DSSSB TGT PART(A+B)

MATHS

GROUP THEORY

Part -8

LIVE

25/07/2024 04:30PM

$\Rightarrow$ Symmetric (S_n) is a group under Composition?

Q. Find Identity and inverse of each element of S_2?

Q. $g = (1 \quad 2 \quad 3 \quad 5) \in S_5$ Then $g^{-1} = ?$

(a) $(5 \quad 3 \quad 2 \quad 1)$

(b) $(1 \quad 2 \quad 3 \quad 5)$

(c) $(2 \quad 1 \quad 5 \quad 3)$

(d) None of these

- Note $\rightarrow$ If $f = (a_1, a_2, a_3, \ldots, a_r) \in S_n r \leqslant n$ then $f^{-1}(ar, a_{r-1}, a_{r-2} \ldots a_2, a_1)$

Q $f = (1 \quad 2 \quad 3) \in S_3$
$g = (1 \quad 3 \quad 2) \in S_3$ Then $f \cdot g = ?$

(a) I

(b) (123)

(c) (132)

(d) None of these

Q. $f = (1 \quad 2) ES_3$
$g = (1 \quad 3) \in S_3$ Then find f.g and $g \cdot f$

(a) (132),(123)

(b) (123), (321)

(c) (213), (123)

(d) None of these

Q. In the group $G = \{\{2, 4, 6, 8\}, 10\}$ the identity element is $=$?

(a) 2

(b) 4

(c) 6

(d) 8

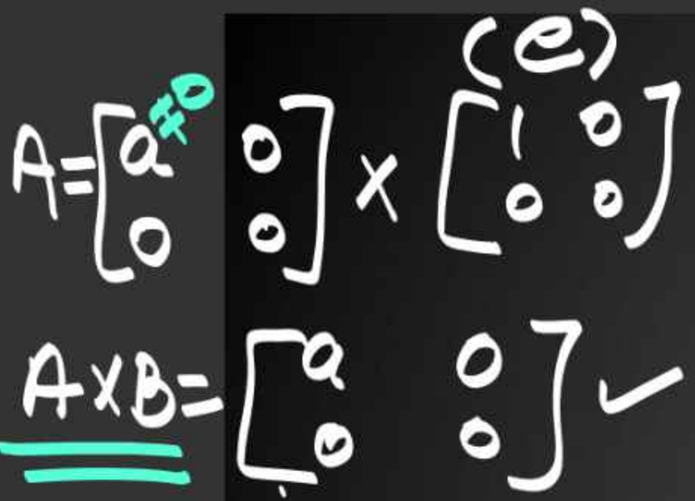
Q. Let G be a group with binary operation defined by $a * b = a + b + 1 \; \forall ab \in G$. Inverse of element ' c ' of G is -

(a) $2 + c$

(b) $2 - c$

(c) $-2 + c$

(d) $-2 - c$



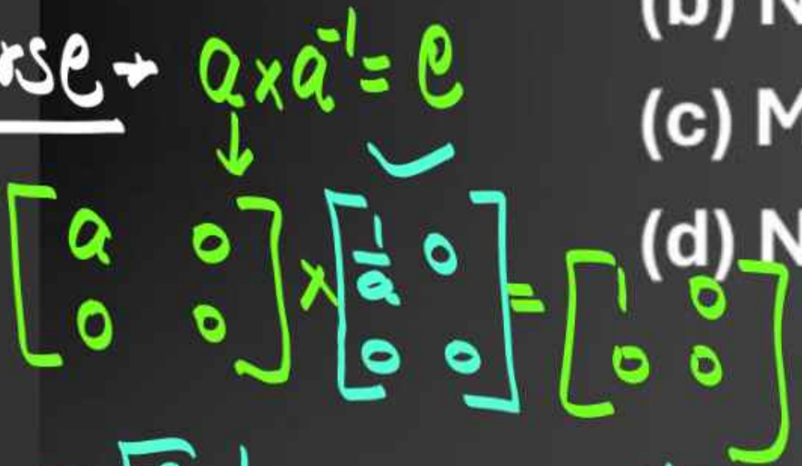
Q. $G = \left\{ A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix};\ 0 \neq a \in \mathbb{R} \right\}$ is group with Respect to multiplication?

