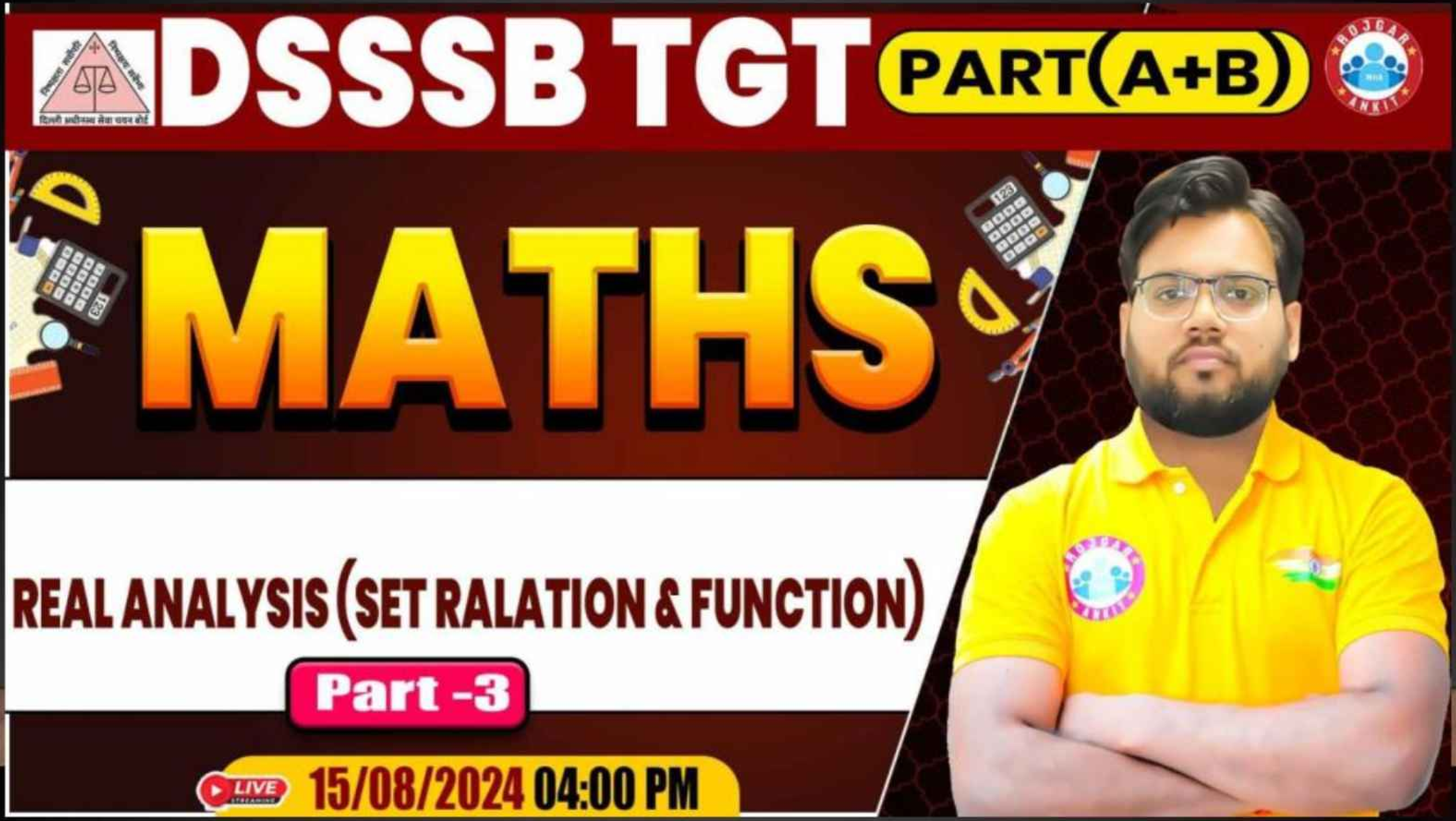
DSSSB TGT
PART (A+B)
MATHS
REAL ANALYSIS (SET RALATION & FUNCTION)
Part -3
LIVE
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1. Set

A well-defined list or collection of things is called a Set.

Sets are gencrally expressed by capital letters of English alphabet $A, B, C, \ldots.$ etc. and their elements are expressed by small letters a, b, c, d, x, y etc.

If a is an element of a set A and b is not an element of a set B. then symbolically we write it as $a \in A, b \notin B$.

