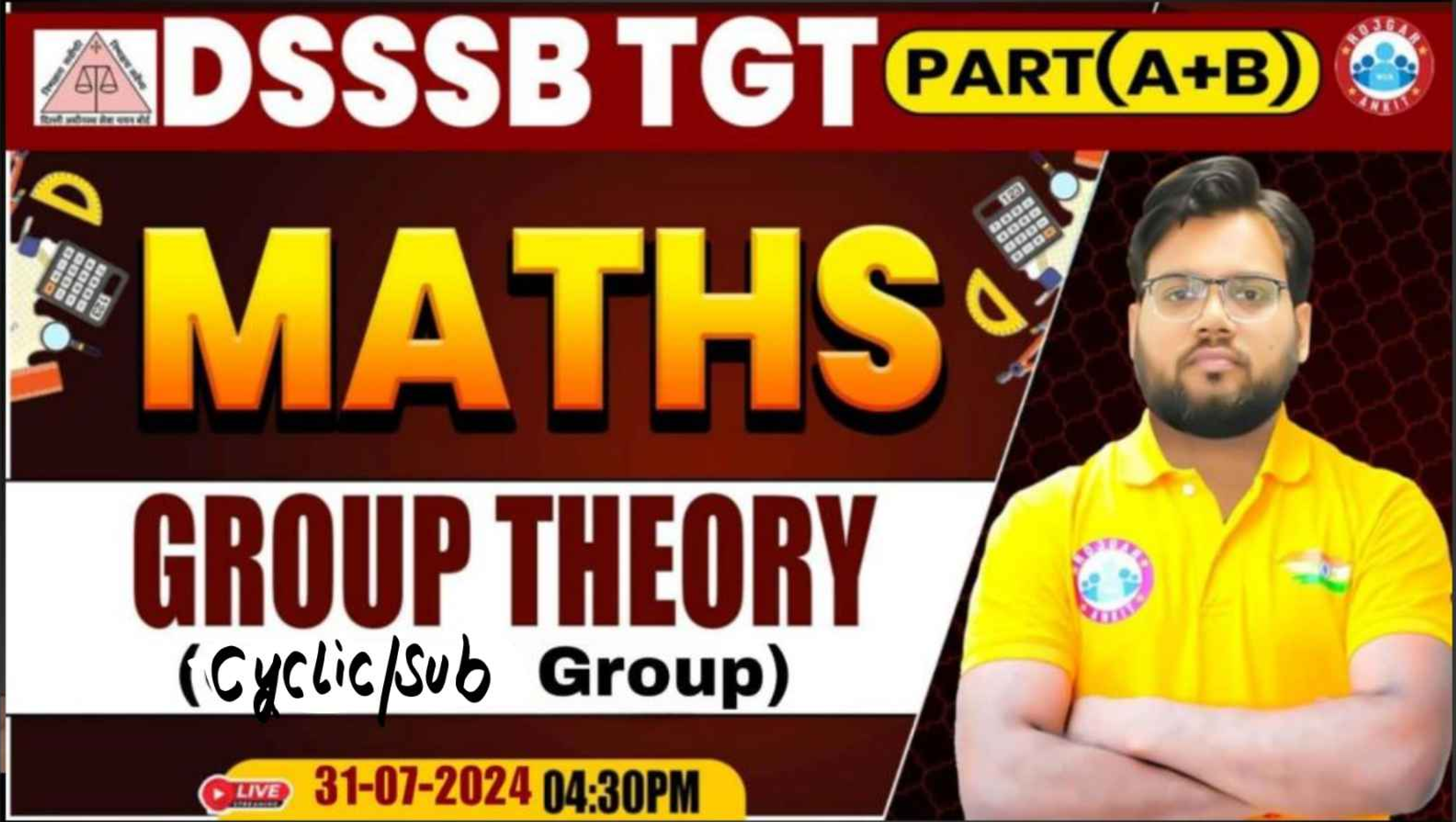
DSSSB TGT
PART(A+B)
MATHS
GROUP THEORY
(Cyclic/Sub Group)
LIVE
31-07-2024 04:30PM

1 Abelian Group → A group $(G,*)$ is said to be a abelian group If $x*y=y*x\ \forall x,y\in G$ under given operation *.

1. (z,t) is abelian?
2. $(Q,+),(\mathbb{R},+),(C,+),(Z_n,+)$.
3. $(Q_1^*\cdot\cdot),(\mathbb{R}_1^*\cdot),(C_0^*\cdot)$
4. z^*
5. $(N,\cdot)$ is a abelian group or not?

<u>Note</u>. If every element of G has self inverse then G is abelian, but converse need not to be true.

$\Rightarrow GL_{n}(\mathbb{F})$ is abelian?

Group and Abelian Group

Property			Explanation
Abelian Group	Group	Closure	$a, b \in G$, then $(a \cdot b) \in G$
		Associative	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all $a, b, c \in G$.
		Identity element	$(a \cdot e) = (e \cdot a) = a$ for all $a, e \in G$
		Inverse element	$(a \cdot a') = (a' \cdot a) = e$ for all $a, a' \in G$
	Commutative		$(a \cdot b) = (b \cdot a)$ for all $a, b \in G$

Example

Question: Is (Z, +) a group?

Solution:

$$Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

CAIN Property	Explanation	Satisfied?
Closure	If a, b $\in$ G, then $(a \cdot b) \in G$.	
Associative	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for all a, b, c $\in$ G.	
Identity element	$(a \cdot e) = (c \cdot a) = a$ for alla $\in$ G.	
Inverse element	$(a \cdot a') = (a^\circ \cdot a) = e$ for alla, $a^\circ \in G$.	
Commutative	$(a \cdot b) = (b \cdot a)$ for all a, b $\in$ G.	

