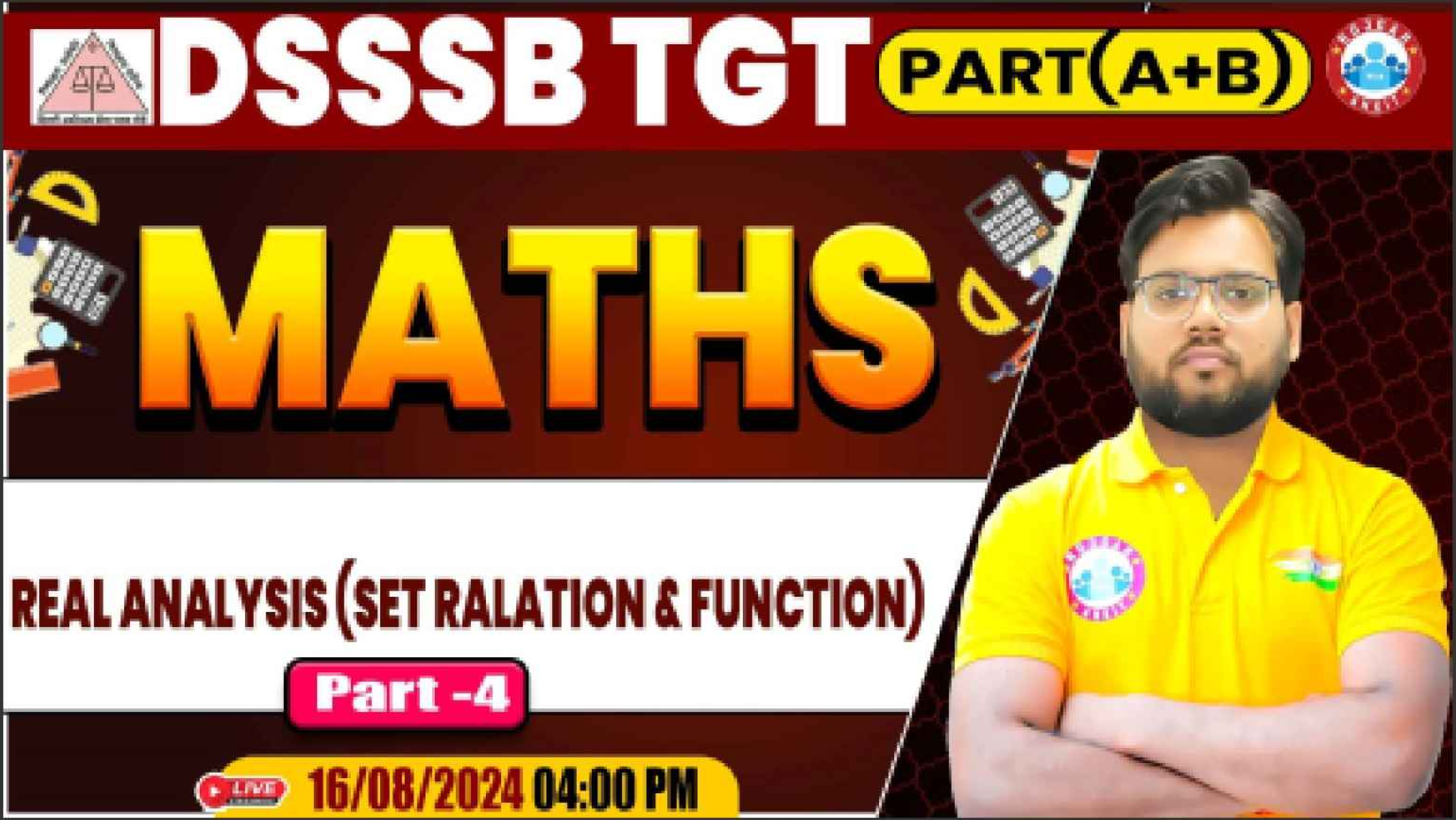
DSSSB TGT
PART(A+B)
MATHS
REAL ANALYSIS (SET RALATION & FUNCTION)
Part -4
LIVE
16/08/2024 04:00 PM

Inverse Relation

$$R^{-1} = \{(b, a) \mid (a, b) \in R\}$$

Obviously

There is a Relation R

$A = \{2, 4, 5\}$ to $B = \{1, 2, 3, 4, 6, 8\}$

$aRb \Leftrightarrow$ "a is divisor of b"

