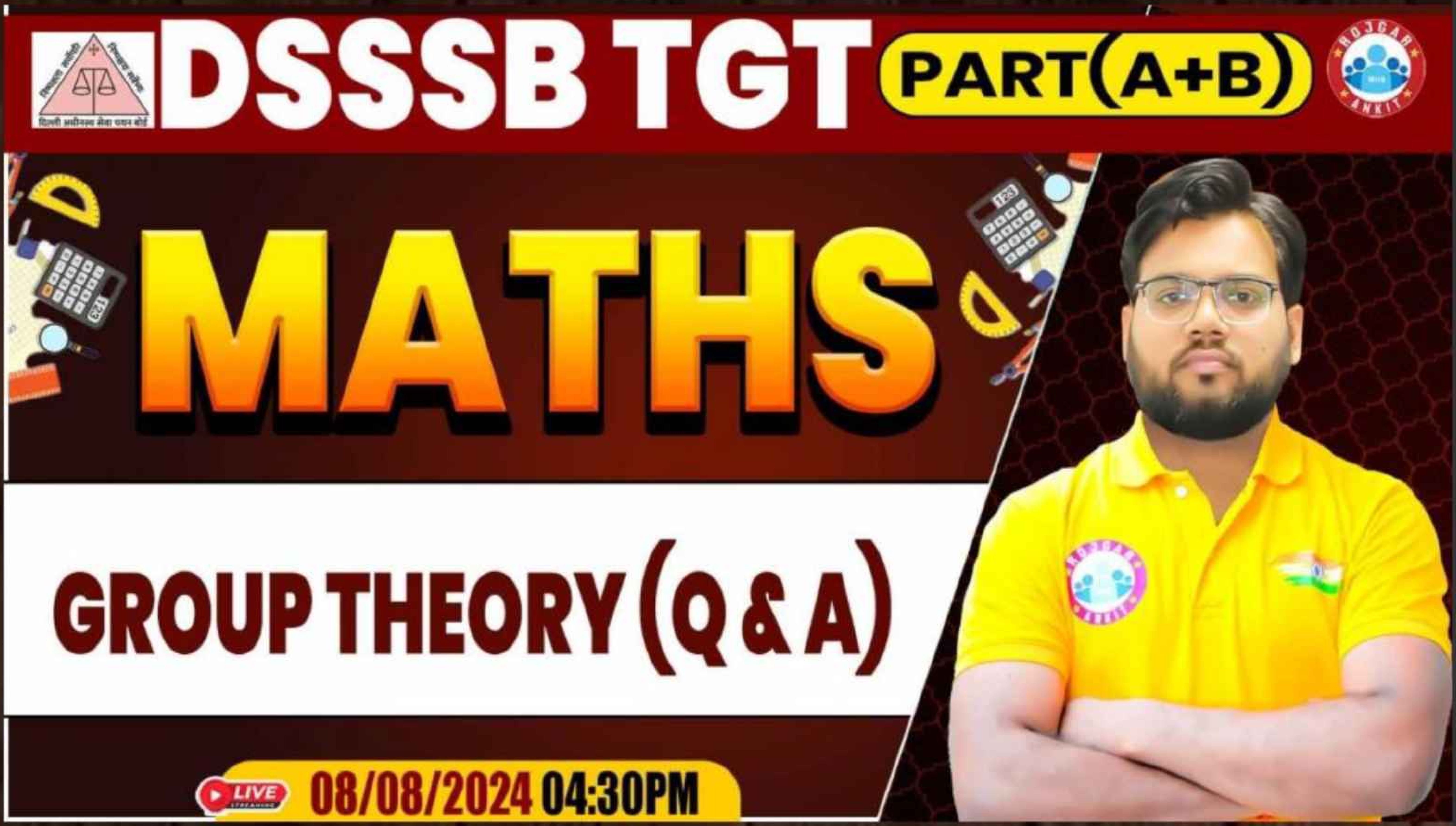
DSSSB TGT
PART (A+B)
ROJGAR ANKIT
MATHS
GROUP THEORY (Q & A)
LIVE
08/08/2024 04:30PM

