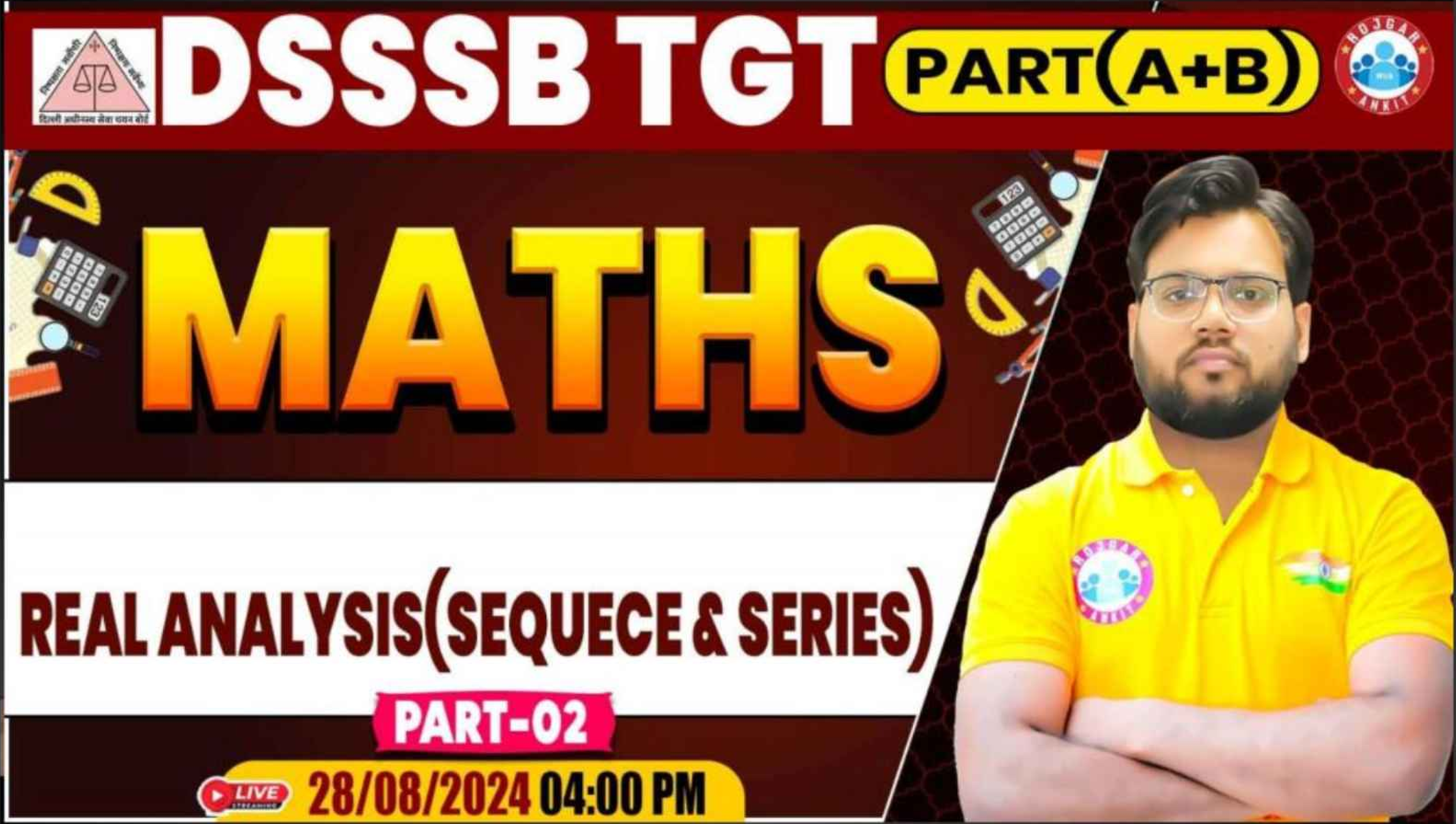
DSSSB TGT
PART(A+B)
MATHS
REAL ANALYSIS(SEQUECE & SERIES)
PART-02
LIVE
28/08/2024 04:00 PM

⇒ Convergent Sequence →

$\langle u_n \rangle \rightarrow$ **sequence;** $n \in N$

$\lim_{n \to \infty} u_n = l$ | $u_n \to l$ at $n = \infty$

l → unique finite

If Limit of the sequence exists [unique finite] Then the Sequence is Convergent.

Otherwise → Divergent ($+\infty$, $-\infty$)

Otherwise → Oscillatory (Not unique Limit)

Q. $\langle u_n \rangle = \left\langle \frac{1}{n} \right\rangle$, जहाँ $n \in N$, कैसी sequence होगी?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

$+\infty$

$-\infty$

$\langle 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \text{------} \rangle$

$\lim_{n\to\infty} \Rightarrow \left\langle \frac{1}{n} \right\rangle$

$\Rightarrow \frac{1}{\infty} = 0$ [unique finite]

Converges to Zero

Q. $\langle u_n \rangle = \left\langle \frac{1}{2n-1} \right\rangle, n \in N$, **कैसी sequence होगी?**

✓ **(a) Convergent**

(b) Divergent

(c) Oscilatory

(d) None of these

$$\left\langle \frac{1}{1}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7} \cdots \right\rangle$$

$$\lim_{n \to \infty} u_n$$

$$\frac{1}{2\infty - 1}$$

$$\Rightarrow \frac{1}{\infty} = 0$$

unique, finite

Converge to zero (unique)

$$\frac{1}{0} = \infty$$

$$\boxed{\frac{1}{\infty} = 0}$$

Q. $\langle u_n \rangle = \left\langle \frac{3n^2+5n}{9n^2+3n} \right\rangle$, **जहाँ** $n \in N$, **कैसी sequence होगी?**

