



DSSSB TGT

PART (A+B)



MATHS

GROUP THEORY (ISOMORPHISM)



05/08/2024 04:30PM



Types of Morphism

If f is one-one $\rightarrow$ 1 Monomorphism (एकैकी समाकारिता)

Onto

$\rightarrow$ 2 Epimorphism (अच्छादक समाकारिता)

One-one onto

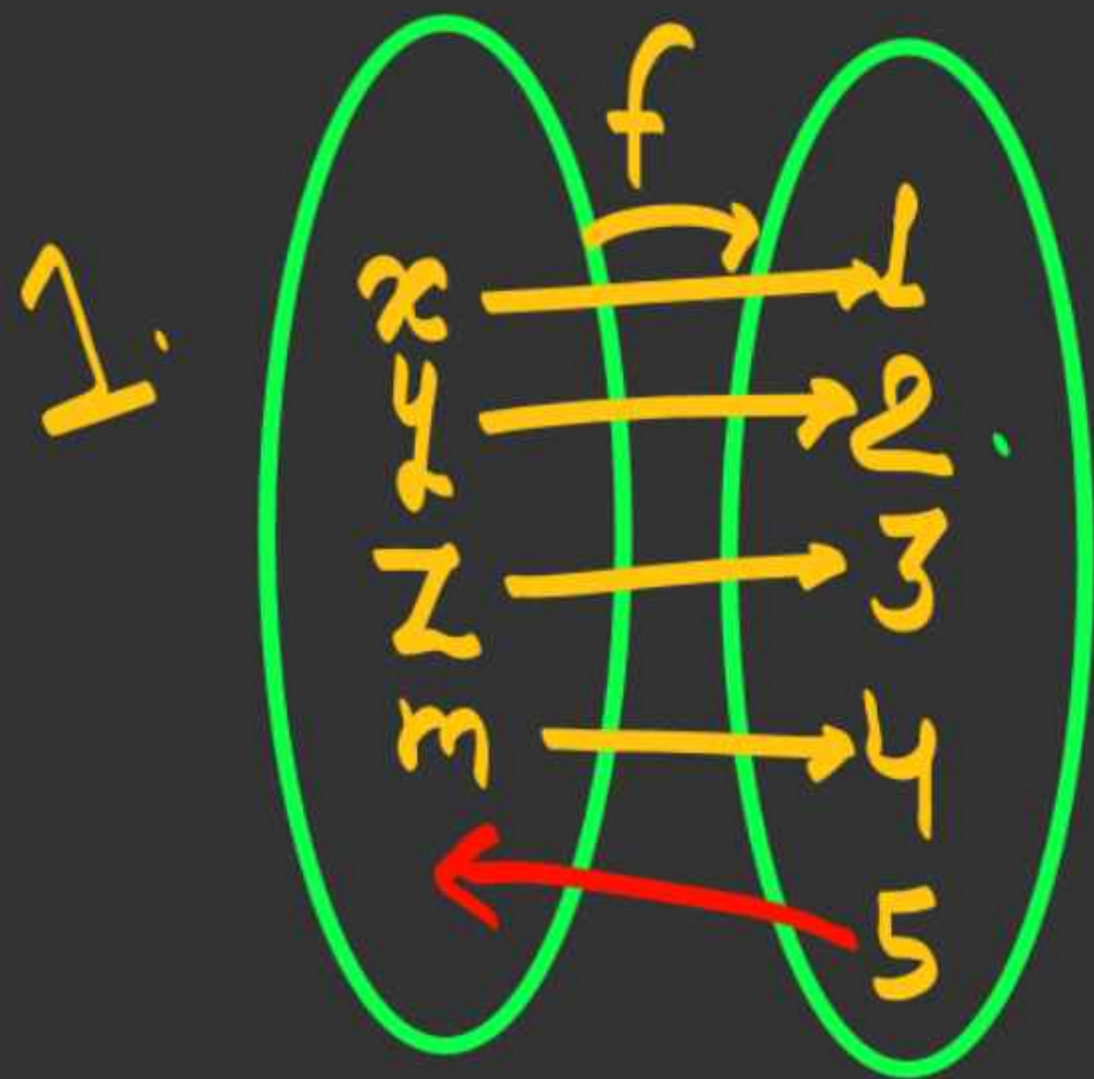
$\rightarrow$ 3 Isomorphism (तुल्यकारिता)

