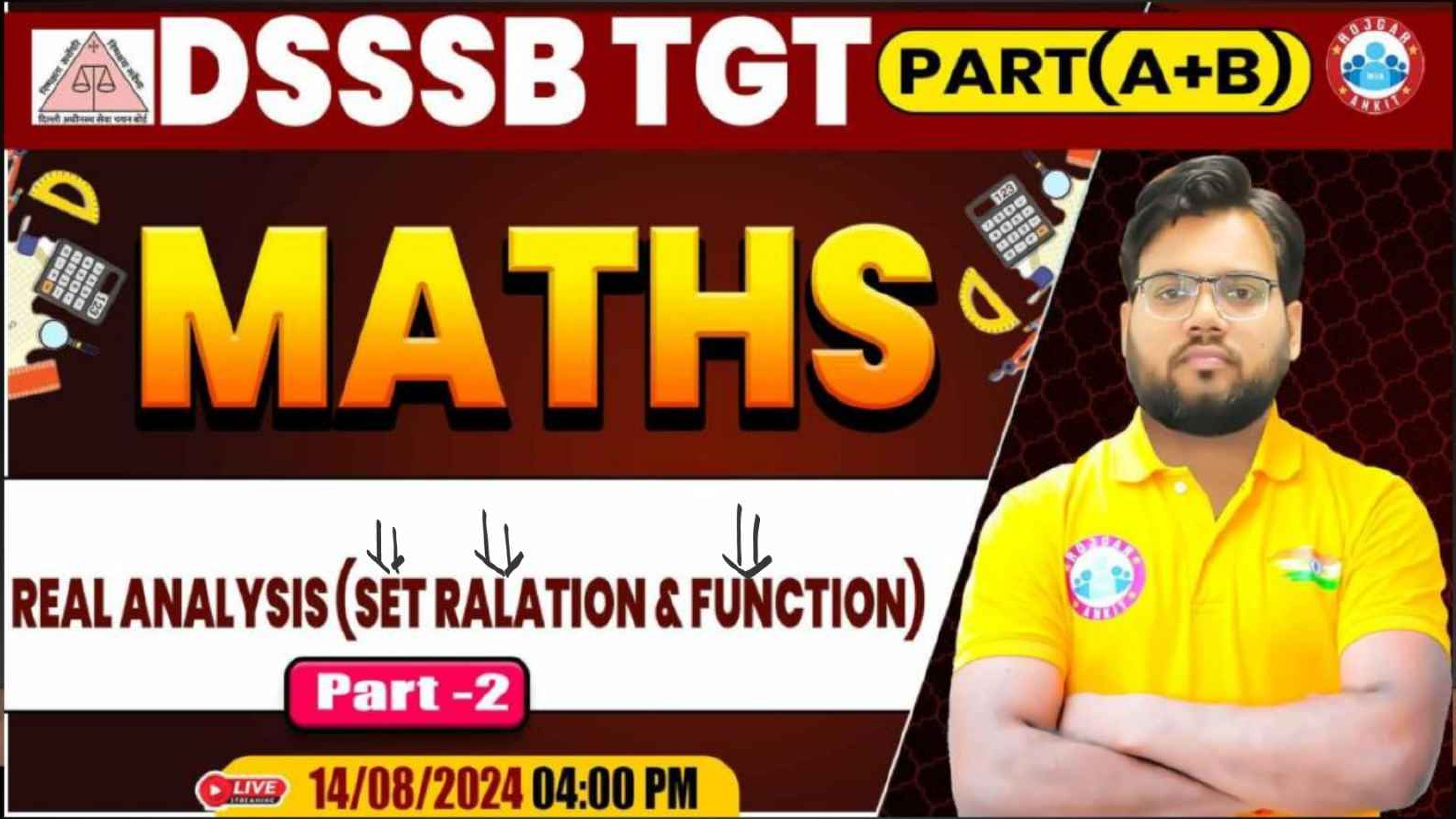

DSSSB TGT
PART(A+B)
MATHS
REAL ANALYSIS (SET RALATION & FUNCTION)
Part -2
LIVE
14/08/2024 04:00 PM

1. Set

A well-defined list or collection of things is called a Set.

Sets are gencrally expressed by capital letters of English alphabet $A, B, C, \ldots$ etc. and their elements are expressed by small letters a, b, c, d, x, y etc.

If a is an element of a set A and b is not an element of a set B. then symbolically we write it as $a \in A, b \notin B$.