(i) $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$

(ii) domain of R^{-1} = range of R

(iii) range of R^{-1} = domain of R

$R^{-1} = ?$

$\Rightarrow$ (iv) $(R^{-1})^{-1} = R$

$\Rightarrow R \Rightarrow \{(2,2), (2,4), (2,6), (2,8), (4,4), (4,8)\}$

Domain $\Rightarrow \{2, 4\}$

Range $\Rightarrow \{2, 4, 6, 8\}$

$\Rightarrow R^{-1} = \{(2,2), (4,2), (6,2), (8,2), (4,4), (8,4)\}$

There is a relation R

$A = \{1, 2, 3, 4, \ldots 30\}$ to $B = \{1, 2, 3, 4, \ldots 20\}$

$aRb \Leftrightarrow a = 4b$ then $R^{-1} = ?$

$R = \{(4,1), (8,2), (12,3), (16,4), (20,5), (24,6), (28,7)\}$

Domain of $R = ?$

Range of $R = ?$

$R^{-1} = \{(1,4), (2,8), (3,12), (4,16), (5,20), (6,24), (7,28)\}$

Domain of $R = \{4, 8, 12, 16, 20, 24, 28\}$

Range of $R = \{1, 2, 3, 4, 5, 6, 7\}$

Types of Relations defined on a Set

(i) Identity Relation: A relation R defined on a set A is called the identity relation on A if every element of A is related to itself and only to itself under R.

$R = \{(4,4), (2,2), (3,3)\}$

This is denoted by I_A

Thus

$$I_A = \{(a, a) \mid a \in A\}$$

(ii) Reflexive Relation: A relation R defined on a set A is reflexive if every element of A has R-relation with itself. Thus

$$R \text{ is reflexive on } A \begin{cases} \Leftrightarrow aRa \, \forall a \in A \\ \Leftrightarrow (a, a) \in R \end{cases}$$

Note: On a set A, its identity relation I_A is reflexive but any reflexive relation on A may not be the identity relation.

x, y are two sets –

$xRy = 1 + xy > 0$ is it Reflexive Relation?

$x = y$

$1 + x \cdot x > 0$

$1 + x^2 > 0$ ✓

Types of Relations defined on a Set

(iii) Symmetric Relation: A relation R defined on a set A is called symmetric relation if for any elements $a, b \in R$, whenever a is R-related to b, then necessarily b must be R-related to a. Hence

$R = \{(1,4), (4,1)\}$

$$R \text{ is symmetric on } A \begin{cases} \Leftrightarrow aRb \Rightarrow bRa, a, b \in A \\ \Leftrightarrow (a, b) \in R \Rightarrow (b, a) \in R \\ \Leftrightarrow R = R^{-1} \end{cases}$$

$a \neq b$

(iv) Antisymmetric Relation: A relation R defined on a set A is called antisymmetric relation if for any elements $a, b \in R, a \neq b$ whenever a is R-related to b, then b should not be R-related to a. Hence

$$R \text{ is antisymmetric on } A \begin{cases} \Leftrightarrow a \neq b, aRb \Rightarrow b\not{R}a \\ \Leftrightarrow aRb \wedge bRa \Rightarrow a = b \\ \Leftrightarrow (a, b) \in R \wedge (b, a) \in R \Rightarrow \boxed{a = b} \end{cases}$$

(v) Transitive Relation: A relation R defined on a set A is called transitive relation if for any elements $a, b, c \in R$, whenever a is R-related to b and b is R-related to c, then a should be R-related to c.

Hence

$$R \text{ is transitive on } A \begin{cases} \Leftrightarrow aRb \wedge bRc \Rightarrow aRc \\ \Leftrightarrow (a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R \end{cases}$$

⇒ In the set L of all lines in a place
a Relation R defined by
$x R y \Leftrightarrow x \parallel y$

Equivalence Relation

A relation R defined on a set A is an equivalence relation, if

(i) R is reflexive i.e., $aRa \, \forall a \in A$ (a, a)

(ii) R is symmetric i.e., $aRb \Rightarrow bRa$

(iii) R is transitive i.e., $aRb \wedge bRc \Rightarrow aRc$

$x \; y \quad y \; z \quad x \; z$

Note: Void relation on a set is symmetric, antisymmetric and transitive but it is not reflexive. So void relation on a set is not an equivalence relation.

Some Properties of Equivalence Relation

(i) If R is an equivalence relation on A, then R^{-1} is also an equivalence relation on A, i.e., inverse relation of an equivalence relation is also an equivalence relation.

(ii) If R and S are two equivalence relations on A, then $R \cap S$ is also an equivalence relation on A, i.e., intersection of two equivalence relations is also an equivalence relation.

(iii) If R and S are two equivalence relations on A, then $R \cup S$ may not be an equivalence relation on A, i.e. union of two equivalence relations may not be an equivalence relation.