$\mathcal{C} = \mathcal{C}'$

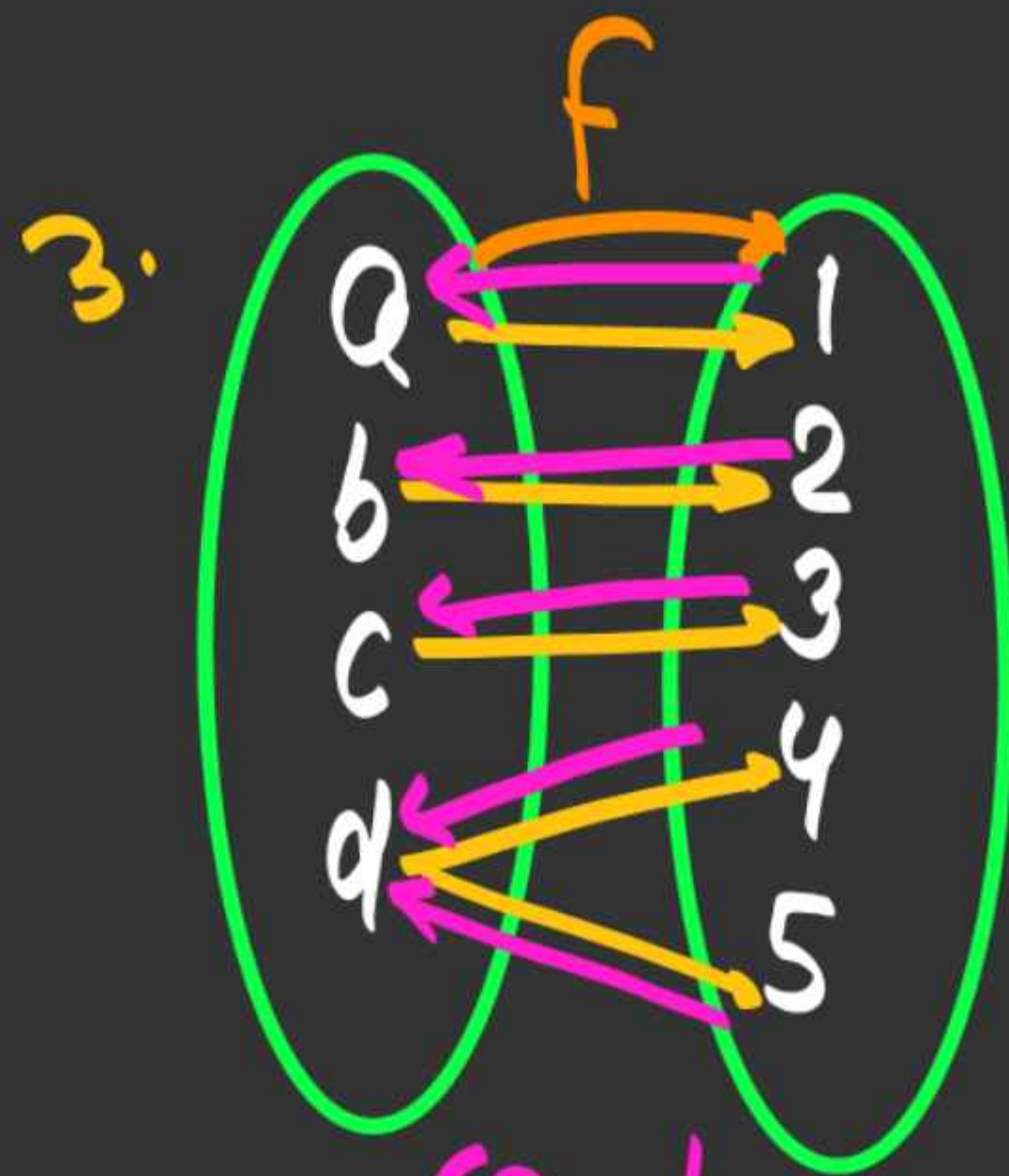
$\rightarrow$ 4 Endomorphism (अन्तराकारिता)

