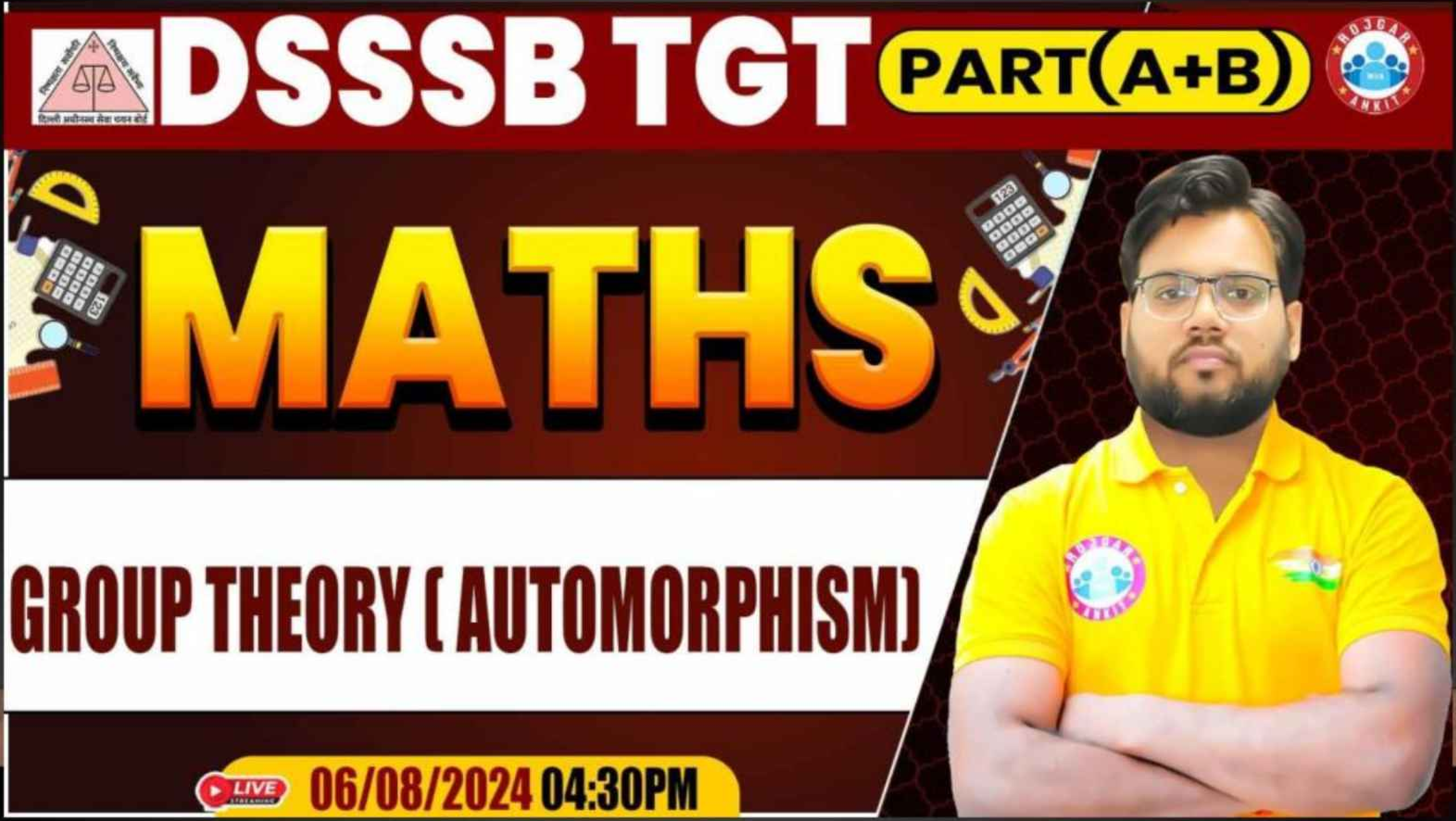

DSSSB TGT
PART (A+B)
MATHS
GROUP THEORY (AUTOMORPHISM)
LIVE
06/08/2024 04:30PM

Trivial Group Homomorphism :-

$f: G \to G'$ defined by $f(x) = e'$

(0 1)

e' is identity of G'

Q. $f: Z_8 \to Z_6$ defined by $f(x) = 0$ is?

(a) Trivial Homo.

(b) Non-Trivial Homo.

(c) Not Homo.