2. Representation of a Set

Roster Form: In this form a set is represented by listing all or some of its elements. The clements are separated by commas and enclosed in curly brackets {}

Set Builder Form: In this form we use a letter x to represent an arbitrary element, write specific properties say $P(x)$, satisfied by elements of the set and the set is represented as

$$\{x \mid P(x)\} \text{ or } \{x: P(x)\}$$

Set of natural numbers

Set of whole numbers

Set of integers

Set of rational numbers

Set of irrational numbers

Set of real numbers

Set of Complex numbers

3. Void Set or Null Set

If a set has no element, then it is called a void or null set and it is expressed as ϕ or {}

Finite and Infinite Sets

A set whith is cither empty or has a tinite number of difigent elements is called a finite set and aset which is not finte is called an in finite set.

In a linite set A , the number ol its different elements is called its order and it is expressed as

$$n(A) \text{ or } o(A)$$

Notes :

(i) For every set $A, n(\Lambda) \geq 0$ and $n(A) \in W$.

(ii) If $n(A) = 1$, then A is called singleton set.

Example. $A = |x|x \in Z, |x| \leq 2$

Equal and Equivalent Sets

Two sets A and B are said to be equal sets if every element of A is in B and every element of B is in A

$$A = B \Leftrightarrow x \in A \Rightarrow x \in B \wedge x \in B \Rightarrow x \in A$$

Two finite sets A and B are said to be equivalent sets.

$(A \sim B)$ if they have equal number of elements

$$A \sim B \Leftrightarrow n(A) = n(B)$$

Subset

If every elemen of aset A is in B, then A is acalida subset of B which is erpresed as

1. $A \subset B$

Hence

$$A \subset B \Leftrightarrow x \in A \Rightarrow x \in B$$

If it is a subset of B, the B s said to bea smper set of A Funther if $A \subset B$ but $A \neq B$ then it is called a proper substi of B.

Property of Subset

For ainy sets A, B, C

(i) $A \subset A$

(ii) $\phi \subset A$

(iii) $A = B \Leftrightarrow A \in B \wedge B \subset A$

(iv) $A \subset B \wedge B \subset C \Rightarrow A \subset C$

Intervals as Infinite Subsets of R

(i) Closed Interval:

$$[a, b] = \{x \mid x \in \mathbf{R}, a \leq x \leq b\}$$

(ii) Open Interval:

(a, b) or $]a, b[= \{x \mid x \in \mathbf{R}, a < x < b\}$

(iii) Semi-open (closed) Interval:

$$(a, b] = \{x \mid x \in \mathbf{R}, a < x \leq b\}$$
$$[a, b) = \{x \mid x \in \mathbf{R}, a \leq x < b\}$$

Interval $(-\infty, \infty)$ represents the set of real numbers R or real line.

- The number $(b-a)$ is called the length of any of the abowe intertals.

SETS & RELATIONS

Power Set

The set of all subsets of a set A is called the Power set of A and symbolically it is expressed as

Hence

$$P(A) \text{ or } 2^A$$

$$P(A) = \{X \mid X \subset A\}$$

Further if A is a finte set of order n, then it can be easily seen that the number of all subsets of A is 2^n and so $n(P(A)) = 2^n$.

Hence

$n(A) = n$ $\Rightarrow$ Total number of subsets of $A = 2^n$

$\Rightarrow n(P(A)) = 2^n, n(P(A)) \geq 1$

Properties of Union and Intersection Opcrations

If A, B, C are any three sets, then

(i) $\left.\begin{matrix} A \cup A = A \\ A \cap A = A \end{matrix}\right\}$ (Idempotent laws)

(ii) $\left.\begin{matrix} A \cup \phi = A \\ A \cap \phi = \phi \end{matrix}\right\}$

(iii) $\left.\begin{matrix} A \cup U = U \\ A \cap U = A \end{matrix}\right\}$

(iv) $\left.\begin{array}{l} A \cup B = B \cup A \\ A \cap B = B \cap A \end{array}\right\}$ (Commutativity)

(v) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associativity)

$(A \cap B) \cap C = A \cap (B \cap C)$

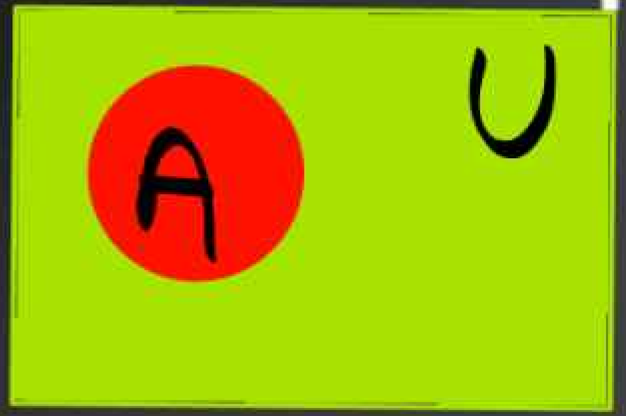
(vi) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ & $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)\}$

(vii) $\left.\begin{array}{l} A \subset A \cup B, B \subset A \cup B \\ A \cap B \subset A, A \cap B \subset B \end{array}\right\}$

Complement of a Set

Let A be a subset of a universal set U, then the set $U - A$ is called the complement set of A and it is denoted by A' or A^C.