one-one onto & $\mathcal{C} = \mathcal{C}'$

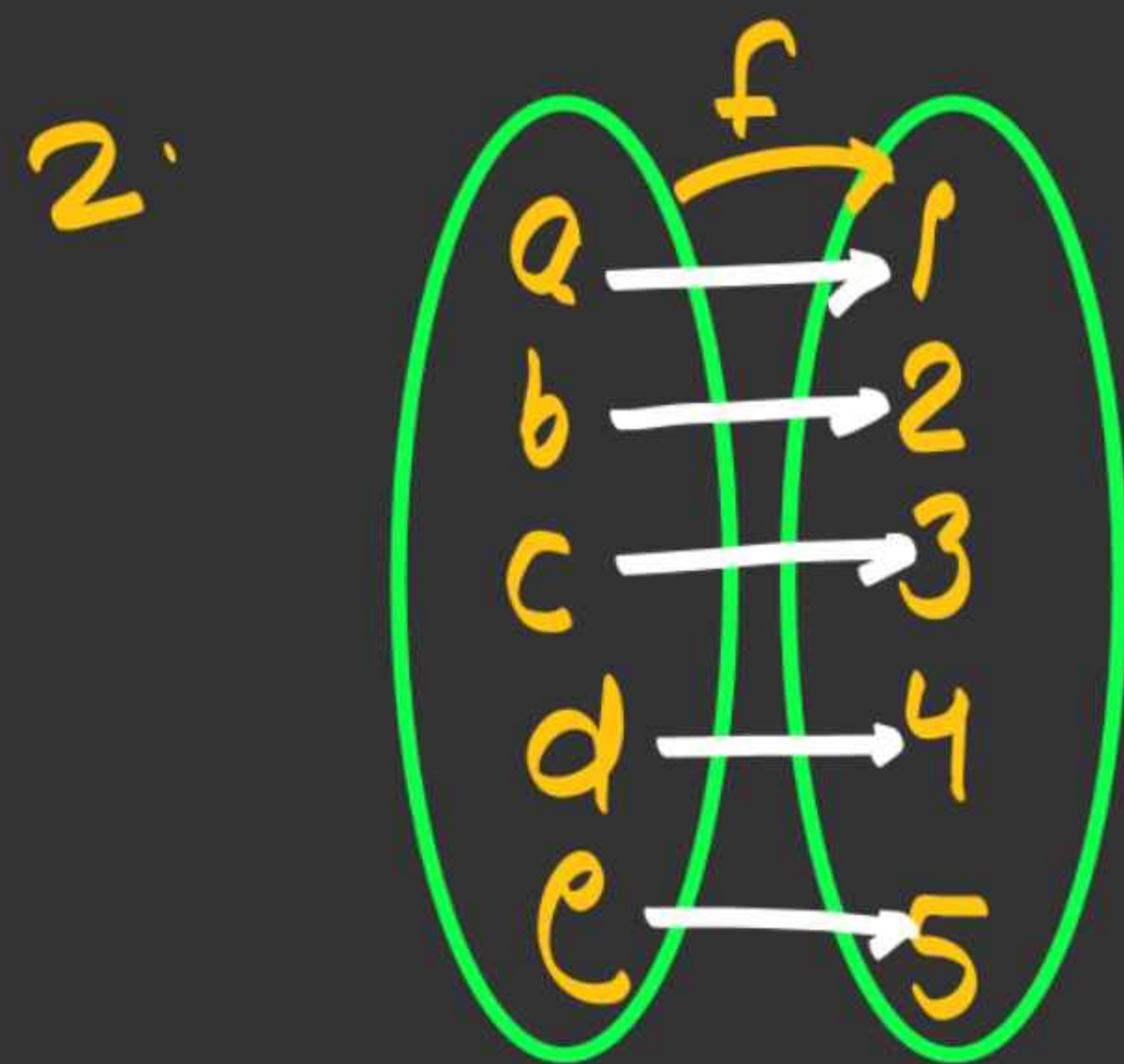
$\rightarrow$ 5 Automorphism (स्वाकारिता)



(one-one)



(Onto)



one one & onto

(तुल्यकारीता) **Isomorphism** $\rightarrow$ A mapping $f; G \rightarrow G'$ is said to be isomorphism if

1 **f is homomorphism** $\rightarrow f(x \cdot y) = f(x) * f(y); \forall x, y \in G$