1. If $G = (Z, +)$ and $H = (3Z, +)$ so, the quotient group $\frac{G}{H}$ will be

(a) $\{H, H+1\}$

(b) $\{H, H+2\}$

(c) $\{H+1, H+2\}$

(d) $\{H, H+1, H+2\}$

$G = Z$, $H = 3Z$

$\frac{G}{H} = \frac{Z}{3Z} \approx Z_3$

$= O(Z_3)$

$\Rightarrow 3$

$\frac{G}{H} = \{H + a \mid a \in G\} \Rightarrow H+0, H+1, H+2, H+3$

2. If $G = [(1, -1, i, -i]]$ is a multiplication group and $H = [\{1, -1\}],$ $\cancel{Gisasubsetof}$,

H is subset of G

then the order of the quotient group $\frac{G}{H}$ is

(a) 4

(b) 2

(c) 1

(d) 3

$o\left(\frac{G}{H}\right) = \frac{o(G)}{o(H)}$ If group is finite

$\frac{4}{2} = 2$

3. If $(G, {}^*)$ is a group in which $(a^*b)^2 = a^2b^2 \forall a, b \in G$ then

(a) G is a cyclic group

(b) G is not commutative group

(c) G is commutative group

(d) G is both cyclic and commutative

$(a*b)*(a*b) = (a*a)*(b*b)$

$a^{-1}(a*b)*(a*b).b^{-1} = a^{-1}(a*a)*(b*b)b^{-1}$

$(a^{-1}a*b)*(a*bb^{-1}) = (a^{-1}a*a)*(b*b.b^{-1})$

$(e*b)*(a*e) = (e*a)*(b*e)$

$\boxed{b*a = a*b}$

$\boxed{a*b = b*a}$

4. If the set is $\{1, 2, 3, 4, 5, 6\}$ the group for binary operation multiplication modulo 7, then the inverse of element 5 is:

(a) 3

(b) 4

(c) 5

(d) 6

$e=1$

$5 \times 4 = \frac{20}{7} = R = 6$

$5 \times 5 = \frac{25}{7} = R = 4$

$5 \times 6 = \frac{30}{7}, R = 2$

$5 \times 3 = \frac{15}{7} = R = 1$

$G = \{1, 2, 3, 4, 5, 6, X_7\}$

$\boxed{a \times a^{-1} = e}$ $\boxed{e = 1}$

$5 \times \underline{3} = 1$

3

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5. The generator of the cyclic group determined by the following table is:

$O(G)=4$

$O(G)=O\langle g \rangle = 4$

	a	b	c	d
a	c	a	d	b
b	a	b	c	d
c	d	c	b	d
d	b	d	d	c

$a \times a = C = a^2$

$a^3 = a^2 \times a \Rightarrow C \times a = d$

$a^4 = a^3 \times a$
$\Rightarrow d \times a = b$ ✓

$b \times b = b \quad O(b)=2$

$b^3 = b^2 \times b \Rightarrow b \times b = b$

(a) a (b) b

(c) c (d) d

6. If $O(a) = m, O(b) = n$ where a and b are elements of any Abeli group G, then

(a) $O(ab) = \sqrt{m}$

(b) $O(ab) = \sqrt{mn}$

(c) $O(ab) = \sqrt{n}$

(d) $O(ab) = L.C.M.$ Is of M and N

$O(a) = 6$

$O(b) = 8$

a, b are the elements of a abelian group

$O(ab) = LCM(6, 8)$

$= 24$

7. What is the reciprocal of permutation $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$?

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

8. Each infinite cyclic group has/have

(a) A Generator

~~(b)~~ Two Generator

(c) Three Generator

~~(d)~~ Infinite Generator

$(Z, +)$

$\Downarrow$

$(a, -a)$

9. If each element of a group ' G ' is its own inverse, then group ' G ' is?

(a) Abelie

(b) Cyclic

(c) Finite

(d) Infinite

10. There are inappropriate subgroups in any group.

Improper
(अनुचित)

(i) G (itself)

(ii) $\{e\}$

(a) 1

(b) 2

(c) 3

(d) Depends on the given group

11. Matrix set $S = \left\{\begin{bmatrix} x & -x \\ -x & x \end{bmatrix}, 0 \neq x \in R\right\}$ forms a group for matrix multiplication whose identity element is?

