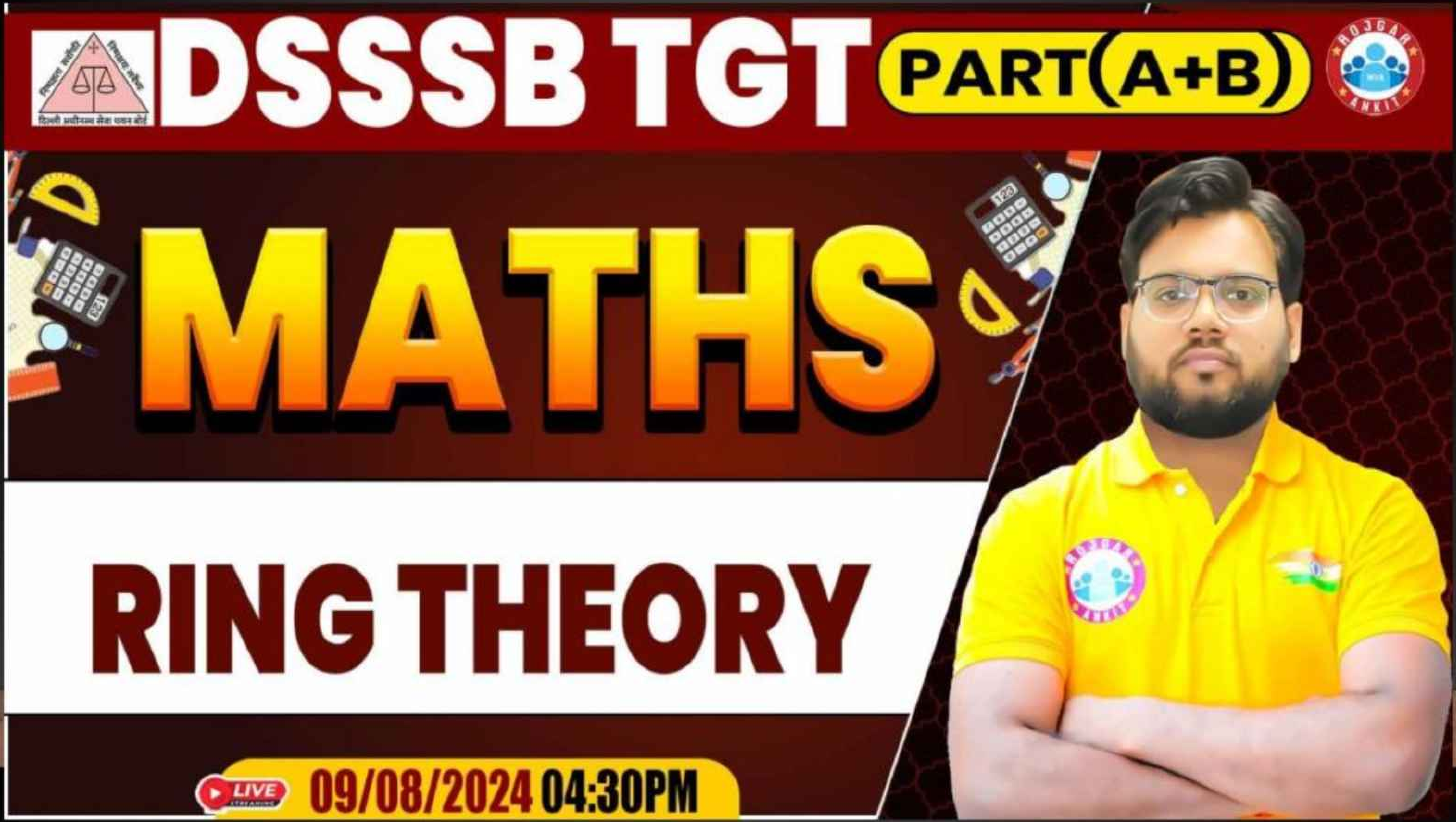
DSSSB TGT
PART (A+B)
MATHS
RING THEORY
LIVE
09/08/2024 04:30PM

RING :-

➢ Types of Ring-

(i) Ring with Unity -

(ii) Commutative Ring -

(iii) Division Ring –

Sub Ring –

Ring with Zero Divisor – A ring is said to be Ring with Zero divisor, If ∃

$$\underbrace{2 \times_3 3}_{\text{Non-Zero}} = 0$$

$a \neq 0, b \neq 0$

$$\boxed{a \cdot b = 0}$$

$a \times b = 0$; $a = 0$ या $b = 0$

Non Zero   Non-Zero

$$A = \begin{array}{cc} & \text{Non Zero} \\ \end{array}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

i.e. $\rightarrow$ $[\{0,1,2,3,4,5\}, +_6, \times_6]$ is a Ring with zero divisor?

$$2 \times_6 3 = 0$$

$$3 \times_6 4 = 0$$

Ring with zero divisor element $\Rightarrow 2, 3, 4$

Zero divisor in a Ring
वलय में शून्य के भाजक ⇒

Q ⇒ $[\{0,1,2,3,4,5\}, +_6, \times_6]$

Zero divisor in a Ring → 2,3,4

Ring without zero divisor

A ring is said to be a ring without zero divisor if has no zero divisor.