2 **f is one-one** —

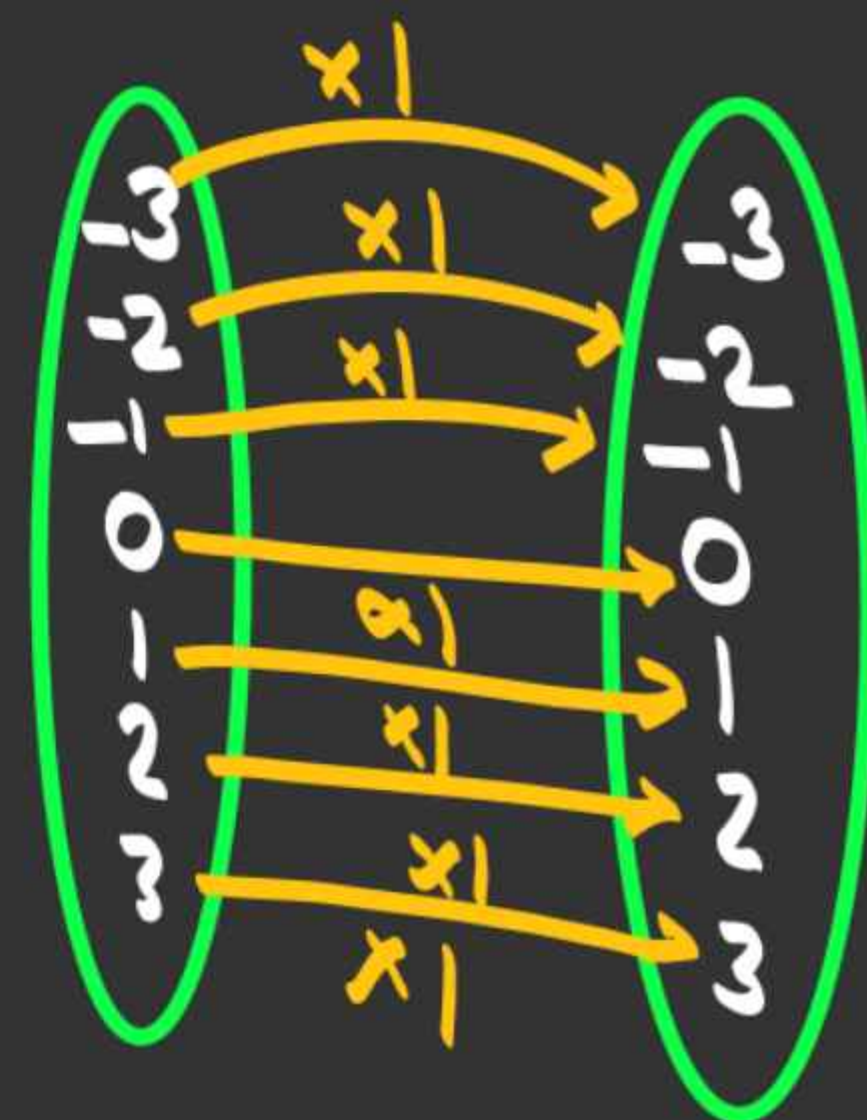
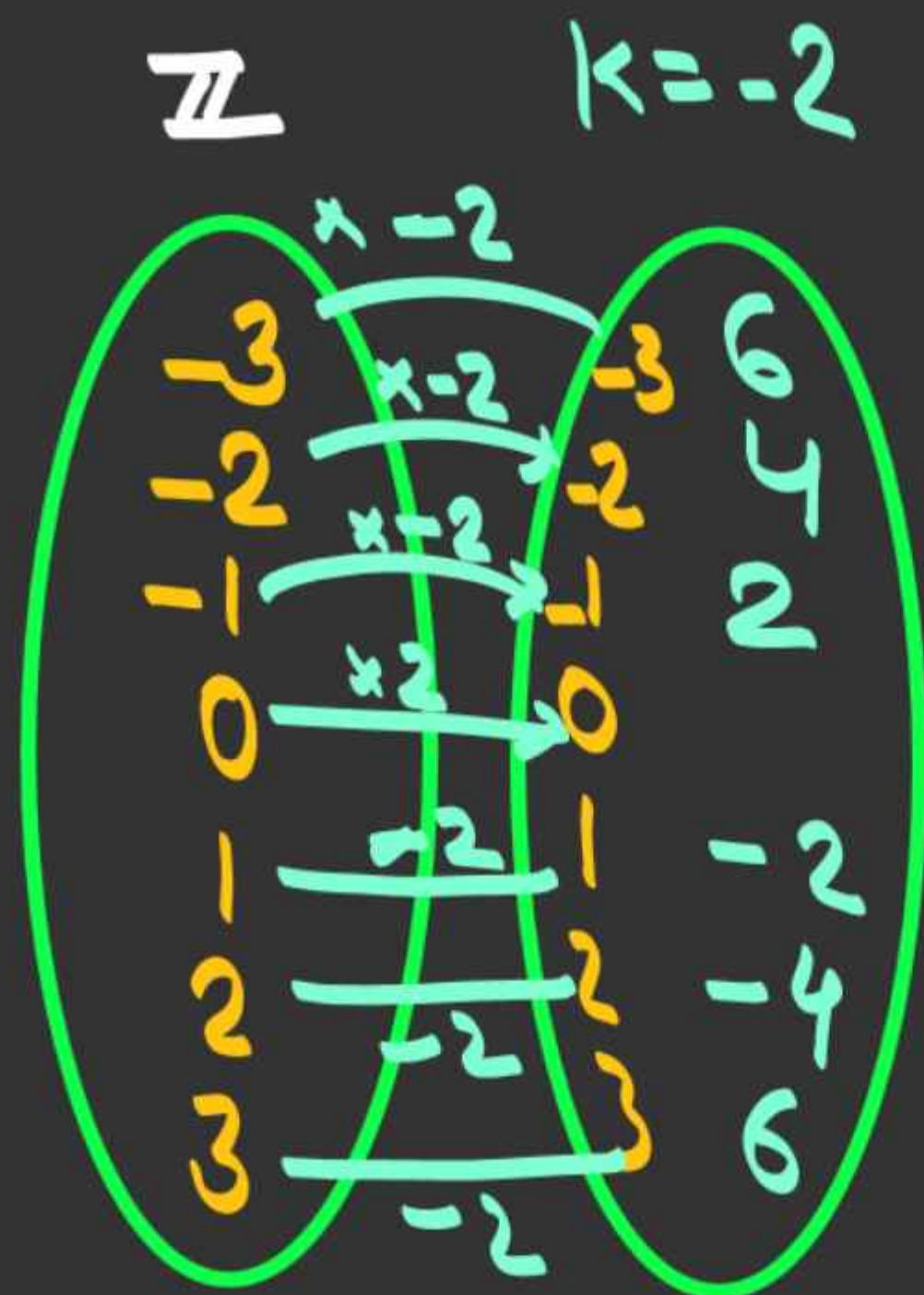
3 **f is onto** ✓