$\boxed{a \times e = a}$

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{bmatrix}$

$\begin{bmatrix} x & -x \\ -x & x \end{bmatrix} \times \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{x}{2}+\frac{x}{2} & -\frac{x}{2}-\frac{x}{2} \\ -\frac{x}{2}-\frac{x}{2} & \frac{x}{2}+\frac{x}{2} \end{bmatrix}$

$\begin{bmatrix} x & -x \\ -x & x \end{bmatrix} = \begin{bmatrix} x & -x \\ -x & x \end{bmatrix}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} m & n \\ o & p \end{bmatrix} = \begin{bmatrix} am+bo & an+bp \\ cm+do & cn+pd \end{bmatrix}$$

12. Let G be the group of order 30 . Suppose H and K are normal subgroups of order 3 and 6 respectively, then the order of the group $\frac{G}{HK}$ is:

(a) 2

(b) 3

(c) 5

(d) 10

$$O\left(\frac{G}{HK}\right) \Rightarrow \frac{30}{6} \Rightarrow 5$$

$$\underset{\sim}{H}K = \frac{O(H) \cdot O(K)}{O(H \cap K)} \Rightarrow \frac{\cancel{3} \times 6}{\cancel{3}} = 6$$

14. If G is a group where order of G is 13 , then G will be.

(a) An Abelian group

(b) A cyclic group

(c) Abelian and cyclic groups

(d) Neither Abelian nor cyclic groups

17. Suppose $G = (Z, +)$ is a group of integers for addition operations and let $H = 3z = \{3x \mid x \in Z\}$ is a subset of G, then the order of quotient group $\frac{G}{H}$ is.

(a) 2

(b) 3

(c) 4

(d) Infinite

18. **What is the order of Alternating group A_n:**

(a) $\lfloor n$

(b) n

(c) $\underline{n-1}$

(d) $\frac{1}{2}\lfloor n$

$O(S_n) = n!$

$O(A_n) = \frac{n!}{2}$

19. The group $G = \{(2, 4, 6, 8); X_{10}\}$ has identity elements.

(a) 2

(b) 4

(c) 6

(d) 8

20. If each element of a group G is its own inverse, then G is?

(a) Normal

(b) Finite

(c) Abelie

(d) Infinite

In addition

$\frac{G}{H} = \{a+H \mid a \in G\}$

$\{H+a \mid a \in G\}$

$o\left(\frac{G}{H}\right) = \frac{o(G)}{o(H)} \Leftarrow$

$\boxed{\textbf{Factor Group/Quotient Group:}}$ If H is $\underbrace{\text{normal}}$ subgroup of G then the set $\frac{G}{H} = \{aH \mid a \in G\} = \{Ha \mid a \in G\}$ is group with operation $(aH) \cdot (bH) = abH$.

$= (Ha) \cdot (Hb) = Hab$

(1) If H is normal subgroup of G then $\frac{G}{H} = \{aH \mid a \in G\}$ is group? yes

(2) In Case of $\boxed{\text{finite}}$ then group Order of $\frac{G}{H}$ is

(3) $\frac{G}{H}$ is Subgroup of G ? No

21. $\dfrac{1\mathbb{Z}}{12\mathbb{Z}} \approx ?$

(a) $\mathbb{Z}_8$

(b) $\mathbb{Z}_{10}$

(c) $\mathbb{Z}_{12}$

(d) $\mathbb{Z}_{18}$

$O\left(\dfrac{\mathbb{Z}}{12\mathbb{Z}}\right) = ?$

$O(Z_{12}) = 12$

$\boxed{O(Z_n) = n}$

$\dfrac{\mathbb{Z}}{m\mathbb{Z}} \approx \mathbb{Z}_m$

$\mathbb{Z}_{\frac{12}{1}}$

$1>0,\ 12>0$

$\frac{12}{1}$ ✓

22. $G = \mathbb{Z}$ and $H = 6\mathbb{Z}$ then Order of $\frac{G}{H}$? $\Rightarrow \frac{Z}{6Z}$ $\Rightarrow O = 6$