2. Representation of a Set

Roster Form: In this form a set is represented by listing all or some of its elements. The clements are separated by commas and enclosed in curly brackets {}

Set Builder Form: In this form we use a letter x to represent an arbitrary element, write specific properties say $P(x)$, satisfied by elements of the set and the set is represented as

$$\{x \mid P(x)\} \text{ or } \{x: P(x)\}$$

Set of natural numbers

Set of whole numbers

Set of integers

Set of rational numbers

Set of irrational numbers

Set of real numbers

Set of Complex numbers

3. Void Set or Null Set

If a set has no element, then it is called a void or null set and it is expressed as ϕ or $\{\}$

Finite and Infinite Sets

A set whith is cither empty or has a tinite number of difigent elements is called a finite set and aset which is not finte is called an in finite set.

In a linite set A , the number ol its different elements is called its order and it is expressed as

$$n(A) \text{ or } o(A)$$

Notes :

(i) For every set $A, n(\Lambda) \geq 0$ and $n(A) \in W$.

(ii) If $n(A) = 1$, then A is called singleton set.

Example. $A = |x|x \in Z, |x| \leq 2$

Equal and Equivalent Sets

Two sets A and B are said to be equal sets if every element of A is in B and every element of B is in A

$$A = B \Leftrightarrow x \in A \Rightarrow x \in B \wedge x \in B \Rightarrow x \in A$$

Two finite sets A and B are said to be equivalent sets.

$(A \sim B)$ if they have equal number of elements

$$A \sim B \Leftrightarrow n(A) = n(B)$$

Subset

If every elemen of aset A is in B, then A is Called subset of B which is Expressed as

1. $\underbrace{A \subset B}$

Hence

$\boxed{A \subset B}$

$A \subset B \Leftrightarrow x \in A \Rightarrow x \in B$

$A = \{1, 2, 3, 4, 5\}$

$B = \{x \mid x \in N\}$

$\boxed{A \subset B}$

$A = \{x \mid x \in N\} = \{1, 2, 3, 4, \ldots \infty\}$

$B = \{x \mid x \in W\} = \{0, 1, 2, 3, 4, \ldots \infty\}$

If it is a subset of B, the B is said to be a Super set of A Further if $A \subset B$ but $A \neq B$ then it is called a proper Subset of B.

Property of Subset

For any sets A, B, C

(i) $A \subset A$

(ii) $\phi \subset A$

(iii) $A = B \Leftrightarrow A \subset B \land B \subset A$

(iv) $A \subset B \land B \subset C \Rightarrow A \subset C$

স্বাভা

পূর্ণ পূর্ণাঙ্ক

$N \subset W \subset Z$

$\boxed{N \subset Z}$

$N \subset W \subset Z \subset Q \subset R$

$N \subset R$ ✓

$N \subset Q$ ✓

$W \subset Q$ ✓

$Z \subset R$ ✓

$W \subset R$ ✓

Intervals as Infinite Subsets of R

(i) Closed Interval:

2,3,4,5,6,7,8,9,10

$[a,b]$ $[a, b] = \{x \mid x \in \mathrm{R}, \underset{2}{a} \leq x \leq \underset{10}{b}\}$

(ii) Open Interval:

3 4 5·8 9

( ) (a, b) or $]a, b[= \{x \mid x \in \mathrm{R}, \underset{2}{a} < x < \underset{10}{b}\}$

(iii) Semi-open (closed) Interval:

$$(a, b] = \{x \mid x \in \mathrm{R}, a < x \leq b\}$$

$$[a, b) = \{x \mid x \in \mathrm{R}, a \leq x < b\}$$

Interval $(-\infty, \infty)$ represents the set of real numbers R or real line.

- The number $(b - a)$ is called the length of any of the above intervals.

SETS & RELATIONS

Power Set

The set of all subsets of a set A is called the Power set of A and symbolically it is expressed as

Hence

$$P(A) \text{ or } 2^A$$

$$P(A) = \{X \mid X \in A\}$$

Further if A is a finte set of order n, then it can be easily seen that the number of all subsets of A is 2^n and so $n(P(A)) = 2^n$.

$A = \{1,2,3\}$

Total Subset $\Rightarrow 2^n$

$= 2^3 = 8$

Hence

$n(A) = n \quad \Rightarrow$ Total number of subsets of $A = 2^n$

$\Rightarrow n(P(A)) = 2^n, n(P(A)) \geq 1$

$\Rightarrow \{(1,2,3), \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$

$\Rightarrow 8$

1,2,3,4}

2,3,4,5}

B = {1,2,3,4,5} (Univeral Set)

