Coset :- Let G be a group and H is Subgroup of G and $a \in G$. The set $a.H = \{a.h / \in H\}$ is called Left coset of H in G.

ऐसे ही $H.a = \{h.a / h \in H\}$ is Called Right Coset of H is G.

Q. If $G = \mathbb{Z}$, find all coset of H in G ?

A. 4 Distinct coset
B. 3 Distinct coset
C. Indentical coset
D. None of these.

Properties on Corsets: Let H be a subgroup of G and a, b ∈ G.

Then,

(i) $a \in aH$ (i.e. coset of every subgroup always non-empty)

(ii) $aH = H$ iff $a \in H$.

(iii) $aH = bH$ or $aH \cap bH = \emptyset$ (null set) cosets are either identical or disjoint.

(iv) $aH = bH$ iff $a^{-1}b$ be $\in H$ or Ha Hb iff $ab^{-1} \in H$.

(v) $|ah| = |bH|$

(vi) $aH = Ha$ iff $H = aHa^{-1}$.

(vii) aH (coset of G) is need not be subgroup of G.

(viii) aH is a subgroup of G iff $a \in H$.

(ix) H is subgroup of G, then, H itself is right as well as left coset of G by e (identity element) as $He = eH = H$.

(x) If H is subgroup of G. Then, $H H = H$.

Note :

1. If G is abelian group. Then, every left coset of H in G is right coset of H in G.

2. If G is cyclic group. Then, every left coset of H in G is right coset of H in G.

Index of a Subgroup: The index of a subgroup H of a group G is defined as the number of distinct right (or left) coset of H in G.

3. If G be a Finite Group, Then $i_G(H)$

Q. If $G = Z_{20}$ and $H = \langle 4 \rangle$ then $i_G(H)$?

A. 1

B. 2

C. 3

D. 4

Q. $G = Q_4$, $H = \{\pm 1, \pm i\}$, find all Distinct cosets of H in Q_4

A. 3

B. 2

C. 0

D. None of these

Q. $G = \mathbb{Z}$ and $H = 6\mathbb{Z}$ then $i_G(H) = ?$

A. 4

B. 6

C. 1

D. infinite

Q. $G = 2\mathbb{Z}$ and $H = 10\mathbb{Z}$ then $i_G(H) = ?$

A. 0

B. 2

C. 5

D. infinite

Q. $G = 3\mathbb{Z}$ and $H = 15\mathbb{Z}$ then $i_G(H) = ?$

A. 0

B. 2

C. 5

D. infinite

Q. Let H is subgroup of G and $a \in H$. then

A aH is subgroup of G.

B. aH is need not be subgroup of G.

C. aH is not subgroup of G.

D. None of these

Q. Let H is subgroup of G and $a \in H$. then,

A aH is subgroup of G.

B. aH is not subgroup of G.

C. aH is need not be subgroup of G.

D. None of these

Q. How many Distinct cosets of H = {I, (123) ,(132)} in S_3 ?

A 1

B. 3

C. 2

D. Infinite

Q. Let G be an Abelian group of order 10. Let $S = \{g \in G : g^{-1} = g\}$.

Then the number of non-indentity elements in S is

A 5

B. 2

C. 1

D. 0

Homomorphism : Let $(G_1, \cdot)$ and $(G_2, *)$ are two groups . A mapping $f : G_1 \rightarrow G_2$ is said to be homomorphism if

$$f(x \cdot y) = f(x) * f(y) \ \forall \ x, y \in G_1$$

$G_1 \rightarrow G_2$

Q. If $f : \mathbb{Z}_{10} \rightarrow \mathbb{Z}_{10} : f(x) = 2x$ is homomorphism ? Yes

same

$(Z_{10}, +)$

$Z_{10} \Rightarrow O(2) \Rightarrow 5$

$Z_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$O(2) = 5$

Q. IF $f : \mathbb{Z}_6 \to \mathbb{Z}_4$ defined by $f(x) = 3x$ is homomorphism ?

A. Yes, homomorphism

B. No, Not homomorphism

C. May be homomorphism

D. None of these

Z_6 = {0,1,2,3,4,5}

Z_4 ⇒ {0,1,2,3}

Z_4 ⇒ O(3) ⇒ 4 ✓

Z_6 → O(0) ⇒ 1

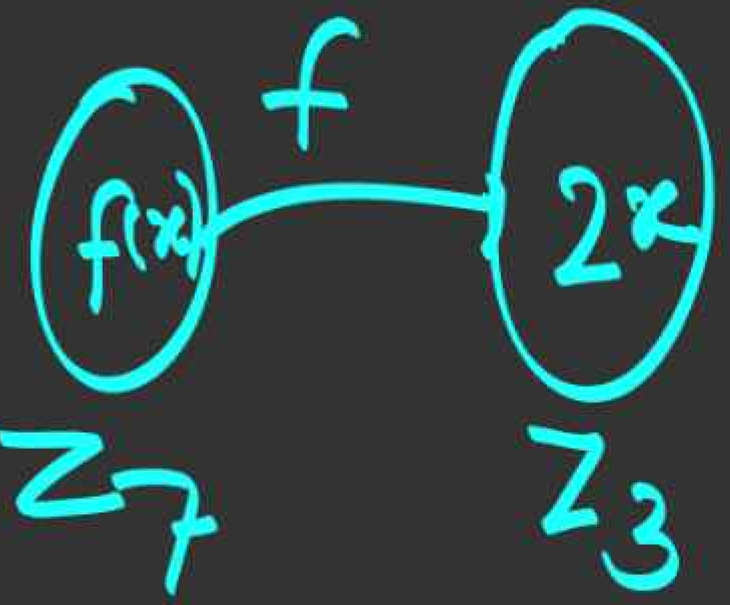
O(1) ⇒ 6

O(2) ⇒ 3

O(3) ⇒ 2

O(4) ⇒ 3

O(5) ⇒ 6



$\Rightarrow f: \mathbb{Z}_7 \to \mathbb{Z}_3 \ni f(x) = 2x$, Hmp?