(d) None of these

0 is identity of Z_6

$Z_6 = \{0, 1, 2, 3, 4, 5\}$

Onto homomorphism → **A mapping $f: G \to G'$ is said to be onto homomorphism.**

$$(G, \cdot) \to (G', *)$$

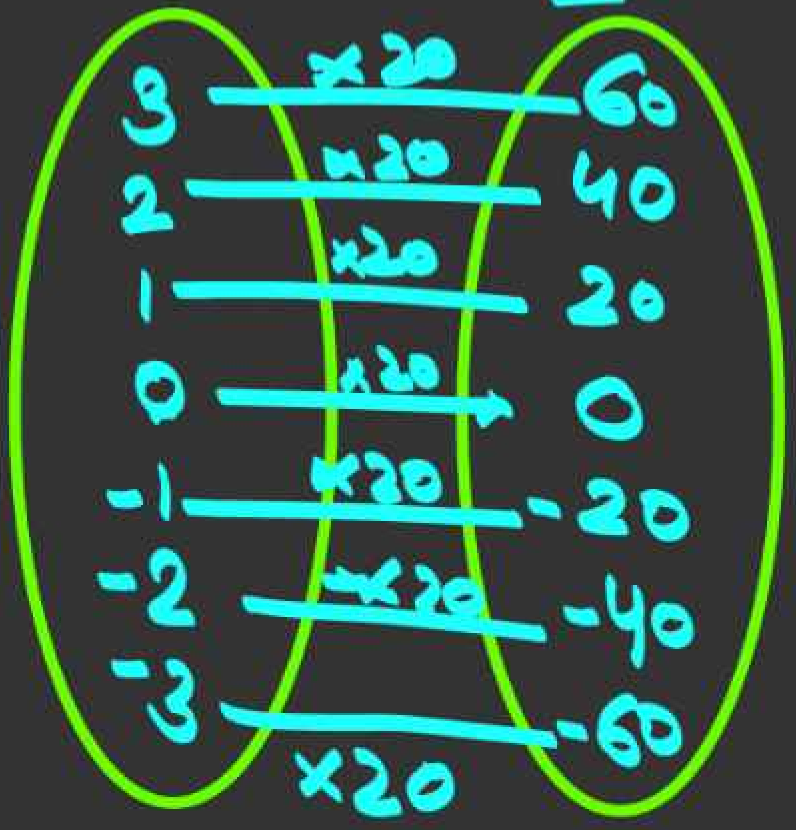
if

1. f is homomorphism → $f(x \cdot y) = f(x) * f(y) \quad \forall\, x, y \in G$

2. f is onto

$f(20) = 20 \cdot x$

Z ✓



One one

$\Rightarrow Z \to Z$

onto →

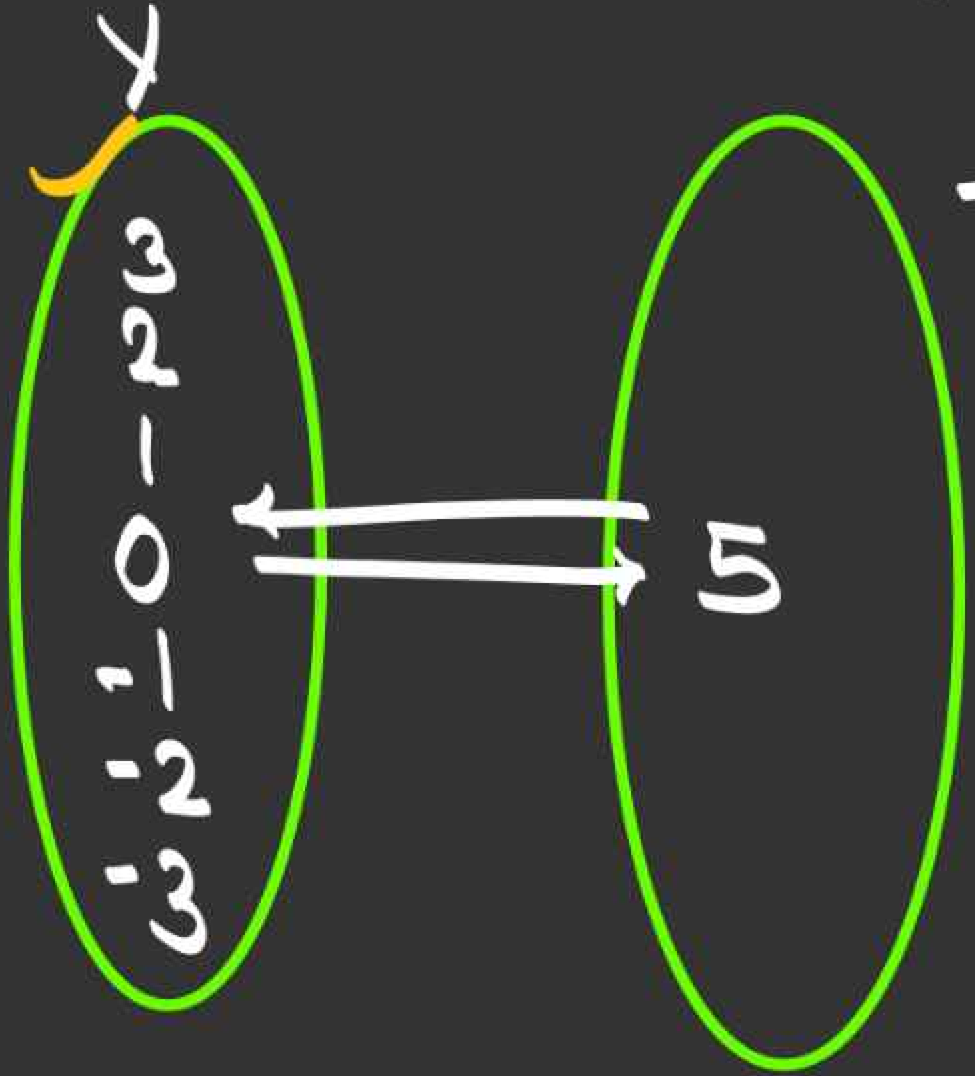
$f(x) = k(x)$

↳ जहाँ $k = Z$

$fx = 20x = 7$

$x = \frac{7}{20}$

$\frac{7}{20} \notin Z$



$fx \Rightarrow 1 \cdot x = 5$

$\boxed{x = 5}$

$fx = -1 \cdot x = 20$

$x = -20$ ✓

Number of Onto homomorphism →

From $f: \mathbb{Z}_m \to \mathbb{Z}_n$; If $\frac{m}{n}$ then, onto homomorpism is $\phi(n)$.

From $f: Z_{16} \to Z_8$

$\frac{16}{8} = 2$

$\phi(8)$

Q. $f: \mathbb{Z}_6 \to \mathbb{Z}_4$, is onto homomorphism?

(a) No, its Trivial homo.

(b) Not onto homo.

(c) Yes, onto homo.

(d) None of these.

$\mathbb{Z}_m \to \mathbb{Z}_n$

$\frac{m}{n}$ ✓

$\phi(n)$

$\mathbb{Z}_6 \to \mathbb{Z}_4$

$\frac{6}{4}$ ✗

Q. How many onto Homomorphism from $f: \mathbb{Z}_{20} \to \mathbb{Z}_{10}$?

(a) 10

(b) 20

(c) 5

(d) 4

$\frac{20}{10}$ ✓

No. of onto Homomorphism $\Rightarrow \phi(10) = 2 \times 5$

$\phi \to \cancel{10} \times \frac{1}{2} \times \frac{4}{5}$

$\Rightarrow 4$

Q. How many onto homomorphism $f\mathbb{Z} \to \mathbb{Z}$?

(a) Exactly 2 onto Homo.

(b) 5 Homo.

(c) Exactly 4 on to Homo.

(d) None of these

Q. $f: U(7) \to U(6)$. Number of onto homo.?

(a) 2 onto homo.

(b) 3 onto homo.

(c) 1 onto homo.

(d) None of these.

$U(6)$

$$\boxed{U(m \cdot n) \approx U(m) \cdot U(n) \qquad \gcd(m,n) = 1}$$