Thus



$$A' = \{x \mid x \in U, x \notin A\}$$

$$A' / A^c \Rightarrow U - A$$

Properties of Product of Sets

If A, B, C, D are any sets, then following properties can easily be established:

(i) $n(A \times B) = n(A) \cdot n(B)$, where A, B are finite sets.

(ii) $A \times B = \phi \Leftrightarrow A = \phi$ or $B = \phi$

(iii) $A \times B \neq B \times A$

(iv) $A \times B = B \times A \Leftrightarrow A = B$

(v) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(vi) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(vii) $A \times (B - C) = (A \times B) - (A \times C)$

(viii) $A \subset B \Rightarrow A \times A = (A \times B) \cap (B \times A)$

(ix) $A \subset B, C \subset D \Rightarrow (A \times C) \subset (B \times D)$

$(x) (A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Cartesian Product of Sets

The cartesian product $A \times B$ of two sets A, B is the set of all those ordered pairs in which first element belongs to A and second element belongs to B. Thus

In particular

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$
$$A^2 = A \times A = \{(a, b) \mid a, b \in A\}$$

Q. If $A = \{1, 3\}, B = \{0, 2, 4\}$, then

$A \times B = \{(1,0), (1,2)$

$B \times A =$

$A^2 =$

Some Properties of Difference and Complement Sets

$A = \{1, 2, 3, 4, 5\}$

$B = \{3, 4, 5, 6, 7, 8\}$

$A - B = \{1, 2\}$

$B - A = \{6, 7, 8\}$

If A, B, C are any sets, then

(i) $A - A = \phi, A - \phi = A, \phi - A = \phi$

(ii) $A - B \subset A, B - A \subset B$

(iii) $(A - B) \cup B = A \cup B$

(iv) $(A - B) \cap B = \emptyset$

(v) $A - B \neq B - A$

(vi) $(A - B) - C \neq A - (B - C)$

(vii) $(A - B) \cap (B - A) = \phi$

(viii) $\phi' = U, U' = \phi$

(ix) $(A')' = A$

(x) $A \cup A' = U$

(xi) $A \cap A' = \phi$

(xii) $A \subset B \Rightarrow B' \subset A'$

(xiii) $A - B = A \cap B' = B' - A'$

(xiv) $A \cap (B - C) = (A \cap B) - (A \cap C)$

$A = \{1,2,3,4,5,6\}$

$B = \{2,4,6,7\}$

$A - B = \{1,3,5\}$

$B - A = \{7\}$

$(A-B) \cap (B-A) = \phi$

$A = \{2,4,6\}$

$A' = \{1,3,5\}$

$A \cup A'$
$= \{1,2,3,4,5,6\}$

$A \cap A' = \{\ \}$

$(xv) A - (B \cup C) = (A - B) \cap (A - C)$

(xvi) $A - (B \cap C) = (A - B) \cup (A - C)$

(xvii) $\left.\begin{array}{l}(A \cup B)' = A' \cap B' \\ (A \cap B)' = A' \cup B'\end{array}\right\}$

Properties of Symmetric Difference Operation

If A, B, C are any sets, then

(i) $A \triangle A = \phi$

(ii) $A \Delta \phi = A$

(iii) $A \triangle B = B \triangle A$

(iv) $A \triangle (B \Delta C) = (A \triangle B) \Delta C$

(v) $A \triangle B = A \triangle C \Rightarrow B = C$

(vi) $A \cap (B \triangle C) = (A \cap B) \Delta (A \cap C)$

A∆B

Note: For any two sets A, B, sets $(A - B), (B - A)$ and $A \cap B$ are disjoint sels,

संबंध

Relation → Let A and B be two sets. Then a Relation R from set A to set B is a subset of $A \times B$. Then R is a relation from A to $B \Rightarrow R \subseteq A \times B$.

$$R \subset A \times B$$

⇒ **If A and B are finite sets of Order m and n Respectively, then the number of subset of $A \times B$ is 2^{mn} ?**

Hence

Total number of Relation defined from A to $B = 2^{mn}$.