$\mathbb{Z}_3 \Rightarrow \{0, 1, 2\}$

$O(2) \Rightarrow 3$

$\mathbb{Z}_7 = \{0, 1, 2, 3, 4, 5, 6\}$

$O(0) = 1$ $\quad O(4) = 7$

$O(1) = 7$ $\quad O(5) = 7$

$O(2) = 7$ $\quad O(6) = 7$

$O(3) = 7$

$2+2+2 = \frac{6}{3} = 0$

{No, not a Homomorphism}

Q. IF $f : \mathbb{Z}_6 \rightarrow \mathbb{Z}_4$: defined by f (x) = 2x is homomorphism ?

A. Yes, homomorphism
B. No, Not homomorphism
C. May be homomorphism
D. None of these

Q. IF $f : \mathbb{Z}_4 \to \mathbb{Z}_6 : f(x) = 2x$ is homomorphism ?

A. Yes, homomorphism

B. No, Not homomorphism

C. May be homomorphism

D. None of these

$O(Z_n) = n$

$Z_n = \{0, 1, 2, --- n-1\}$

$Z_5 = \{0, 1, 2, 3, 4\}$

$Z_3 = \{0, 1, 2\}$

Note :-

(1) $f : \mathbb{Z}_m \to \mathbb{Z}_n$ defined by $f(x) = ax$ if $O(a) = k$ in $\mathbb{Z}_m$ and $\mathbb{Z}_n$ has elements of order k. then, $f(x) = ax$ is homomorphism.

(2) If $f : \mathbb{Z}_m \to \mathbb{Z}_n$, Number of homeomorphism = g . c . d (m,n).

(3) $f : \mathbb{Z} \to \mathbb{Z} : f(x) = kx$ is homomorphism. (i.e. Has Infinite Number of homomorphism)

(4) $U(P^r)_{p \neq 2} \approx Z_{\emptyset(p^r)}$: p is prime.

$\phi \Rightarrow 3^2 - 3^1$

$9 - 3 = 6$

$9 \rightarrow 3^2$

$\phi \rightarrow \not{9}^{\,3} \times \frac{2}{\not{3}}$

$\Rightarrow 6$

$$U(9)$$

$$U(3^2) = Z_{\phi 3^2} = Z_{\phi 9} = Z_6$$

$O(U(49)) = ?$

$U(p) = Z_{p-1}$

$U(47) = Z_{47-1} = Z_{46} = O(Z_{46}) = 46$

$U(49) \Rightarrow$

$= Z_{42}$

$U(7^2) = Z_{\phi 49}$

$O(U(49)) = O(Z_{42}) = 42$

$49 \to 7^2 - 7^1$

$\Rightarrow 49 - 7 = 42$

(5) $U(m,n) \approx U(m) \cdot U(n)$: g.c.d(m,n) = 1.

(6) $U(p) \approx Z_{p-1}$ p- अभाज्य संख्या $O(U(7)) = Z_{7-1} = Z_6 = 6$

(7) If $f : \mathbb{Z}_m \to \mathbb{Z}_n \times \mathbb{Z}_k$ number of homomorphism = g.c.d(m,n) x g.c.d(m,k).

(8) $f : \mathbb{Z}_m \times \mathbb{Z}_l \to \mathbb{Z}_n \times \mathbb{Z}_k$, number of homomorphism [illegible]

gcd (m,n) x gcd (m,k) x gcd (l,n) x gcd (l,k)

Q. IF $f : \mathbb{Z}_{12} \rightarrow \mathbb{Z}_8 : f(x) = 2x$ is homomorphism ?

A. Yes, homomorphism

B. No, Not homomorphism

C. May be homomorphism

D. None of these

$Z_{12} = \{0, 1, 2, 3, 4, --11\}$

$O(3) = 4$

Z_8

$O(2) \Rightarrow 4$

$\mathbb{Z}_{40} \to \mathbb{Z}_{25}$

Q. IF $f : \mathbb{Z}_{25} \to \mathbb{Z}_{40}$ There are How many homomorphism?

A. 40 homomorphism

B. 5 homomorphism

C. 25 homomorphism

D. 15 homomorphism

$m = 25$, $n = 40$

g.c.d ⇒ 5

$m = 40, 25$

gcd = 5

Q. $f : U(13) \rightarrow U(9)$, How many homomorphism?

A. 13 homomorphism

B. 19 homomorphism

C. 8 homomorphism

D. 6 homomorphism

$U(13) \rightarrow U(9)$

$Z_{12} \rightarrow Z_6$

No. of Homomorphism $\Rightarrow \gcd(12, 6)$ = 6

$U(p) = Z_{p-1}$

$U(3^2) \rightarrow Z_{\phi 9}$