B = {2,3,4}

B = {1}

A = {5}

ΔB = (A−B) ∪ (B−A) = {1,5}

1. Union of Sets → A ∪ B {जो A में है वो और B में है वो भी}

2. Intersection of Sets → A ∩ B {A व B के Common Ele

3. Difference of Sets → A − B , B − A

4. Symmetric Difference of Sets → A Δ B

$A \Delta B = (A-B) \cup (B-A) = (A \cup B)$

$A = \{1, 2, \cancel{3}, \cancel{4}\}$

$B = \{\cancel{3}, \cancel{4}, 5, 6, 7, 8, 9, 10, 11, 12\}$

$(A - B) = \{1, 2\}$

$B - A = \{5, 6, 7, 8, 9, 10, 11, 12\}$

(Symmetric Difference) $A \triangle B = (A - B) \cup (B - A)$

$= \{1, 2, 5, 6, 7, 8, 9, 10, 11, 12\}$

Properties of Union and Intersection Operations

If A,B,C are any three sets, then

$A=\{1,2,3,4\}$
$A=\{1,2,3,4\}$

$A\cup A=\{1,2,3,4\}$
$A\cap A=\{1,2,3,4\}$

(i) $\left.\begin{array}{l} A\cup A = A \\ A\cap A = A \end{array}\right\}$ (Idempotent laws)

$A=\{1,2,3\}$, $B=\{\ \}$ $A\cup B=\{1,2,3\}$

(ii) $\left.\begin{array}{l} A\cup \phi = A \\ A\cap \phi = \phi \end{array}\right\}$

$U \to$ Universel set

(iii) $\left.\begin{array}{l} A\cup U = U \\ A\cap U = A \end{array}\right\}$

(iv) $\left.\begin{matrix} A \cup B = B \cup A \\ A \cap B = B \cap A \end{matrix}\right\}$ **(Commutativity)**

(v) $(A \cup B) \cup C = A \cup (B \cup C)$ **(Associativity)**

$(A \cap B) \cap C = A \cap (B \cap C)$

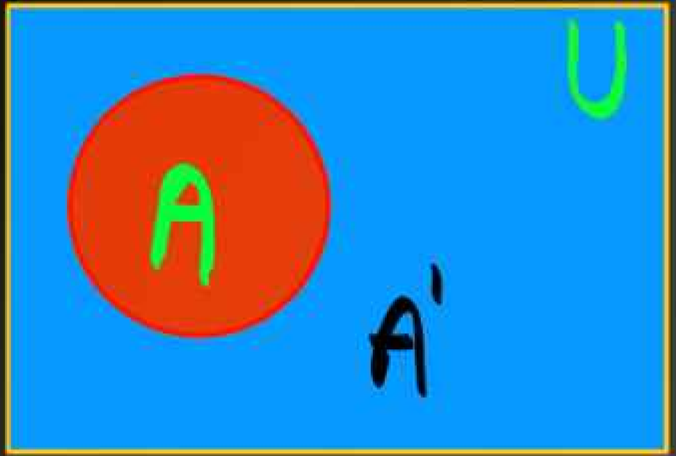
(vi) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $\& A \cap (B \cup C) = (A \cap B) \cup (A \cap C)\}$

(vii) $\left.\begin{matrix} A \subset A \cup B, B \subset A \cup B \\ A \cap B \subset A, A \cap B \subset B \end{matrix}\right\}$

Complement of a Set

Let A be a subset of a universal set U, then the set $\boxed{U - A}$ is called the complement set of A and it is denoted by A' or A^C.

$A \subset U$



$(U - A)$

Thus

$$A^C \;/\; A' = \{x \mid x \in U, x \notin A\}$$

$U = \{1,2,3,4,5,6,7,8,9\}$

$A = \{2,4,6,8\}$

$A' = \{1,3,5,7,9\}$

$A = \{1, 3, 5, 7, 9, \ldots\}$, $B = \{2, 4, 6, 8, \ldots\}$

$C = \{1, 4, 9, 16, 25, 36 \ldots\}$

Universal set $\rightarrow U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, \ldots\}$

$U = N$

$$A = \{1, 9, 16, 25\}$$

$$B = \{2, 3, 4, 5\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 9, 16, 25\}$$

Properties of Product of Sets

If A, B, C, D are any sets, then following properties can easily be established:

(i) $n(A \times B) = n(A) \cdot n(B)$, where A, B are finite sets.

(ii) $A \times B = \phi \Leftrightarrow A = \phi$ or $B = \phi$

(iii) $A \times B \neq B \times A$

(iv) $A \times B = B \times A \Leftrightarrow A = B$

(v) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(vi) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(vii) $A \times (B - C) = (A \times B) - (A \times C)$

(viii) $A \subset B \Rightarrow A \times A = (A \times B) \cap (B \times A)$

(ix) $A \subset B, C \subset D \Rightarrow (A \times C) \subset (B \times D)$

$(x) (A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

$3\}$

$\}$

$(1,4),(1,5),(2,4),(2,5),(3,4),(3,5$

In particular

$A \times B =$

$A^2 = A$

Q. If $A = \{1,3\}, B = \{0,2,4\}$, then

$A \times B = \{(1,0),(1,2),(1,4),(3,0),(3,2),(3,4)\}$

$B \times A = \{(0,1),(2,1),(4,1),(0,3),(2,3),(4,3)\}$

$A^2 =$