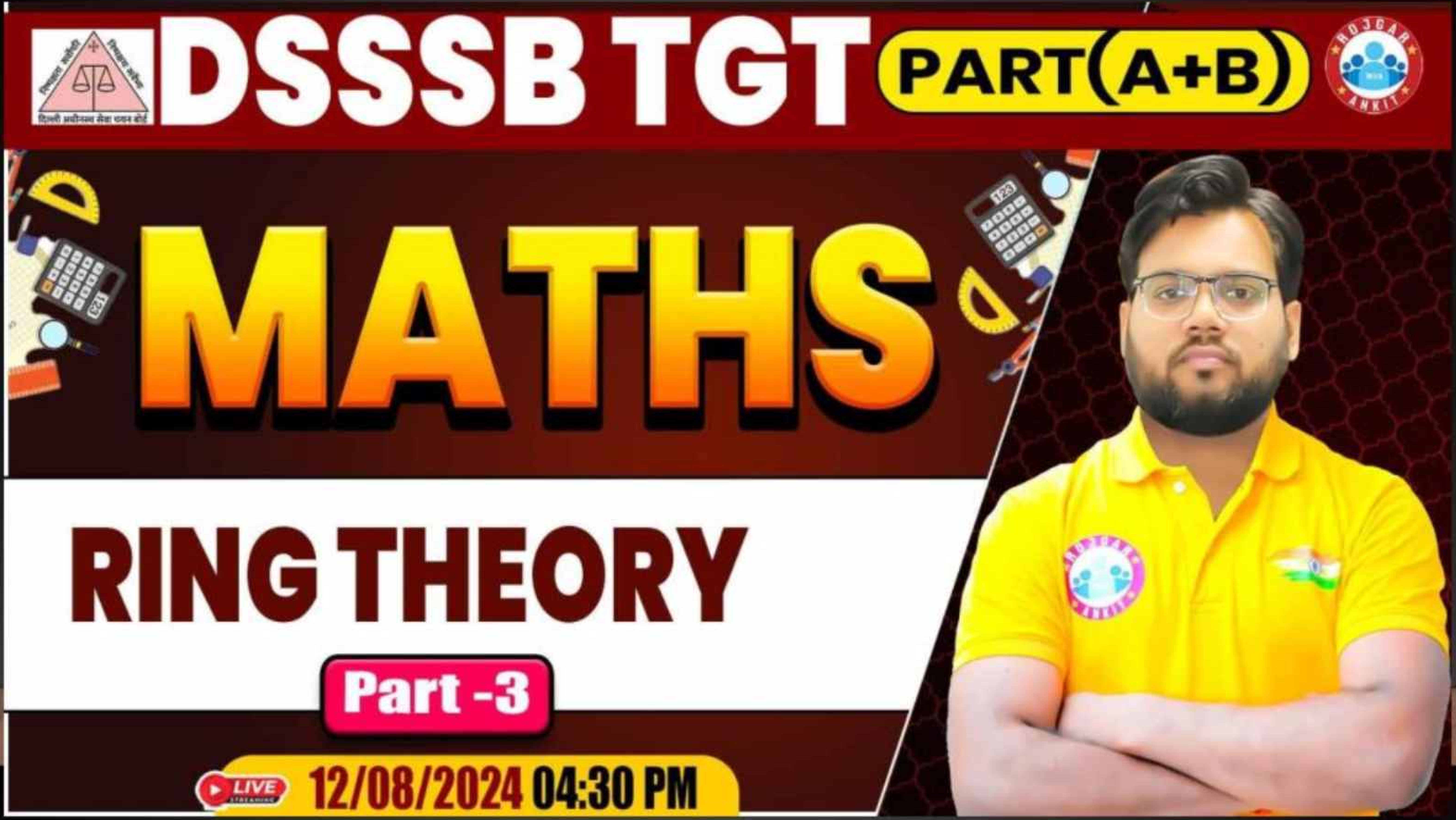
DSSSB TGT
PART(A+B)
MATHS
RING THEORY
Part -3
LIVE
12/08/2024 04:30 PM

