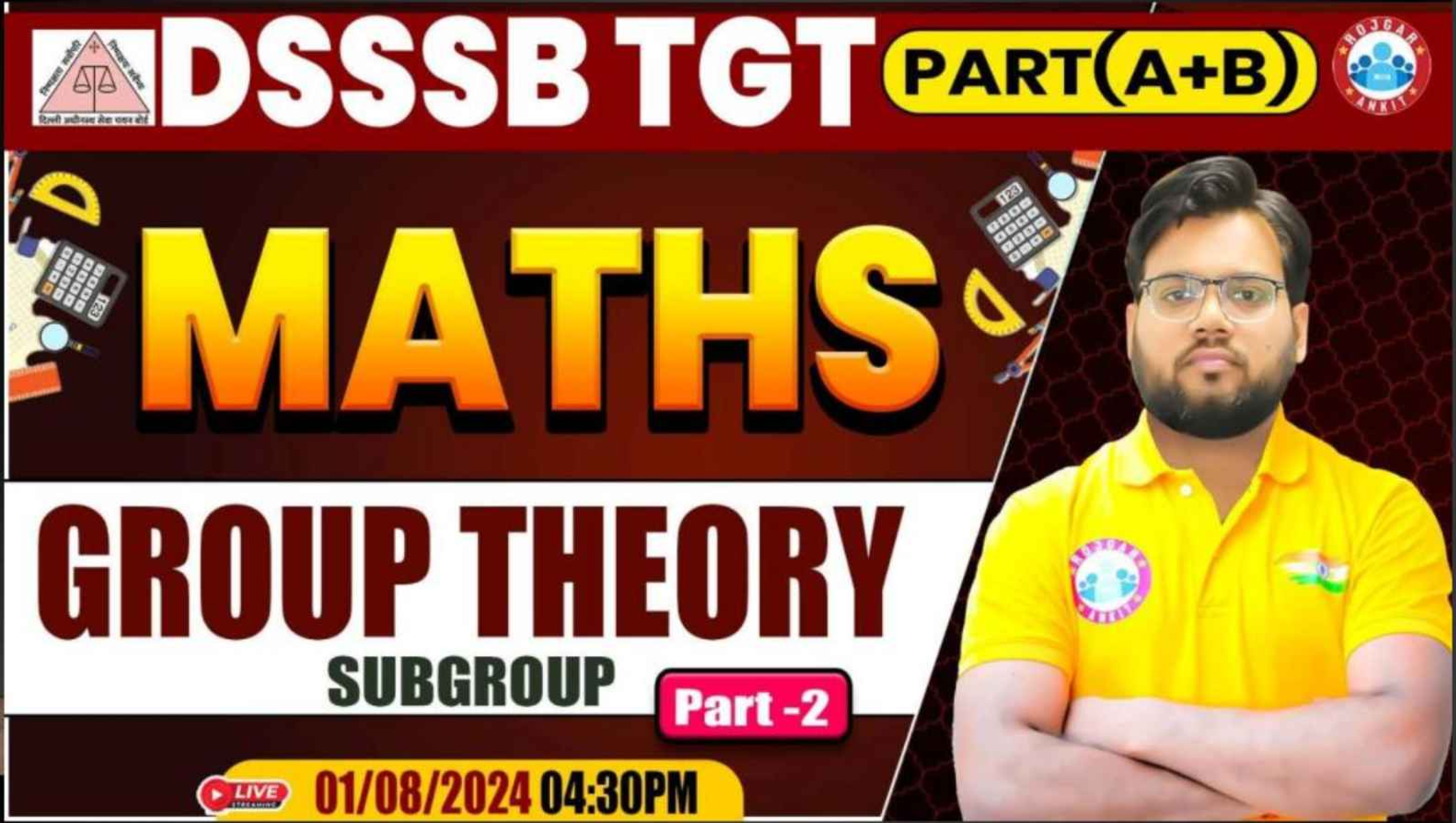
DSSSB TGT
PART(A+B)
MATHS
GROUP THEORY
SUBGROUP
Part -2
LIVE
01/08/2024 04:30PM

**1 Abelian Group → A group $(G,*)$ is said to be a abelian group If $x * y = y * x \ \forall x, y \in G$ under given operation *.**

1. $(z, t)$ is abelian?
2. $(Q, +), (\mathbb{R}, +), (C, +), (Z_n, +)$.
3. $(Q_1^* \cdot\cdot), (\mathbb{R}_1^* \cdot), (C_0^* \cdot)$
4. $z^*$
5. $(N, \cdot)$ is a abelian group or not?