$(G, \cdot)$ $(G, *)$

$$f(x \cdot y) = f(x) * f(y)$$

$$\# f: \mathbb{Z} \rightarrow \mathbb{Z} \rightarrow$$

$$f(x) = kx$$



Q. $f: \mathbb{Z} \rightarrow \mathbb{Z}$, How many isomorphism?

- (a) Exactly 3 isomorphism
- (b) 5 isomorphism
- (c) Exactly 2 isomorphism
- (d) None of these

$$f(x) = kx$$

$k=1$
$k=-1$

→ Same

Q. Find number of homomorphism in

$f: \mathbb{Z}_{16} \rightarrow \mathbb{Z}_{30} ?$

(a) 16 Homomorphism

(b) 30 Homomorphism

(c) 2 Homomorphism

(d) 8 Homomorphism

$$\gcd(16, 30) \Rightarrow 2$$

$$\mathbb{Z}_{16} \rightarrow \mathbb{Z}_{30}$$

Q. $f: \mathbb{Z}_2 \times \mathbb{Z}_4 \rightarrow \mathbb{Z}_8$, how many homomorphism?

- (a) 2 homomorphism
- (b) 4 homomorphism
- ✓ (c) 8 homomorphism
- (d) 16 homomorphism

No of homomorphism $\rightarrow \overset{2}{\text{gcd}}(2, 8) \times \overset{4}{\text{gcd}}(4, 8)$
 $2 \times 4 = 8$

