle \frac{3n^2+5n}{9n^2+3n} \right\rangle$, जहाँ $n \in N$, कैसी sequence होगी?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

Q. $\langle u_n \rangle = \langle 2^n \rangle$, जहाँ $n \in N$, कैसी sequence होगी?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

Q. $\langle u_n \rangle = \left\langle \frac{3n^2+5n}{9n+3} \right\rangle$, जाएँ $n \in \mathcal{N}$, कैसी sequence होगी?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

Q $\rightarrow \langle x_n \rangle = \langle 1-(n)^2 \rangle$, जाँ $n \in N$, कैस sequence होगे?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

Q. $\langle x_n \rangle = \langle (-1)^n \cdot n \rangle$, जाएँ $n \in N$, कैसी sequence है?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

Q. $\langle u_n \rangle = \langle (-1)^n \rangle$, जहाँ $n \in N$, कैसी sequence है?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

Theorem –1:- **Every Convergent sequence Converges to a unique limit.**

Or

A sequence can not converge to more than one limit.

Theorem-2:- Every Convergent sequence is bounded but not conversely.

Theorem-3:- A sequence $\langle a_n \rangle$ is convergent if given $\varepsilon > 0$, there exist a positive integer m such that $|a_n - a_m| < \varepsilon$ whenever $n \geqslant m$.

NULL SEQUENCE:- A sequence $\langle a_n \rangle$ is said to be a null sequence if $\langle a_n \rangle$ converges to zero.

Infinite series :-

CONVERGENCE AND DIVERGENCE OF AN INFINITE SERIES ALL TESTS

1. GEOMETRIC SERIES → गुणोत्तर श्रेणी
2. COMPARISON TESTS
3. P-SERIES TEST OR HYPER HARMONIC SERIES
4. CAUCHY'S INTEGRAL TEST C.I.T.
5. D'ALEMBERT'S RATIO TEST
6. CAUCHY'S ROOT TEST
7. Raabe's TEST
8. LOGARITHMIC TEST

9. DE MORGAN'S AND BERTRAND'S TEST

10. LEIBNITZ'S TEST FOR THE CONVERGENCE OF ALTERNATING SERIES

11.ABSOLUTE CONVERGENCE

12.CONDITIONAL CONVERGENCE

13.ABEL'S TEST (For Arbitrary Series)

14.GAUSS TEST

15.DIRICHLET'S TEST

16. Harmonic Series

17. Binomial Series

1. Geometric series.

⇒ The geometrical series $a + ar + ar^2 + ar^3 + \cdots + \infty$.

$a \rightarrow$ पहला पद

$r \Rightarrow$ सार्वनिष्पात (Common Ration)

$\frac{U_{n+1}}{U_n} = \text{Constant}$

(i) Converges If $\rightarrow$ $\boxed{-1 < r < 1}$

(ii) Diverges If $\rightarrow r \geqslant 1$

(iii) Oscillates If $\rightarrow r \leqslant -1$

(iv) Oscillates finitely If $\rightarrow r = -1$

(v) Oscillates Infinitely If $\rightarrow r < -1$

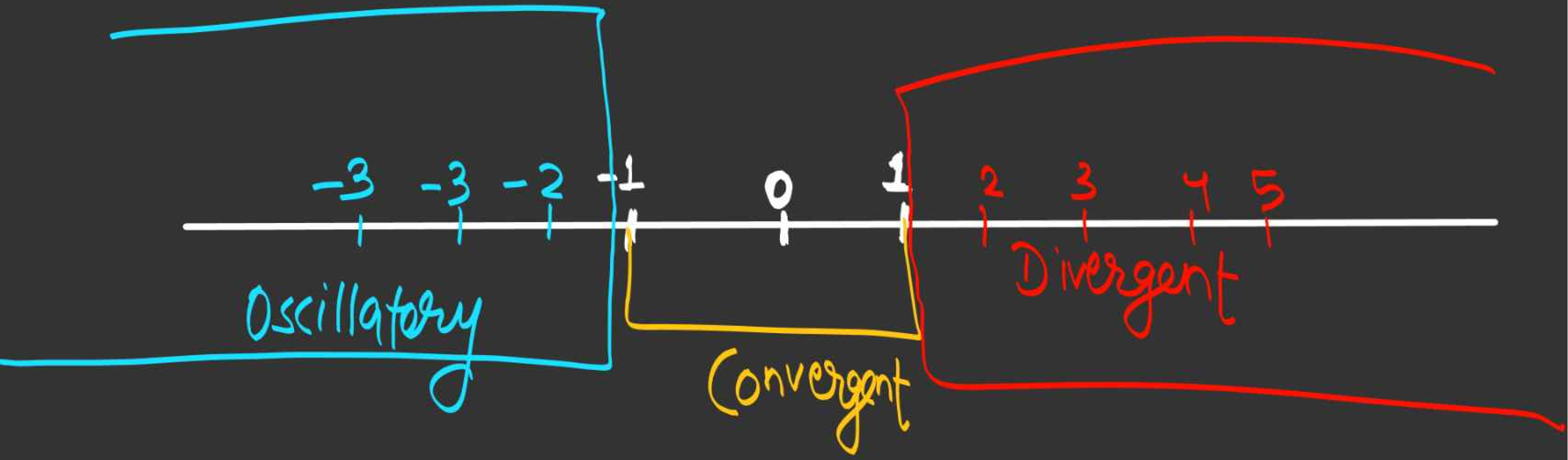
$$\Rightarrow 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+ - - - - \infty$$