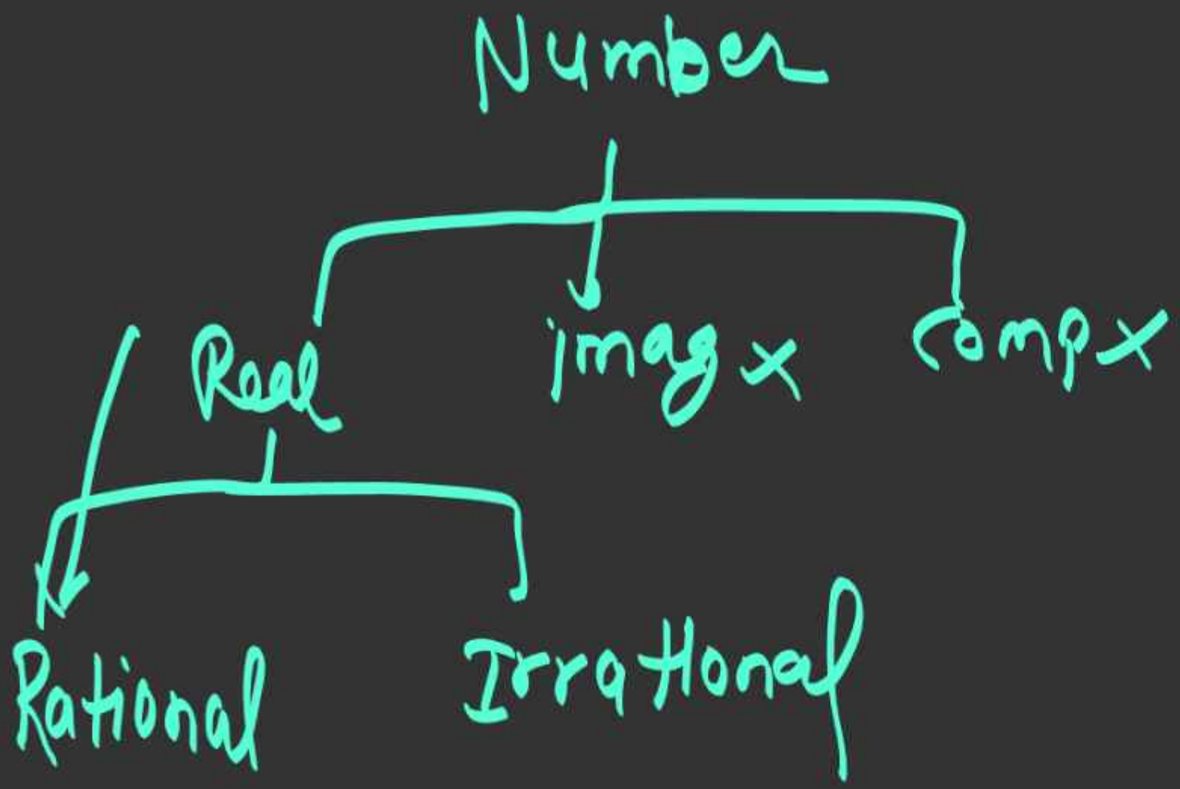
(a) Yes, it is a group

(b) No, it is not a group

(c) Monoid

(d) None of these




Number
Real
imagx
compx
Rational
Irrational

Q. $G = \left\{ A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} ; 0 \neq a \in \mathbb{Z} \right\}$ is group?

$0 \neq a$

$a \in \mathbb{Z}$ → integer: +ve, -ve

(a) yes

(b) No

(c) Monoid

(d) None of these

$e = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Inverse $\Rightarrow$ $a^{-1} = \frac{1}{a}$

$\frac{1}{2}, \frac{1}{3}, \frac{1}{400} \notin \mathbb{Z}$

under multiplication

Q. $G = Q^-$ → set of all negative rational numbers & $a \cdot b = \frac{ab}{3}$, then ~~(G, ·)~~ (G, ∘) is group?

(a) Yes

~~(b)~~ No

(c) Monoid

(d) None of these

① Closure →

$\forall\ a, b \in G^-$

$a \cdot b \in G$

-2 -3

$a \cdot b = \frac{ab}{3} = \frac{-2 \times (-3)}{3} \Rightarrow +2 \notin G^-$

$\bar{Q}$ ⇒ Negative Rational No.