$U(7) \approx Z_{7-1} \Rightarrow Z_6$

$\Rightarrow f: Z_6 \to Z_2$

$\frac{6}{2}$ ✓ $\qquad \phi(2) \Rightarrow 1$

$U(6) = U(2 \times 3)$

$\Rightarrow U(2) \cdot U(3)$

$\Rightarrow \underbrace{Z_1 \times Z_2}$

$\Rightarrow Z_2$

Q. How many Onto Homomorphism from $f: \mathbb{Z}_{15} \rightarrow \mathbb{Z}_{15}$?

(a) 15

(b) 8

(c) 12

(d) 10

$\frac{15}{15}$ ✓

No. of onto Homomorphism ⇒ $\phi(15)$

$15 = 3 \times 5$

$\phi \rightarrow 15 \times \frac{2}{3} \times \frac{4}{5}$

$\Rightarrow 8$

Note → (a) An onto homomorphism from $f; G \to G'$ is called Epimorphism.

(b) A one-one homomorphism from $f: G \to G'$ is Called Monomorphism.

(c) A homomorphism from a group G to it self is called an Endomorphism of G.

$G \longrightarrow G$

Q. How many epimorphism are possible from $\mathbb{Z}_{12}$ to $\mathbb{Z}_6$?

(a) 0

(b) 6

(c) 4

(d) 2

$\frac{12}{6} = \checkmark$

No of Epimorphism $= \phi(6) = 2 \times 3$

$\phi \rightarrow \cancel{6} \times \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}}$

$\Rightarrow 2$

Isomorphism $\rightarrow$ A mapping $f; G \rightarrow G'$ is said to be isomorphism if

1. F is homomorphism $\rightarrow f(x \cdot y) = f(x) * f(y); \forall x, y \in G$
2. f is one-one $\quad Z \rightarrow Z$

Infinite

3. f is onto

Exactly 2

Q. $f:\mathbb{Z} \to \mathbb{Z}$, How many isomorphism?

(a) Exactly 3 isomorphism

(b) 5 isomorphism

(c) Exactly 2 isomorphism

(d) None of these

Note→ (1) No. of isomorphism $f: \mathbb{Z}_n \to \mathbb{Z}_m$ are $\phi(m)$

$f = Z_n - Z_m$

(2) Number of isomorphism from $T: \mathbb{Z}_n \to \mathbb{Z}_n$

= Number of onto homomorphism

$= \phi(n)$

So $T: Z_n \to Z_n$ has exactly $\phi(n)$ Isomorphism.