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

$$\lim_{n\to\infty}\left[\frac{\cancel{n}(3n+5)}{\cancel{n}(9n+3)}\right]$$

$$\lim_{n\to\infty}\left[\frac{\cancel{n}\left(3+\frac{5}{n}\right)}{\cancel{n}\left(9+\frac{3}{n}\right)}\right]$$

$$\frac{3+\frac{5}{\infty}}{9+\frac{3}{\infty}} \Rightarrow \frac{3+0}{9+0} \Rightarrow \frac{3}{9} \Rightarrow \frac{1}{3}$$

Unique, finite

Converges to $\frac{1}{3}$

Q. $\langle u_n \rangle = \langle 2^n \rangle$, **जहाँ** $n \in N$, **कैसी sequence होगी?**

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

$\langle 2^1, 2^2, 2^3, 2^4, 2^5 - - - - - \rangle$

$\lim_{n \to \infty} [2^{\infty}]$

$\Rightarrow \infty$

Q. $\langle u_n \rangle = \left\langle \frac{3n^2+5n}{9n+3} \right\rangle$, जाएँ $n \in \mathcal{N}$, कैसी sequence होगी?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

$$\lim_{n\to\infty}\left[\frac{3n^2+5n}{9n+3}\right]$$

$$\lim_{n\to\infty}\left[\frac{\cancel{n}(3n+5)}{\cancel{n}\left(9+\frac{3}{n}\right)}\right]$$

$$\lim_{n\to\infty}\left[\frac{3n+5}{9+\frac{3}{n}}\right] = \infty$$

$+\infty$

$-\infty$

$$\frac{\cancel{n}\left(3+\frac{5}{n}\right)}{\cancel{n}\left(\frac{9}{n}+\frac{3}{n^2}\right)}$$

$$\Rightarrow \frac{3+0}{0+0} \Rightarrow \frac{3}{0} = \infty$$

Q $\rightarrow \langle x_n \rangle = \langle 1-(n)^2 \rangle$, **जाँ** $n \in N$, **कैस sequence होगे?**

(a) **Convergent**

(b) **Divergent**

(c) **Oscilatory**

(d) **None of these**

$\langle 0, -3, -8, -15, -24, -35, \ldots \rangle$

$\lim_{n \to \infty} [1-(n)^2]$

$(1-\infty)$

$= -\infty$

Q. $\langle x_n \rangle = \langle (-1)^n \cdot n \rangle$, जाएँ $n \in N$, कैसी sequence है?

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

$\langle -1, 2, -3, 4, -5, 6, -7, 8, -9, 10 \ldots \rangle$

$$\lim_{n \to \infty} \left((-1)^{\infty} \cdot \infty \right) \begin{cases} \infty \\ -\infty \end{cases}$$

Infinitely Oscillatory Sequence

Q. $\langle u_n \rangle = \langle (-1)^n \rangle$, **जहाँ** $n \in N$, **कैसी sequence है?**

(a) Convergent

(b) Divergent

(c) Oscilatory

(d) None of these

$\langle -1, 1, -1, 1, -1, 1, -1, 1 \cdots\cdots \rangle$

$\lim_{n \to \infty} \langle (-1)^n \rangle \to 1, -1$

$\{-1, 1\}$

Finitely oscillating Sequence

Theorem –1:- **Every Convergent sequence Converges to a unique limit.**

Or

A sequence can not converge to more than one limit.

Unique Limit

Theorem-2:- Every Convergent sequence is bounded but not conversely.

Theorem-3:- A sequence $\langle a_n \rangle$ is convergent if given $\varepsilon > 0$, there exist a positive integer m such that $|a_n - a_m| < \varepsilon$ whenever $n \geqslant m$.

NULL SEQUENCE:- A sequence $\langle a_n \rangle$ is said to be a null sequence if $\langle a_n \rangle$ converges to zero.

Ex→ $\left\langle \frac{1}{n} \right\rangle, \left\langle \frac{1}{2n} \right\rangle, \left\langle \frac{1}{2^n} \right\rangle, \left\langle \frac{1}{n^2} \right\rangle$

$= \frac{1}{\infty} \quad \frac{1}{\infty} \quad \frac{1}{2^\infty} \quad \frac{1}{\infty^2}$

$\Rightarrow 0 \quad 0 \quad 0 \quad 0$

Infinite series :-

$$\sum_{n=1}^{\infty} U_n \Rightarrow U_1 + U_2 + U_3 + U_4 + \cdots + U_n + \cdots \infty$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \Rightarrow 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots \frac{1}{\infty}$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n} \Rightarrow \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots \infty$$

Convergent Series	Divergent Series	Oscillatory series
$\lim_{n\to\infty} S_n = l$ (Unique finite) Ex→ $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\cdots\infty$ गुणोत्तर श्रेणी $a=1$ $r=\frac{1}{2}$ $S_n \Rightarrow \frac{a}{1-r}$ $\lim_{n\to\infty} S_n \Rightarrow 2$ (unique)	$\lim_{n\to\infty} S_n = l$ → $+\infty$, → $-\infty$ Ex→ $1^2+2^2+3^2+4^2+\cdots+\infty$ $=\infty$	$\lim_{n\to\infty} S_n = l$ (not unique Limit)