$\Rightarrow SLn\ (\mathbb{F})$ is abelian?

$\boxed{n=1} \rightarrow SL_1(\mathbb{F}) \rightarrow$ Abelian

$\boxed{n\geq 2} \rightarrow \left.\begin{array}{l} SL_2(\mathbb{F}) \\ SL_q(\mathbb{F}) \end{array}\right\}$ Not an abelian group.

Q. IF G is a group of even order, then it has an element $a \neq e$, satisfying:-

e = identity

[UP PGT]

(a) $a^2 = e$

(b) $a^3 = e$

(c) $a^5 = e$

(d) $a^7 = e$

$Q_4 \Rightarrow \{\pm 1, \pm i, \pm j, \pm k$, $i^2 = j^2 = k^2 = -1$

$O(Q_4) = 8$ (Even order group)

$\boxed{e = 1}$

$a = -i$

$a^4 = (-i)^4 = 1 = e$

$\boxed{a^4 = e}$

$O(-1) = 2$

$O(i) = 4$

$O(-i) = 4$

$a = -1$

$a^2 = 1 = e$

Q $\Rightarrow S_n$ is abelian? (under Composition)

$O(S_n) = n!$

$2!$

S_1, S_2 > Abelian

$S_1 = \{I\}$ $\quad I \cdot I = I \cdot I$

$S_2 = \{I, (1\,2)\}$

$\Rightarrow I \cdot (1\,2) = (1\,2)\, I$

$S_3, S_4, S_5, \ldots - S_{99} \ldots$ Non abelian

$\boxed{n \geq 3}$ $\rightarrow$ Non

$O(S_3) = 3! = 6$

$S_3 = \{I, (1\,2), (2\,3), (1\,3), (1\,3\,2), (1\,2\,3)\}$

Q $\rightarrow D_n$ is abelian?

Cyclic Group → A group G is said to be cyclic group If $\exists$ an element $a \in G$, such that every element of G is generated by a.

$$G = \langle a \rangle = \{a^n \mid n \in \mathbb{Z}\}$$

$$a^n \Rightarrow \underbrace{a+a+a+a+a+a+\cdots}_{n \text{ times}}$$

$$a^n = \underbrace{a \times a \times a \times a \times a \times a \times a \cdots}$$

Theorem 1 - A finite group G is cyclic if and only if it has an element, which has the same order as that of the group itself.

$$O(G) = O(a) \ \backslash\& \ a \in G$$

$(Z_8, +)$

$O(Z_8) = 8$

$Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\} = \langle 1 \rangle, \langle 3 \rangle, \langle 5 \rangle, \langle 7 \rangle$

$e = 0$

$3+3+3+3+3+3+3+3 = 0$

$3+3+3 = \frac{9}{8} = 1$

$3+3+3+3+3+3 = \frac{18}{8} = 2$

$3+3+3+3+3+3+3+3+3 \Rightarrow \frac{27}{8} \Rightarrow 3$

$O(0) = 1$

$O(1) = 8$

$O(2) = 4$

$O(3) = 8$

$O(4) = 2$

$O(5) = 8$

$O(6) = 4$

$O(7) = 8$

$\bar{1} + \bar{1} + \bar{1} + \bar{1} + \bar{1} + \bar{1} + \bar{1} + \bar{1}$

$3+3+3+3+3+3+3+3 = 24$

8 ϕ Eular

Order 8 के साथ Co-prime

$(8,1)\ (8,3),\ (8,5)\ (8,7)$

$(Z, +)$

$Z = \{0, \pm 1, \pm 2, \pm 3, - - - - - - \infty\}$

$1, -1$

$O(2) = \infty$

$O(3) = \infty$

$1+1=2$

$1+1+1=3$

$1+1+1+1=4$

$1-1=0$

$1-1-1=-1$

$1-1-1-1=-2$

Q → $(\mathbb{Z}, +)$ is cyclic? Yes

$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \text{------} \infty\}$

$\langle 1 \rangle, \langle -1 \rangle$

Q → How many exactly generator of $\mathbb{Z}$?

$(\mathbb{Z}, +)$

(a) Exactly 1

(b) Exactly 2 $\langle 1 \rangle, \langle -1 \rangle$

(c) Exactly 3

(d) None of these

Q) IF Group G is finite and $O(a) = O(G)$

Then $a \in G$

(a) a is generator of G

(b) a is not be generator of G

(c) a is need not be generator of a

(d) None of these

$G = Z_4 = \{0, 1, 2, 3\}$

$O(Z_4) = 4$

$O(0) = 1$

$O(1) = 4$

$O(2) = 2$

$O(3) = 4$

Q. If $O(a) = O(G)$ Then

(a) a is generator of G

(b) a is not be generator of G

(c) a is need not be generator of G

(d) None of these.

$\langle 1 \rangle, \langle -1 \rangle$

$(\mathbb{Z}, +)$

$O(\mathbb{Z}, +) = \infty$

Subgroup → $\phi \neq H \subset G$ is said to be subgroup of G.

If H is itself group with Binary operation defined on G.

Q. $n\mathbb{Z}$ is subgroup of $\mathbb{Z}$, when $n \in \mathbb{Z}$?

(a) $n\mathbb{Z}$ is not subgroup of $\mathbb{Z}$

(b) $n\mathbb{Z}$ need not be subgroup of $\mathbb{Z}$

(c) $n\mathbb{Z}$ is subgroup of $\mathbb{Z}$

(d) None of these