$a = 1$

$-1 < r < 1$

(Cgt series)

$r \Rightarrow \frac{1}{2}$


-3
-3
-2
-1
0
1
2
3
4
5
Oscillatory
Convergent
Divergent

Q. The series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$, is-

$-1 < r < 1 \longrightarrow$ (a) Convergent

(b) Divergent

(c) Oscillating

(d) None of these

$a = 1$

$r = -\frac{1}{3} \Rightarrow -0.333$

-0.33

$-1 \quad 0 \quad 1$

Q. The series $1 + 5 + 25 + 125 + \cdots$, is.

(a) Convergent

(b) Divergent

(c) Oscillating

(d) None of these

$r = 5 > 1$

$\boxed{r = 5}$

Q. The series $(-2) + (-2)^2 + (-2)^3 + (-2)^4 + \cdots$, is-

(a) Convergent

(b) Divergent

(c) Oscillating

(d) None of these

$r < -1$

$$r = \frac{(-2)^2}{(-2)} = -2$$

$$= \frac{-2 \times \cancel{-2}}{\cancel{-2}}$$

Q. $\frac{3}{1} + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \cdots\cdots$ is-

(a) Convergent

(b) Divergent

(c) Oscillating

(d) None of these

$r = \frac{1}{2}$

- Cgt 1 # $1-\frac{1}{9}+\frac{1}{81}-\frac{1}{729}+ \cdots \infty$ Convergent

$r=-\frac{1}{9}$ $r=-1<-\frac{1}{9}<1$

- Osc 2 ⇒ $1-9+81-729+ \cdots \infty$

$r=-9$ $r<-1$ Ocsillating infinitely

- Dgt 3 ⇒ $1+9+81+729+ \cdots \infty$

$r=9$ $r>1$ Divergent

- Dgt 4 ⇒ $-1-9-81-729- \cdots \infty$

$r=9$ $r>1$ Divergent

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ में यदि $p>1$ तो cgt
$p \le 1$ Dgt

Comparison test

If $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ are two positive terms series

(i) $\sum_{n=1}^{\infty} V_n$ is convergent then $\sum_{n=1}^{\infty} u_n$ is also Convergent-

ये दिया गया होगा।

If $\text{Lim}_{n\to\infty} \frac{u_n}{V_n} = l <^{\text{finite}}_{\text{P+ve}}$ $u_n \leqslant V_n$

$$\frac{U_n}{V_n} = l <^{\text{finite}}_{\text{positive}}$$

$$\boxed{U_n \leqslant V_n}$$

$$\sum_{n=1}^{\infty} \frac{n}{\underbrace{(n+1)(2n+1)}} = U_n$$

$$\sum_{n=1}^{\infty} V_n = \sum_{n=1}^{\infty} \frac{n}{n^2}$$

$$V_n = \sum_{n=1}^{\infty} \frac{1}{n}$$

$V_n = \frac{}{n^3} \Rightarrow \frac{}{n^2}$

$\sum_{n=1}^{\infty} V_n = \sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow V_n$ is

यदि इसपर $\lim_{x\to\infty}$ apply करने पर finite व positive मान आए तो V_n के convergent होने के कारण ये (U_n) भी convergent होगा।

$\cancel{n}$

$(n+1)(2n+1)$

$\frac{1}{n^2}$

$\frac{n^2}{2n^2+3n+1} \Rightarrow \lim_{n\to\infty} \frac{\cancel{n^2}}{\cancel{n^2}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)}$

positive

(ii) $\sum_{n=1}^{\infty} V_n$ **is divergent then** $\sum_{n=1}^{\infty} U_n$ **is also divergent.**

If $\mathbf{Lim}_{n\to\infty} \frac{u_n}{v_n} = l \sum_{i+ve}^{\text{Finite}} u_n \geqslant V_n$

$\lim_{n\to\infty} \frac{u_n}{V_n} = l$ finite / positive

$\boxed{U_n \geq V_n}$

• P -Series Test or Hyper Harmonic Series →

The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $\quad n \in N$

(i) **Converges If** $\Rightarrow P > 1$

(ii) **Diverges If** $\Rightarrow P \leqslant 1$

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

Ex → $\sum_{n=1}^{\infty} u_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$ $\quad$ Cgt or Dgt ? $\quad p$

$p=2$

Convergent by p-series test $(\because p > 1)$

Ex- $\sum_{n=1}^{\infty} \frac{1}{n^3 \log n}$ कैसी series है? u_{n_1}

(a) Convergent

(b) Divergent

(c) Oscillating

(d) None of these

$$\sum_{n=1}^{\infty} \frac{1}{n^3 \log n}$$

$$V_n \Rightarrow \frac{1}{n^3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3} \rightarrow$$

Cgt by p-series Test ($\because p>1$)

$$\lim_{n\to\infty} \frac{u_n}{V_n}$$

$$\Rightarrow \frac{\frac{1}{\cancel{n^3}\log n}}{\frac{1}{\cancel{n^3}}} \Rightarrow \lim_{n\to\infty} \Rightarrow \frac{1}{\log n}$$

$$\Rightarrow \frac{1}{\infty} \Rightarrow 0$$

→ finite

→ positive