$m = O(A) = 3$ $n = O(B) = 3$

$A = \{1, 2, 3\}$, $B = \{4, 6, 9\}$

No. of Relation ⇒ $2^{mn} \Rightarrow 2^{3 \times 3} \Rightarrow 2^{9}$

$A=\{1,2,3,4,5,6,7,8\}$, $B=\{1,2,3,4\}$

$aRb \Rightarrow a=2b$, then $R=?$

$a\in A$, $b\in B$

$a=2b$

$R=\{(2,1),(4,2),(6,3),(8,4)\}$

Domain $=(2,4,6,8)$

Range $\Rightarrow(1,2,3,4)$

$A = \{1, 2, 3, \ldots 70\}$, $B = \{1, 2, 3, \ldots 100\}$

$xRy \Rightarrow x = y^2$ where $x \in A$, $y \in B$

$R = ?$

$R = \{(1,1), (4,2), (9,3), (16,4), (25,5), (36,6), (49,7), (64,8)\}$

Domain = (1, 4, 9, 16, 25, 36, 49, 64)

Range = (1, 2, 3, 4, 5, 6, 7, 8)

x ← (4, 16)

16, 4

$x = y^2$

$A = \{2, 4, 6, 8, 10, \ldots, 100\}$, $B = \{0, 1, 2, 3, 4 \ldots 100\}$

$2^2 = 4$

$aRb = a^2 = b$ where $a \in A$
$b \in B$

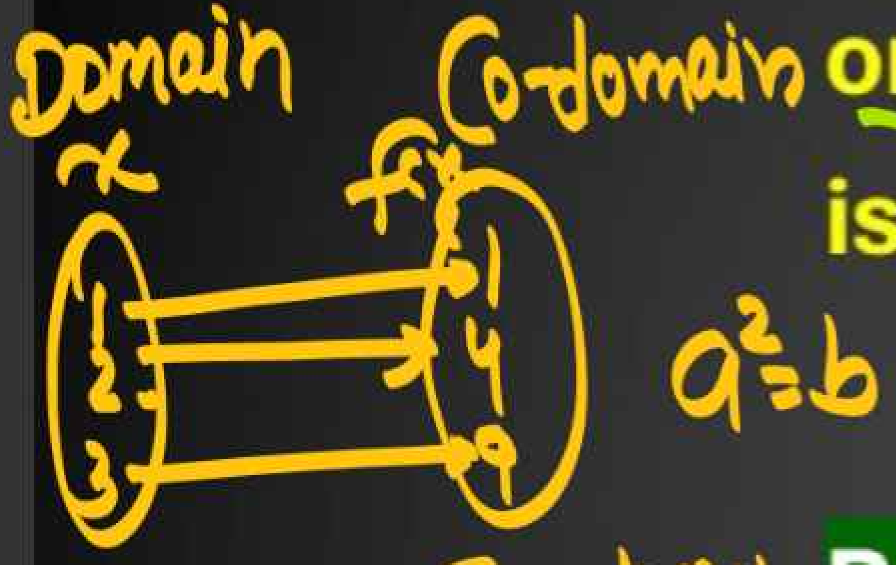
$R = ?$

$R = \{(2, 4), (4, 16), (6, 36), (8, 64), (10, 100)\}$

Domain ⇒ $\{2, 4, 6, 8, 10\}$

Range ⇒ $(4, 16, 36, 64, 100)$

Domain → The set of all first elements of ordered pair in relation R from a set A to set B is called Domain of R.



first element

Range → The set of all second element in Relation R from set A to set B.

Co-domain

(1,1), (2,4), (3,9)

Range = (1,4,9)

⇒ The whole set B is called co-domain of relation R.

⇒ Range ⊆ Co-Domain.

Trivial Relation on a Set

The following two relations are called trivial relations which are defined as follows:

(i) Void (or Empty) Relation:

A relation R defined on as set A is called its void or empty relation if $R = \phi$, i.e., if no element of A is related to any element of A.

(ii) Universal Relation:

A relation R defined on a set A is called its universal relation, if you $R = A \times A$, i.e., if every element of A is related to every element of A.

Inverse Relation

$$R^{-1} = \{(b, a) \mid (a, b) \in R\}$$

Obviously

(i) $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$

(ii) domain of R^{-1} = range of R

(iii) range of R^{-1} = domain of R

(iv) $\left(R^{-1}\right)^{-1} = R$

$R = (a, b)$ — a → Domain, b → Range

$R^{-1} = (b, a)$ — b → Domain, a → Range

$R \to (a, b)$

$R^{-1} = (b, a)$

$(R^{-1})^{-1} = (a, b) = R$