$N = \{1, 2, 3, \ldots \infty\}$

Q. If N is a set of natural numbers, * is an operation such that $a * b = a + b + ab \; \forall a, b \in N$, then identity element for * is:

यदि N प्राकृत संख्याओं का एक समुच्चय है, ' *' एक संक्रिया इस प्रकार है कि $a * b = a + b + ab \forall a, b \in N$, तब * के लिए तत्समक अवयव है:

(a) 1 (b) 0

(c) 2 (d) Non-existent/अस्तित्वहीन

Identity ⇒ $b = e$

$a * e = a$

$a + e + ae = a$

$ae = -e$

$a = -1$

$ae + e = 0$ $e(a+1) = 0$

$e = 0 \notin N$

Q. If $(G,*)$ is a group and $x * y = x + 2y - 3 \forall x, y \in G$, then inverse of x in the group is.

यदि $(G,*)$ एक समूह है और $x * y = x + 2y - 3 \forall x, y \in G$, तो x का समूह में व्युत्क्रम है-

(a) $\frac{2x+9}{4}$

(b) $\frac{9-2x}{4}$

(c) $\frac{x-3}{4}$

(d) $\frac{x+2}{4}$

Identity → $y=e$

$a * e = a$

$a + 2e - 3 = a$

$2e = 3$

$e = \frac{3}{2}$

Inverse → $a * a^{-1} = e$

$a + 2a^{-1} - 3 = \frac{3}{2}$

$2a^{-1} = \frac{3}{2} + 3 - a$

$2a^{-1} = \frac{3+6-2a}{2} \Rightarrow \frac{9-2a}{2}$

$a^{-1} = \frac{9-2a}{4}$

put $a = x$

$x^{-1} = \frac{9-2x}{4}$

$O(a^{20}) = 3$

Q. Let ' a ' be an element of a group and $O(a) = 30, O(a^{18})$ is equal to -

मान लीजिए ' a ' एक समूह का अवयव है और $O(a) = 30, O(a^{18})$ बराबर है-

(a) 2

(b) 5

(c) 6

(d) 10

$$O(a^{18}) = \frac{30}{HCF(18, 30)}$$

$$= \frac{30}{6}$$

$$\Rightarrow 5 \text{ Ans.}$$

$$O(a^n) = \frac{O(a)}{\gcd(n, O(a))}$$

Identity →

$a * e = a$

$\Rightarrow \frac{ae}{3} = a$

$e = 3$

Inverse → $a * a^{-1} = e$

$\frac{a \times a^{-1}}{3} = 3$

$a^{-1} = \frac{9}{a}$

Q. Let * be the binary operation defined on the set of positive rational number Q^+ by the rule $a * b = \frac{ab}{3}$ $\forall a, b \in Q^+$ Then, inverse of $4 * 6$ is?

मान लीजिए * धनात्मक परिमेय संख्या Q^+ के समुच्चय पर नियम $a * b = \frac{ab}{3} \forall a, b \in Q^+$ द्वारा परिभाषित द्विआधारी संक्रिया है, तो $4 * 6$ का व्युत्क्रम क्या है?

$4 * 6 \Rightarrow \frac{4 \times 6}{3} \Rightarrow 8$

(a) $\frac{3}{8}$ (b) $\frac{2}{3}$

(c) $\frac{9}{8}$ (d) $\frac{3}{2}$

Q. In the multiplication group $\{1, -1, i, -i\}$ where $i^2 = -1$; the inverse of i^5 is?

गुणन समूह $\{1, -1, i, -i\}$ में जहाँ $i^2 = -1$; i^5 का व्युत्क्रम क्या है?