Ring Homomorphism

$f : (R, +, \cdot) \rightarrow (S, +, \cdot)$

$f(x+y) = f(x) + f(y)$

$f(x \cdot y) = \underbrace{f(x) \cdot f(y)}_{\boxed{f^2 = f}}$

idempotent

$T(x+y) \neq T(x) + T(y)$

$T(x \cdot y) \neq \underbrace{T(x) \cdot T(y)}_{\boxed{T^2 = T}}$

Trivial Ring homomorphism

$T(x) = 0 \cdot x$

$T = 0$

$T(x+y) = 0(x+y) = 0 \cdot x + 0 \cdot y = T(x) + T(y)$

$T(x \cdot y) = 0(x \cdot y) \Rightarrow \underbrace{0(x) \cdot 0}_{0^2 = 0 \checkmark} \cdot (y) \Rightarrow T(x) \cdot T(y)$

Q. $T:(z,+,\cdot)\rightarrow(z,+,\cdot)$ **defined** $T(x)=0x$ **is Ring homomorphism?** Yes

Trivial Ring Homomorphism $T(x)=0x$

Q. $T:(Z,+,\cdot)\rightarrow(Z,+,\cdot)$ **defined by** $T(x)=\mathbf{1\cdot x}$

[redacted] **is Ring homomorphism?**

$\boxed{\text{Yes}}$

$T(x)=1\cdot x$

$T=1$ ✓

$Z=\{0,1,2,3\ldots\infty\}$

$2,3$

$T(2+3)=1(2+3)$

$\Rightarrow 5\Rightarrow T(2)+T(3)$

$\Rightarrow 1\cdot2+1\cdot3$

$=5$ ✓

$T(x+y)=1(x+y)=1\cdot x+1\cdot y\Rightarrow T(x)+T(y)$ ✓

$T(x\cdot y)=1\cdot(x\cdot y)=\underbrace{1\cdot(x)\cdot 1}(y)\Rightarrow T(x)\cdot T(y)$ ✓

$1\cdot(2\cdot3)=6$

$1^2=1$ ✓

$1(2)\cdot1(3)$

$2\times3=6$

Q. $T: (z,+,\cdot) \to (z,+,\cdot)$ **defined by** $Tx = 3x$ **is Ring homomorphism?** No

$\mathbb{Z} = Z = \{0, \pm 1, \pm 2 \ldots \infty\}$

$\underbrace{2,3}$

$T(x+y) = 3(x+y) = 3(x) + 3(y) \Rightarrow T(x) + T(y)$

$3(2+3) = 3\cdot 2 + 3\cdot 3$

$6 + 9$

$\boxed{15 = 15}$

$T(x\cdot y) = 3(x\cdot y) = \underbrace{3(x)\cdot 3}(y) =$

$3^2 \neq 3$

$T(x\cdot y) = T(x)\cdot T(y)$

$3(2\cdot 3) = 3\cdot 2 \times 3\cdot 3$

$18 \neq 54$

$n \to n$

- $f: Z_n \to Z_n$; Then Number of Ring Homomorphism = Number of idempotent element = 2^d.

where $d \to$ No of prime divisor of n

Idempotent Element $\to a^2 = a$

Q. $f: z_6 \to Z_6$. How many Ring homomorphism? $\Rightarrow 2^d$

(a) Exactly 4 Ring Homomorphism

(b) only 2

(c) exactly 3

(d) None of these

$Z_n \to Z_n$

$\Rightarrow 2^d$

$d \to$ अभाज्य गुणनखण्ड n के

$6 \to 2 \times 3$

$d = 2$

No. of Ring Hom$^m \to 2^d = 2^2$

Q. How many Ring Homomorphism from $T: Z \to Z$?

ⓐ 2 ⓑ 3 ⓒ 99 ⓓ ∞

$Z = \{0, \pm 1, \pm 2, \pm 3, \pm - - - - \infty\}$.

