DSSSB TGT PART(A+B)

MATHS

RING THEORY

LIVE

09/08/2024 04:30PM

RING :-

(वलय)

$(R, +, \cdot)$

$a, b \in R$

R1. $(R, +)$ is an abelian group.

- Closure ✓ $a+b \in R$
- Assosiativity ⇒ $a+(b+c)=(a+b)+c$
- Identity → $a+e=a=e+a$
- Inverse → $a+a^{-1}=e$
- Commutative Property ⇒ $a+b=b+a$

R2. $(R, \cdot)$ is a semi-group.

- Closure
- Assosiativity

] Semi-group

R3. Distributive property $a, b, c \in R$

$a \cdot (b+c) = a \cdot b + a \cdot c$ [Left D.L.]

$(a+b) \cdot c = a \cdot c + b \cdot c$ [Right D.L.]

$(\mathbb{Z},+,\cdot)$, $(\mathbb{Q},+,\cdot)$, $(\mathbb{C},+,\cdot)$ All forms a Ring.

$(R,+,\cdot)$ क्या ये एक Ring है? R→Real Numbers

$(R,+)$ is an abelian group. ✓

$(R,\cdot)$ is a semi group. ✓

Closure ✓ $5+7=12 \in R$

Assosiativity → ✓ $2+(3+5)=(2+3)+5$

Identity → $0 \in R$

Inverse → $5+(-5)=0 \in R$

Commutative property ⇒ $5+(-9)=-9+5$

$-4=-4$ ✓

Distributive property → $a\cdot(b+c)=a\cdot b+a\cdot c$

$1\cdot(2+3)=1\times2+1\times3$

$5=5$ ✓

C

$\Rightarrow Z_4 = [\{0,1,2,3\}, +_4, \times_4]$ क्या एक Ring है?

$\Rightarrow (Z_4, +)$ is an abelian? ✓ yes

$\Rightarrow (Z_4, \cdot)$ Semi group. ✓ yes

$\Rightarrow$ Distributive ✓✓

Closure $\Rightarrow 3+2 \Rightarrow \frac{5}{4} \Rightarrow 1$ ✓

Assosiativity → ✓

Identity → $0 \in Z_4$

Inverse $\Rightarrow 2+2 = \frac{4}{4} = 0$ ✓

Commutative → $a+b = b+a$

$3+2 = 2+3$

$5 = 5$

➢ Types of Ring-

(R U) **(i) Ring with Unity -** तत्समक वलय →

A $(R,+,\cdot)$ is said to be Ring with unity If its multiplication identity exists.

$$e \cdot a = a = a \cdot e$$

$(Z,+,\cdot)$ क्या RU होगा?

Identity in case of Multiplication = 1

$$Z = \{0, \pm 1, \pm 2, \pm 3, \ldots\ldots \pm \infty\}$$

(ii) Commutative Ring - $(R,+,\cdot)$ is said to be Commutative Ring if its Multiplication Composition exists.

(क्रमविनिमेय वलय)

$$a \cdot b = b \cdot a \quad , \forall a, b \in R$$

*CRU ⇒ Commutative Ring with Unity ⇒ $(Z,+,\cdot)$

$\not$CRU $\Longrightarrow$ $(Q,+,\cdot), (R,+,\cdot), (C,+,\cdot)$ ⋯

Square Matrix $\Rightarrow (n \times n)$

✓ RU

Ring with unity

✗ CRU

$a \cdot b \neq b \cdot a$

$$a = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

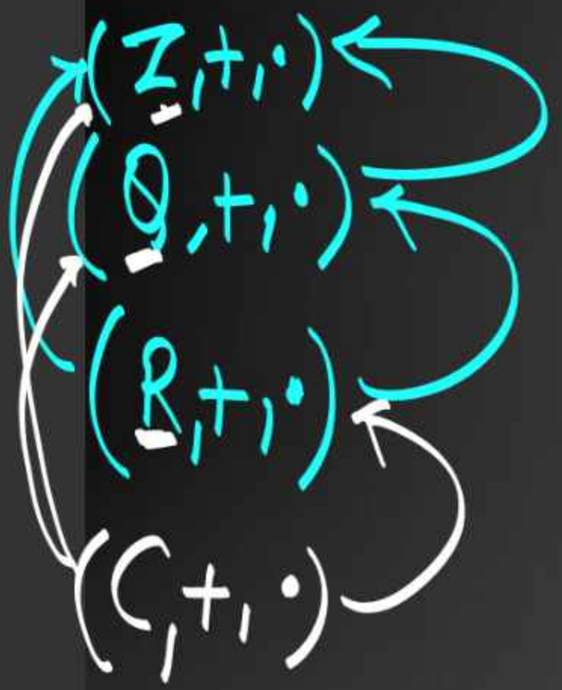
$$ab = \begin{bmatrix} 11 & 4 \\ 25 & 10 \end{bmatrix} \neq ba = \begin{bmatrix} 9 & 14 \\ & \end{bmatrix}$$

(iii) Division Ring – If in Ring $(R, +, \cdot)$ $\exists$ $a^{-1} \in R$ for multiplication.

- Ring with unity
- Each of its non-zero element has multiplication inverse.

(उप वलय) → **Sub Ring –** A non-empty subset of Ring

$(R, +, \cdot)$ is said to be Sub-Ring if itself is a ring w.r.t. induced/produce Composition. $(+, \cdot)$



① Improper Subring (Trival Subgroup) ⇒ $R, \{0\}$ Empty Set

② proper Sub ring → A Subring which is not improper

Ring with Zero Divisor –

$2 \times_6 3 = 0$

Non Zero

$2 \times 3 = \frac{6}{6}$ $R = 0$

$a \times b = 0$

या तो $a = 0$ या फिर $b = 0$