Q. $f: \mathbb{Z}_8 \rightarrow \mathbb{Z}_2 \times \mathbb{Z}_4$, how many homomorphism?

(a) 2 homomorphism

(b) 4 homomorphism

(c) 8 homomorphism

(d) 16 homomorphism

No of Homomorphism $\rightarrow \text{HCF}(8, 2) \times \text{HCF}(8, 4)$

$$2 \times 4 = 8$$

Q. $f: \mathbb{Z}_2 \times \mathbb{Z}_4 \rightarrow \mathbb{Z}_2 \times \mathbb{Z}_4$, how many homomorphism?

(a) 8 homomorphism

(b) 4 homomorphism

(c) 32 homomorphism

(d) 12 homomorphism

$$\begin{aligned}\text{No. of homomorphism} &\Rightarrow \text{HCF}(2, 2) \times \text{HCF}(2, 4) \times \text{HCF}(4, 2) \times \text{HCF}(4, 4) \\ &= 2 \times 2 \times 2 \times 4 \\ &= 32\end{aligned}$$

Q. $f: U(11) \rightarrow \mathbb{Z}_3 \times U(11)$, how many homomorphism?

✓ (a) 10

(b) 11

(c) 3

(d) 33

No of Homomorphism

$$\Rightarrow \phi(10, 3) \times \phi(10 \times 10)$$

$$\Rightarrow 1 \times 10$$

$$\Rightarrow \underline{10}$$

prime No.

$$U(\overset{\downarrow}{11}) \rightarrow \mathbb{Z}_3 \times U(11)$$

$$\mathbb{Z}_{(11-1)} \rightarrow \mathbb{Z}_3 \times \mathbb{Z}_{10}$$

$$\mathbb{Z}_{10} \rightarrow \mathbb{Z}_3 \times \mathbb{Z}_{10}$$

Q. $f: U(9) \rightarrow U(27)$, How many homomorphism?

(a) 9

(b) 27

(c) 6

(d) 12

$$\begin{array}{l} 9 \rightarrow 3^2 \\ \downarrow \phi \\ 3^2 \times 2 \\ \Rightarrow 6 \end{array}$$

$$U(3^2) \rightarrow U(3^3)$$

અભાજ્ય

$$\mathbb{Z}_{\phi 9} \rightarrow \mathbb{Z}_{\phi 27}$$

$$\mathbb{Z}_6 \rightarrow \mathbb{Z}_{18}$$

$$\begin{array}{l} 27 \rightarrow 3^3 \\ \downarrow \phi \\ 3^3 \times 2 \\ \Rightarrow 18 \end{array}$$

No. of Homomorphism $\Rightarrow \text{gcd}(6, 18)$
 $\Rightarrow 6$

$$\frac{\mathbb{Z}}{3\mathbb{Z}} \times \frac{\mathbb{Z}}{5\mathbb{Z}} \rightarrow U(11)$$

$$\mathbb{Z}_3 \times \mathbb{Z}_5 \rightarrow \mathbb{Z}_{10}$$

No. of Homomorphism $\rightarrow$

$$\text{HCF}(3, 10) \times \text{HCF}(5, 10)$$

$$1 \times 5 \Rightarrow 5$$

$$\mathbb{Z}_{15} \rightarrow \mathbb{Z}_{10}$$

HCF=5

Q. $f: \frac{\mathbb{Z}}{3\mathbb{Z}} \times \frac{\mathbb{Z}}{5\mathbb{Z}} \rightarrow U(11)$, Number of homomorphism?

(a) 3

(b) 5

(c) 11

(d) 15

Note * यदि

$$\frac{\mathbb{Z}}{m\mathbb{Z}} \cong \mathbb{Z}_m$$

* $\mathbb{Z}_m \times \mathbb{Z}_n$, यदि $\text{HCF}(m, n) = 1$
 $\Rightarrow \mathbb{Z}_{m \times n}$ भी लिख सकते हैं।

$$\frac{\mathbb{Z}}{9\mathbb{Z}} \times \frac{\mathbb{Z}}{5\mathbb{Z}} \rightarrow U(81)$$

No. of Homomorphism?

$$\mathbb{Z}_{45} \rightarrow U(3^4)$$

$$\mathbb{Z}_{45} \rightarrow \mathbb{Z}_{54}$$

HCF = 9

$$\mathbb{Z} \xrightarrow{\phi} 81 \rightarrow 3^4$$

$\phi \rightarrow \frac{81 \times 2}{3} = 54$

$$f: \frac{\mathbb{Z}}{9\mathbb{Z}} \times \frac{\mathbb{Z}}{5\mathbb{Z}} \rightarrow U(41)$$