**Note.** If every element of $G$ has self inverse then $G$ is abelian, but converse need not to be true.

# Group and Abelian Group

| Property | | | Explanation |
|---|---|---|---|
| Abelian Group | Group | Closure | $a, b \in G$, then $(a \cdot b) \in G$ |
| | | Associative | $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all $a, b, c \in G$. |
| | | Identity element | $(a \cdot e) = (e \cdot a) = a$ for all $a, e \in G$ |
| | | Inverse element | $(a \cdot a') = (a' \cdot a) = e$ for all $a, a' \in G$ |
| | Commutative | | $(a \cdot b) = (b \cdot a)$ for all $a, b \in G$ |

**Example**

**Question: Is ( $Z, +$ ) a group?**

**Solution:**

$$Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

| CAIN Property | Explanation | Satisfied? |
|---|---|---|
| Closure | If $a, b \in G$, then $(a \cdot b) \in G$. | |
| Associative | $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all $a, b, c \in G$. | |
| Identity element | $(a \cdot e) = (c \cdot a) = a$ for all $a \in G$. | |
| Inverse element | $(a \cdot a') = (a^{\circ} \cdot a) = e$ for all $a, a^{\circ} \in G$. | |
| Commutative | $(a \cdot b) = (b \cdot a)$ for all $a, b \in G$. | |

$\Rightarrow SL_n(\mathbb{F})$ is abelian?

Q. IF G is a group of even order, then it has an element $a \neq e$, satisfying:-

[UP PGT]

(a) $a^2 = e$

(b) $a^3 = c$

(c) $a^5 = l$

(d) $a^7 = C$

## Q $\Rightarrow$ $S_n$ is abelian? (under Composition)

Q $\rightarrow D_n$ is abelian?

**Cyclic Group** → A group $G$ is said to be cyclic group If $\exists$ an element $a \in G$, such that every element of $G$ is generated by $a$.

$$G = \langle a \rangle = \{a^n \mid n \in \mathbb{Z}\}$$

**Theorem 1** - A finite group $G$ is cyclic if and only if it has an element, which has the same order as that of the group itself.

$$O(G) = O(a) \backslash\& \; a \in G$$

Q $\rightarrow (\mathbb{Z}, t)$ is cyclic?

Q → How many exactly generator of $\mathbb{Z}$ ?

(a) Exactly 1

(b) Exactly 2

(c) Exactly 3

(d) None of these

Q) IF Group $G$ is finite and $O(a) = O(G)$ Then

(a) $a$ is generator of $G$

(b) $a$ is not be generator of $G$

(c) $a$ is need not be generator of $a$

(d) None of these

Q. If $O(a) = O(G)$ Then

(a) $a$ is generator of $G$

(b) $a$ is not be generator of $G$

(c) $a$ is need not be generator of $G$

(d) None of these.

**Subgroup** → $\phi \neq HCG$ is said to be subgroup of $G$.

If $H$ is itself group with Binary operation defined on $G$.

Q. $n\mathbb{Z}$ is subgroup of $\mathbb{Z}$, when $n \in \mathbb{Z}$ ?

(a) $n\mathbb{Z}$ is not subgroup of $\mathbb{Z}$

(b) $n\mathbb{Z}$ need not be subgroup of $\mathbb{Z}$

(c) $n\mathbb{Z}$ is subgroup of $\mathbb{Z}$

(d) None of these

- If $a$ is generator of cyclic group $a$ then $a^{-1}$ is also its generator.

यदि a चक्रीय समूह a का जनरेटर है तो $a^{-1}$ भी इसका जनरेटर है।

$(Z, +)$

$\Downarrow$

$(1, -1)$

$O(\mathbb{R}) = \infty$

- Every $\boxed{\text{Infinite}}$ cyclic group has two and only two generators.

प्रत्येक अनंत चक्रीय समूह में दो और केवल दो जनरेटर होते हैं।

$(\mathbb{R})$

(अनंत)

- Every $\underbrace{\text{subgroup}}$ of a cyclic group is also cyclic.