$$\left\{\begin{array}{l} 0^2 = 0 \\ 1^2 = 1 \end{array}\right\}$$

Q. $T: Q \rightarrow Q$ How many Ring Homomorphism?

a. 1

b. 2

c. 9

d. ∞

$$\left\{ \begin{aligned} 0^2 &= 0 \\ 1^2 &= 1 \end{aligned} \right\}$$

Q. $T: R \to R$ **How many Ring Homo?**

a. 2

b. 9

c. 99

d. 84

$0^2 = \square$

$1^2 = 1$

Q. $T: Z_4 \to Z_{10}$ **defined by** $T(x) = 1 \cdot x$ **is Ring Homo?** NO ✓

order(1) in Z_{10}

$Z_n \to Z_{\underline{m}}$

$T(x) = \underline{1} \cdot x$

$O(1)$ in $Z_{10} = 10$

$Z_4 = \{0, 1, 2, 3\}$

$Z_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

क्या Z_4 में कोई element order 10 का है? नहीं

$\{0, 1, 2, 3\}$

$O(0) = 1$

$O(1) = 4$

$O(2) = 2$

$O(3) = 4$

$T: Z_4 \to Z_{10}$ defined by $Tx = 2x$ is a ring homomorphism?

$O(2)$ in $Z_{10} \Rightarrow 5$

Z_4 has no element of order 5.

So $T: Z_4 \to Z_{10}$; $T(x) = 2x$
is not Ring homm.

Q. $T: Z_4 \to Z_{10}$ **defined by** $T(x) = 5x$ **is Ring Homo?**

$O(5)$ in $Z_{10} \to 2$

$Z_4 = \{0, 1, 2, 3\}$

$O(0) = 1$
$O(1) = 4$
$\boxed{O(2) = 2}$ ✓
$O(3) = 4$

Yes it is a Ring homo^n

Boolean Ring→ A Ring $(R,+,\cdot)$ is said to be boolean Ring if $\underbrace{a^2 = a}$, $\forall a \in R$.

$a^2 = a$

$R = (Z_2, +, \cdot)$ is a boolean group?

$Z_2 = \{0, 1\}$

$0^2 = 0$

$1^2 = 1$

Boolean Ring of order 8 ⇒

$O(Z_2 \times Z_2 \times Z_2) = 2 \times 2 \times 2 = 8$

Q1. $T : Z_4 \to Z_{10}$, How many is Ring homo?

$O(2)=2$

$O(1) = 10$ ✓
$O(5) = 2$ ✓
$O(6) = 5$

$a^2 = a$

$Z_{10} \to \underbrace{2 \times 5}_{d=2}$

No of. Ring hom^m $\to 2^d \Rightarrow 2^2 \Rightarrow 4$

possible $\to 4$

A. 0.x
B. ~~1.x~~
C. $\boxed{0.x \ \& \ 5x}$
D. ~~0x & 6x~~

$O(0)=1$
$Z_4 \ O(0) = 1$

$T(x) = 0 \cdot x$
× $T(x) = 1 \cdot x$
$T(x) = 5 \cdot x$
× $T(x) = 6x$

$5^2 = 25 \pmod{10} = 5$

$6^2 = 36 \pmod{10} \Rightarrow 6$

Q2. $T : Z_5 \rightarrow Z_{10}$, How many is Ring homo?

$O(1) = 10$
$O(5) = 2$
$O(6) = 5$

✓ A. 0.x

B. 1.x

C. 0.x & 5x

D. 0x & 6x

$Z_{10} \rightarrow 10 \rightarrow \underbrace{2 \times 5}_{d=2}$

No of possible

Ring $Hom^m \Rightarrow 2^d \Rightarrow 4$

$T(x) = 0 \cdot x$

$T(x) = 1 \cdot x$

$T(x) = 5x$

$T(x) = 6x$

$1^2 = 1$

$Z_5 = \{0, 1, 2, 3, 4\}$

$O(0) = 1$, $O(2) = 5$, $O(4) = 5$

$O(1) = 5$, $O(3) = 5$