Q. $T: Z_8 \to Z_8$, how many Isomorphism?

(a) 3

(b) 4

(c) 5

(d) None of these.

$\phi(8) \to 2^3$

$\phi \to 8 \times \frac{1}{2}$

$= 4$

Q. If $f: \mathbb{Z}_{18} \to U(19)$, how many Isomorphism? prime

(a) 4 isomorphism

(b) 5 Isomorphism

~~(c)~~ 6 Isomorphism

(d) None of These.

$Z_{18} \to Z_{18}$

No. of Isomorphism $= \phi(18) \to 2 \times 3^2$

$\phi(18) \to 18 \times \frac{1}{2} \times \frac{2}{3}$

$\Rightarrow 6$

Q. If $f: k_4 \to k_4$, How many homomorphism?

CSIR

(a) 10 Homomorphism

(b) 12 Homomorphism

(c) 14 Homomorphism

(d) 16 Homomorphism

$K_4 \Rightarrow Z_2 \times Z_2$

$Z_2 \times Z_2 \to Z_2 \times Z_2$

No. of Homom. $\Rightarrow \gcd(2,2) \times \gcd(2,2) \times \gcd(2,2) \times \gcd(2\times2)$

$2 \times 2 \times 2 \times 2$

$\Rightarrow 16$

Q. If $T: Z_{15} \to Z_{15}$, How many Isomorphism?

(a) 4 isomorphism

(b) 6 isomorphism

(c) 8 isomorphism

(d) None of these

$\phi(15) \to 3 \times 5$

$\phi \to \cancel{15} \times \frac{2}{\cancel{3}} \times \frac{4}{\cancel{5}}$

$\Rightarrow 8$

Q. If : $G \rightarrow G'$ is a homomorphism; where e, e' are identity elements of G and G' respectively, then.

(a) $f(e) = e'$

(b) $f(x^{-1}) = [f(x)]^{-1}$

(c) $f(x^n) = [f(x)]^n$, an integer

(d) All of the above

- **POINTS To be noted -**

1 $U(p^r)_{p \neq 2} \approx Z_{\phi(p^r)}$; p is prime.

Imp 2 $U(p^r)_{p=2} \approx Z_2 \times Z_{2^{r-2}}$.

$U(2^6) = Z_2 \times Z_{2^{6-2}}$

$\Rightarrow Z_2 \times Z_{2^4}$

$Z_2 \times Z_{16}$

3 $f: \mathbb{Z} \to \mathbb{Z}: f(x) = kx$ is homomorphish (i.e. Has Infinite Number of homomorphism)

4 When e is the identity of G and e' is the identity of G' and $f: G \to G'$ is homomorphism, Then If $f(e) = e'$.

5 If $f: G \to G'$ is homomorphism then $f(x^{-1}) = (f(x))^{-1}$.

6 If $f: G \to G'$ is homomorphism and $x \in G$, when G and G' is finite group then

$$\boxed{\frac{\text{order }(x)}{\text{order }(f(x))}.}$$

Automorphism → A mapping $T: \underbrace{G \to G}_{\text{itself}}$ is called Automorphism if.

1 T is homomorphism

2 T is one-one

3 T is onto

- Number of Automorphism from Z_n to Z_n is $\phi(n)$.

Q. Find Number of Automorphism from

$f: U(11) \to U(11)$.

$Z_{10} \longrightarrow Z_{10}$

(a) 5 Automorphism

(b) 2 Automorphism

(c) 4 Automorphism

(d) 8 Automorphism

No. of Automorphism

$\Rightarrow \phi(n)$

$\Rightarrow \phi(10) \Rightarrow 2 \times 5$

$\phi \longrightarrow \cancel{10 \times \frac{1}{2}} \times \frac{4}{5}$

$\Rightarrow 4$

Q. $f: Z_{26} \rightarrow Z_{26}$, How many Automorphism?

(a) 12 Automorphism

(b) 21 Automorphism

(c) 14 Automorphism

(d) 16 Automorphism

$26 \rightarrow 2 \times 13$

$\phi \quad \cancel{26} \times \frac{1}{\cancel{2}} \times \frac{12}{\cancel{13}}$

$= 12$