$(\mathbb{Z}, +, \cdot)$
$(\mathbb{Q}, +, \cdot)$
$(\mathbb{C}, +, \cdot)$

$\boxed{a \cdot b = 0}$ $a = 0$ या $b = 0$

Integral Domain

→ A ring is said to be integral domain if it is Commutative Ring with unity without zero divisor.

$(\mathbb{Z}, +, \cdot)$
$(\mathbb{Q}, +, \cdot)$
$(\mathbb{R}, +, \cdot)$
$(\mathbb{C}, +, \cdot)$
} Integral Domain

1. Commutative ring $\Rightarrow a \cdot b = b \cdot a$
2. Ring with unity $\Rightarrow \boxed{a \cdot e = a = e \cdot a}$
3. Ring without zero divisor

$\Rightarrow \left[\{Z_3\}, +_4, \times_4\right]$ is it a integral Domain?

$\{0, 1, 2\}$

$a \cdot e = a$

$0 \cdot 1 = 0$
$1 \cdot 1 = 1$
$2 \cdot 1 = 2$

1. Commutative $\rightarrow a \cdot b = b \cdot a$
   $1 \cdot 2 = 2 \cdot 1 = ✓$

2. RU $\rightarrow \boxed{a \cdot e = a = e \cdot a}$ $e = 1$ ✓

3. Ring without zero divisor $\rightarrow$
   $0 \cdot 1 = 0$
   $0 \cdot 2 = 0$
   $0 \cdot 0 = 0$

✓

$a = 0$ या $b = 0$

$a \cdot b = 0$

Field - A ring F is said to be a field, If

1. Commutative $\Rightarrow a \cdot b = b \cdot a$
2. Ring with unity $\Rightarrow a \cdot e = a = e \cdot a$
3. Every non-zero element has multiplicative inverse in Ring.

$\left.\begin{array}{l} (Q, +, \cdot) \\ (R, +, \cdot) \\ (C, +, \cdot) \end{array}\right\}$ Field

$(Z, +, \cdot)$ is not a field.

$Z = \{0, \pm 1, \pm 2, \pm 3 \ldots\ldots \infty\}$

जिनका गुणात्मक प्रति...
है।

Multiplicative Inverse →

$\Rightarrow a \times a^{-1} = e = 1$

$a \times a^{-1} = 1$

$S^{\times}, L^{\times}, F^{\times}, R^{\times}$

Ring के वे Elements ...
Multiplicative inve...
है

$(\check{Z}, +, \cdot)$ में कितने Units हैं?

→ Only Two

$\boxed{e = 1} \Rightarrow (\times)$

$-1 \times -1 = 1$ ✓

$1 \times 1 = 1$

$Z^{\times} = ...$

is a...
w.r.t...

$(\ldots, +, \cdot)$
$(\ldots, +, \cdot)$
$(\ldots, +, \cdot)$

$(\times) \Rightarrow$ गुणा

Units $= \infty$

$\boxed{e = 0} (+) \Rightarrow \infty$ units

$P = \{1,2,3,4,5\}, \times_6$

multiplication identity = 1

$P^{\times} = \{1,5\}$

$$\boxed{\begin{aligned} 1 \times_6 1 &= 1 \\ 5 \times_6 5 &= 1 \end{aligned}}$$

$1 \times 1 \Rightarrow 1$

2 ___

3 ___

4 ___

$5 \times 5 = 1$

$$T(x+y) = T(x) + T(y)$$

Idempotent $\longleftarrow \boxed{T^2 = T}$

$$T(x \cdot y) = T(x) \cdot T(y)$$

$$\phi : R \to R'$$

Let $(R,+,\cdot) \longrightarrow (R',+,\cdot)$ are Ring Morphism if follows both operations –

$$\phi(a+b) = \phi(a) + \phi(b)$$

$$\phi(a \cdot b) = \phi(a) \cdot \phi(b)$$

$\phi^2 =$

...g Idempotent

...ment in Z_6?

a) 2 b) 4 c) 6 d) 8

of R if $\boxed{a^2 = a}$

* No. of Idempotent element i...

$$\boxed{Z_n = 2^d}$$

where d is number of pri...

$6 \to \underbrace{2 \times 3}_{d=2}$

No of idempotent element $\Rightarrow 2^d$

$\Rightarrow 2^2 = 4$

{...2, 3, 4, 5}

$3^2 = 9$

⇒ How many idempotent in Z_{100} ?

a) 2 b) 4 c) 6 d) 8

⇒ No of idempotent ⇒ $2^d \rightarrow 2^2 = 4$

$100 = \underbrace{2^2 \times 5^2}_{d=2}$

#

Trivial Ring Homomorphism → A mapping

$T:(R,+,\cdot)\longrightarrow(S,+,\cdot)$ defined by

$$\boxed{T(x)=0\cdot x}$$

24 में No of prime factor = ?

$24 \Rightarrow 2^3 \times 3^1$

1. कुल गुणनखण्ड ⇒ $(3+1) \times (1+1)$ ✓

$\Rightarrow 8$

2. सम गुणनखण्ड ⇒ $3 \times (1+1) \Rightarrow 6$

3. विषम गुणनखण्ड ⇒ $(1+1) = 2$

4. अभाज्य गुणनखण्ड ⇒ $3+1 \Rightarrow 4$

$24 \rightarrow 1, 2, 3, 4, 6, 8, 12, 24$

2