(a) 1

(b) 6

(c) 10

(d) 4

$G = Z$ $\quad H = 6Z$

$\frac{G}{H} \Rightarrow \{a + H \mid a \in G\}$

$= \{0+H, 1+H, 2+H, 3+H, 4+H, 5+H\}$

$6+H$ $\quad 6+6 \Rightarrow 12$

$Z = \{0, \pm 1, \pm 2, \pm 3, \text{-----} \infty\}$

$6Z = \{0, \pm 6, \pm 12, \pm 18, \pm 24, \text{------} \infty\}$

$(Z, +)$

Z_6

Imp

(1) $\frac{\mathbb{Z}}{m\mathbb{Z}} \approx \mathbb{Z}_m$

(2) $\frac{\mathbb{Z}}{\{0\}} \approx \mathbb{Z}$

$\frac{\mathbb{Z}_{20}}{\{0\}} = \mathbb{Z}_{20}$

(3) $\frac{G}{G} \approx \mathbb{Z}_1$

(4) $\frac{G}{\{e\}} \approx G$

(5) $\frac{\mathbb{Z}}{\mathbb{Z}} \approx \mathbb{Z}_1$

$\frac{n}{k}$

(6) $\frac{n\mathbb{Z}}{m\mathbb{Z}} \approx \mathbb{Z}_{\frac{m}{n}}$ $m>0,\ n>0$

$\boxed{n \mid m} \Rightarrow n$ divides m

k divides n

(7) If $\frac{k}{n}$. Then, $\frac{Z_n}{\langle k \rangle} \approx \mathbb{Z}_k$

$\Rightarrow$

K divides n

$\frac{n}{k}$

(8) If $\frac{k}{n}$. Then, $\frac{U(n)}{U_k(n)} \approx U(k)$

23. if $m > 0$, $m\mathbb{Z}$ is subgroup of $n\mathbb{Z}$. if,

(a) m/n

(b) n/m

(c) $n = m$

(d) None

$m\mathbb{Z}$ is subgroup of $n\mathbb{Z}$

group $\leftarrow G$

Subgroup $\leftarrow H$

$$\frac{G}{H} = \frac{n\mathbb{Z}}{m\mathbb{Z}}$$

$$\frac{m}{n}$$

$$\frac{1\mathbb{Z}}{12\mathbb{Z}} = \mathbb{Z}_{\frac{12}{1}}$$

$$\underline{12}$$

24. Let $G = 2\mathbb{Z}$ and $\mathrm{H} = 8\mathbb{Z}$ then the quotient group $\frac{G}{H}$ is

(a) Z_2

(b) Z_8

(c) Z_4

(d) Not defined

$$\frac{G}{H} \Rightarrow \frac{2Z}{8Z} = Z_{\frac{8}{2}} = Z_4$$

$\frac{8}{2}$ ✓

quotient

25. The factor group of abelian group is abelian, but Converse need not be true.

~~26.~~ The factor group of cyclic group is cyclic, but Converse need not be true.

27. $U_k(n)$ is normal subgroup of $U(n)$ where k/n.

$k/n \quad \frac{n}{k} \Rightarrow k$ divides n

28. Let $G = \mathbb{Z}_4 \times \mathbb{Z}_6$ be a group and H be a subgroup of G generated by $(0, 1)$. Then order of G/H is:

(a) 1

(b) 2

(c) 3

(d) 4

29. $\frac{Z_{12}}{\langle 3 \rangle} \approx \mathbb{Z}_3$ $\quad * \frac{12}{3}$ ✓

(a) Z_3

(b) Z_5

(c) Z_{12}

(d) Not defined

30. $\frac{Z_{20}}{\{0\}} \approx \mathbb{Z}_{20}$

(a) Z_{50}

(b) Z_{20}

(c) Z_{10}

(d) Not defined

31. $\frac{U(20)}{U_{10}(20)} \approx ?$

(a) $U(10)$

(b) $U(20)$

(c) $U(30)$

(d) None of these

$$\frac{U(n)}{U_k(n)}$$

$\frac{20}{10}$ ✓

If k divides n, then $\frac{U(n)}{U_k(n)} = U(k) = U(10)$

32. $\dfrac{u(30)}{u_5(30)} \approx ?$

(a) Z_{10}

(b) Z_{20}

(c) Z_4

(d) None of these

$$\frac{U(30)}{U_5(30)} \approx U(5)$$

$\frac{30}{5}$

$U(K)$

$U(p) \approx Z_{p-1}$; p is अभाज्य संख्या

$\frac{30}{5}$

$\Rightarrow U(5) \approx Z_{5-1} = Z_4$