चक्रीय समूह का प्रत्येक उपसमूह भी चक्रीय होता है।

- Any group of order 3 is cyclic.

क्रम 3 का कोई भी समूह चक्रीय होता है।

$(1, \omega, \omega^2)$

- **Sub-Group (उपसमूह)** → Any non empty sub set $H$ of a group $G$ is called sub-Group or complex of a group.

समूह G का कोई भी गैर-खाली उपसमूह H समूह का उपसमूह या संकुल कहलाता है।

$Z = \{-\infty \ldots -4, -3, -2, -1, 0, 1, 2, 3, 4 \ldots \infty\}$

$(Z, +)$ is a group.

$Z^+ = \{\underbrace{1, 2, 3, 4, \ldots \infty}_{N}\}$

$(Z^+, +)$ No group ✗ $\boxed{e = 0}$

No subgroup ✗

$(\mathbb{R}, +)$ is a group.

$$\boxed{e = 0}$$

<u>Inverse</u> $\rightarrow$ $a + (a^{-1}) = e$

$8 + (-8) = 0$

$$\boxed{-8 \notin \mathbb{R}'}$$

$\mathbb{R}' = 2n$

$\Rightarrow \{0, 2, 4, 6, 8 \ldots \infty\}$

$(\mathbb{R}', +)$ is a group?

1. Closure → $2, 3 \in Z^+$

$2 \times 3 = 6 \in Z^+$ ✓

2. Assosiativity → $1, 2, 3 \in Z^+$ ✓

$1 \cdot (2 \cdot 3) = (1 \cdot 2) \cdot 3$

$6 = 6$ ✓

3. Indentity ⇒ $e = 1$ ✓

4. Inverse → $a \times a^{-1} = e$ → $2 \times \frac{1}{2} = 1$

$1 \times 1 = 1$ ✓

$(Z, \cdot)$ is a group ✗

$(Z^+, \cdot)$ is a group?

$Z^+ = \{1, 2, 3, 4, \ldots\ldots \infty\}$

✗ ✗

$\frac{1}{2} \notin Z^+$ ✗

$\mathbb{Q} \to$ Rational No.

$(\mathbb{Q}, \cdot)$ is a group.

$(\mathbb{Q}^+, \cdot)$ is a subgroup?

4. <u>Inverse</u> $\to a \times a^{-1} = e$

$$4 \times \frac{1}{4} = 1 \Rightarrow \frac{1}{4} \in \mathbb{Q}^+$$

$\mathbb{Q}^+$ is a subgroup of $\mathbb{Q}$.

## • Proper and Improper (or Trivial) Subgroup:

If order of a group is at least 2 or more than 2 then it has atleast two subgroups which are:

यदि किसी समूह का कोटि कम से कम 2 या 2 से अधिक है तो उसके कम से कम दो उपसमूह होंगे जो इस प्रकार हैं:

(Trival) Improper Subgroup

(i) $G$ (itself स्वयं)

(ii) $\{e\}$ ie. the group of identity alone अर्थात केवल पहचान का समूह

The above two subgroups are known as Improper or Trivial subgroups.

उपर्युक्त दो उपसमूहों को अनुचित या ~~[illegible]~~ उपसमूह के रूप में जाना जाता है।

A subgroup other than these two is known as a proper ~~group.~~ Subgroup.

इन दोनों के अलावा एक अन्य उपसमूह को उचित समूह के रूप में जाना जाता है।

$H \subseteq G$

1. If group $G$ is a subgroup $H$ and $a \in G, b \in G$, then:

यदि समूह G एक उपसमूह H है और $a \in G, b \in G$ है, तो:

$\boxed{H \in G}$

~~(a)~~ $a = Hb \Leftrightarrow ab \in H$

~~(b)~~ $Ha = Hb \Leftrightarrow (ab)^{-1} \in H$

(c) $Ha = Hb \Leftrightarrow a^{-1}b \in H$

(d) $Ha = Hb \Leftrightarrow ab^{-1} \in H$

$a = Hb$

$ab^{-1} = Hb^{-1}$

$Ha = Hb$

$H\underbrace{aa^{-1}}_{e} = Hba^{-1}$

$H = Hba^{-1}$

$ba^{-1} = H$

$a^{-1}b$

(iv) $Hab^{-1} = Hbb^{-1}$

$Hab^{-1} = H$

2. The Identical element of the relative group of $G = \{2, 4, 6, 8\}$ with respect to multiplication modulo 10 is:

गुणन मॉड्यूलो 10 के संबंध में G={2,4,6,8} के सापेक्ष समूह का समान तत्व है:

(a) 2

(b) 4

(c) 6

(d) 8

$\boxed{e = ?}$

Identity $= \boxed{a \times e = a}$

$2 \times 6 = \frac{12}{10} = 2$

$2 \times 6 = \frac{12}{10} = R = 2$

$4 \times 6 = \frac{24}{10} = R = 4$

$6 \times 6 = \frac{36}{10} = R = 6$

$8 \times 6 = \frac{48}{10} = R = 8$

3. The Generator of the group $(Z, +)$ is

समूह (Z,+) का जनरेटर है

(a) 0 and 1

(b) -1 and 1

(c) 0 and -1

(d) Generator doe not exist

$o(G) = 6$

$G = \{1, 2, 3, 4, 5, 6\}$

4. A Generator element for a Multiplicative group of finite squares $\{1, 2, 3, 4, 5, 6(\text{mod}7))$ is:

परिमित वर्गों {1,2,3,4,5,6(mod7)) के गुणक समूह के लिए एक जनरेटर तत्व है:

(a) $\overline{2}$

(b) $\overline{3}$

(c) $\overline{4}$

(d) $\overline{6}$

$\boxed{e = 1}$

$G = \langle a \rangle$

$o(G) = o(a)$

$o(2) \Rightarrow 3$

$o(3) \Rightarrow 6$

$o(4) \Rightarrow 3$

$o(6) = 2$

5. The multiplicative group $\{1, -1, i, -i\}$ is a cyclic group, its generators are

गुणक समूह {1,-1,i, -i} एक चक्रीय समूह है, इसके जनरेटर हैं

(a) 1 and $i$

(b) 1 and -1

(c) $i$ and $-i$

(d) only i

$i \times i = -1$

$i \times i \times i = -i$

$i \times i \times i \times i = 1$

$i \times i \times i \times i \times i = i$

$e = 1$

$O(G) = 4$

$O(1) = 1$

$O(-1) = 2$

$O(i) = 4$

$O(-i) = 4$

6. The order of the element $(-\mathrm{i})$ of the multiplicative group $\mathrm{G} = \{1, -1, i, -i\}$ is

गुणक समूह G= {1,-1,i,-i} के तत्व (-i) का क्रम है

(a) 4

(b) 3

(c) 2

(d) 1

7. The number of generators of a cyclic group of order 10 is

क्रम के चक्रीय समूह के जनरेटर की संख्या 10 है

(a) 2

(b) 3

(c) 4

(d) 5

$10 \Rightarrow 2 \times 5$

$\hookrightarrow \cancel{10} \times \frac{1}{\cancel{2}} \times \frac{4}{\cancel{5}}$

$\Rightarrow 4$

$O(G) = 10$

$(1, 3, 7, 9)$

9. The number of Generators of cyclic group of order 8 is:

क्रम 8 के चक्रीय समूह के जनरेटर की संख्या है:

(a) 2

(b) 3

(c) 4

(d) 6

$o(G) = 8$

$(1, 3, 5, 7)$

$8 \to 2^3$

$\to \overset{4}{\cancel{8}} \times \frac{1}{\cancel{2}} = 4$

13. The order of the group S4 [symmetric group] is

समूह S4 [सममित समूह] का क्रम है

$O(S_n) = n!$

(a) 12

(b) 16

(c) 20

(d) 24

$O(S_4) = 4! = 4 \times 3 \times 2 \times 1$
$= 24$

14. The generators of a group $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$ are

समूह $G = \{a, a^2, a^3, a^4, a^5, a^6 = e\}$के जनरेटर हैं

(a) $a$ and $a^5$

(b) $a^2$ and $a^4$

(c) $a^3$ and $a^5$

(d) $a^2$ and $a^3$

15. Let G be an infinite cyclic group, then:

मान लीजिए G एक अनंत चक्रीय समूह है, तो:

(a) G has in finitely many generators

(b) G has one generator

(c) G has two generator

(d) G has three generator