समाकारिताओं की संख्या = ?

$$\mathbb{Z}_9 \times \mathbb{Z}_5 \rightarrow \mathbb{Z}_{40}$$

HCF = 1

$$\mathbb{Z}_{45} \rightarrow \mathbb{Z}_{40}$$

HCF = 5

must be
अभाज्य

Onto homomorphism $\rightarrow$ A mapping $f: G \rightarrow G$ is said to be onto homomorphism.

if

1. f is homomorphism
2. f is onto

Number of Onto homomorphism →

From $f: \mathbb{Z}_m \rightarrow \mathbb{Z}_n$ If $\frac{n}{m}$ then, onto homomorphism is $\phi(n)$.

Homomorphism : Let $(G_1, \cdot)$ and $(G_2, *)$ are two groups . A mapping $f : G_1 \rightarrow G_2$ is said to be homomorphism if

$$f(x \cdot y) = f(x) * f(y) \quad \forall x, y \in G_1$$

Q. If $f : \mathbb{Z}_{10} \rightarrow \mathbb{Z}_{10} : f(x) = 2x$ is homomorphism ?

Q. IF $f : \mathbb{Z}_6 \rightarrow \mathbb{Z}_4$: defined by $f(x) = 3x$ is homomorphism ?

- A. Yes, homomorphism**
- B. No, Not homomorphism**
- C. May be homomorphism**
- D. None of these**

Q. IF $f : \mathbb{Z}_6 \rightarrow \mathbb{Z}_4$: defined by $f(x) = 2x$ is homomorphism ?

- A. Yes, homomorphism**
- B. No, Not homomorphism**
- C. May be homomorphism**
- D. None of these**

Q. IF $f : \mathbb{Z}_4 \rightarrow \mathbb{Z}_6 : f(x) = 2x$ is homomorphism ?

- A. Yes, homomorphism**
- B. No, Not homomorphism**
- C. May be homomorphism**
- D. None of these**

Note :-

(1) $f : \mathbb{Z}_m \rightarrow \mathbb{Z}_n$ defined by $f(x) = ax$ if $O(a) = k$ in $\mathbb{Z}_m$ and $\mathbb{Z}_n$ has elements of order k . then, $f(x) = ax$ is homomorphism.

(2) If $f : \mathbb{Z}_m \rightarrow \mathbb{Z}_n$, Number of homeomorphism = $\text{g.c.d}(m, n)$.

(3) $f : \mathbb{Z} \rightarrow \mathbb{Z} : f(x) = kx$ is homomorphism. (i.e. Has Infinite Number of homomorphism)

(4) $U(P^r)_{p \neq 2} \approx \sum \phi(p^k) : p \text{ is prime.}$

$$\sum \phi(p^k)$$

$p \rightarrow \text{prime No.}$

(5) $U(m.n) \approx U(m) \cdot U(n) : \text{g.c.d}(m,n) = 1.$

(6) $U(p) \approx Z_{p-1}$ $\uparrow$ $\rightarrow$ અમાજ્ય સંરચા

(7) If $f : \mathbb{Z}_m \rightarrow \mathbb{Z}_n \times \mathbb{Z}_k$ number of homomorphism = $\text{g.c.d}(m,n) \times \text{g.c.d}(m,k).$

(8) $f : \mathbb{Z}_m \times \mathbb{Z}_l \rightarrow \mathbb{Z}_n \times \mathbb{Z}_k$, number of homomorphism $\text{gcd}(m,n).$
 $\text{gcd}(1,n). \text{gcd}(1,k)$

Q. IF $f : \mathbb{Z}_{12} \rightarrow \mathbb{Z}_8 : f(x) = 2x$ is homomorphism ?

- A. Yes, homomorphism**
- B. No, Not homomorphism**
- C. May be homomorphism**
- D. None of these**

Q. IF $f : \mathbb{Z}_{25} \rightarrow \mathbb{Z}_{40}$ J How many homomorphism?

- A. 40 homomorphism**
- B. 5 homomorphism**
- C. 25 homomorphism**
- D. 15 homomorphism**

Q. $f : U(13) \rightarrow U(9)$, How many homomorphism?

- A. 13 homomorphism**
- B. 19 homomorphism**
- C. 8 homomorphism**
- D. 6 homomorphism**