(a) 1

(b) -1

(c) i

(d) $-i$

Identity → $e = 1$

Given = $i^2 = -1$ → $i^4 = 1$

पूछा है → inverse of i^5

Inverse →

$i^5 \times x = 1$ → जहाँ x = inverse of i^5

$i^4 \times i \times x = 1$

$x = \frac{1}{i}$

$x = -i$

$\frac{1}{i} = -i \Rightarrow \frac{1 \times i}{i \times i} \Rightarrow \frac{i}{-1} = -i$

Q. Which of the following structures is not group?

निम्नलिखित में से कौन सी संरचना समूह नहीं है?

$\omega^3 = 1$ $(e=1)$

① Identity → $a \times e = a$

② Inverse → $a \times a^{-1} = e$

$\omega \times \omega^2 = 1$

(a) $\{(1, \omega, \omega^2), \bullet\}$

(b) $\{\{1, -1\}, \bullet\}$

(c) $\{\{1, -1\}, +\}$ → Addition

(d) $(\{1, -1, i, -i\}, \bullet)$

→ multiplication

Q. Which of the following is group?

निम्नलिखित में से कौन सा समूह है?

(a) $(\mathbb{Z},*)$ where $a * b = ab; \forall a, b \in \mathbb{Z}$

(b) $(\mathbb{Z},*)$ where $a * b = a - b; \forall a, b \in \mathbb{Z}$

(c) $(\mathbb{R},*)$ where $a * b = ab; \forall a, b \in \mathbb{R}^+$ ⇒ Positive Real

(d) $(\mathbb{Z},*)$ where $a * b = a + b; \forall a, b \in \mathbb{Z}^-$ ⇒ Negative integer

① Closure ⇒ 2, 3

$2 * 3 = 6 \in R^+$

Identity ⇒ $e = 1$

Inverse ⇒ $2 \times \frac{1}{2} = e$

Q. The identity element in the set z of integers under the binary operation * Defined by $a * b = a + b + 1, \forall a, b \in Z$ is? integer

$a * b = a + b + 1, \forall a, b \in Z$ द्वारा परिभाषित बाइनरी ऑपरेशन $*$ के अंतर्गत पूर्णांकों के सेट z में पहचान तत्व क्या है?

(a) 1

(b) -1

(c) 0

(b) 2

Identity ⇒

$a * e = a$

$a + e + 1 = a$

$e = -1$

[Raj TGT]

Inverse

$e = 0$

Inverse → $\bar{2} + \bar{3} = 0$

$\bar{5} = 0$

Q. In the group $Z_5 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}\}$ of residue classes Modulo 5 , with respect to the operation of addition of residue classes modulo 5 , the inverse of $\bar{2}$ is?

$e = 0$

अवशेष वर्गों 5 के समूह $Z_5 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}\}$ में, अवशेष वर्गों 5 के जोड़ के संचालन के संबंध में, $\bar{2}$ का व्युत्क्रम क्या है?

$\bar{1}^{-1} = \bar{4}$ $\bar{1}$ का inverse $\Rightarrow 1 + \bar{4} = 0$

$\bar{3}^{-1} = 2$

$\bar{4}^{-1} = 1$

(a) $\bar{1}$ (b) $\bar{2}$

(c) $\bar{3}$ (d) $\bar{4}$

[DSSSB PGT]

$$Z_6 = \{0, 1, 2, 3, 4, 5\}$$

$$Z_n = \{0, 1, 2, 3, \ldots n-1\}$$

$$Z_n = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \ldots \overline{n-1}\}$$

$$Z_n = \{[0], [1], [2], \ldots [n-1]\}$$

Q. The set $\{\overline{5}, \overline{15}, \overline{25}, \overline{35}\}$ under the binary operation of multiplication of residue classes modulo 40 is?

अवशेष वर्गों के गुणन के बाइनरी ऑपरेशन के तहत $\{\overline{5}, \overline{15}, \overline{25}, \overline{35}\}$ मॉड्यूलो 40 है?

[UP PGT]

(a) not a group

(b) a group with identity $\overline{5}$

(c) A group with identity $\overline{25}$

(d) A group with identity $\overline{35}$

Identity →

$5 \times e = 5$

$5 \times 15 = 75 \pmod{40} = 35$

$5 \times 5 = 25 \pmod{40} = 25$

$5 \times 25 = 125 \pmod{40} = 5$

$e = 25$

$5 \times 25 \Rightarrow 125 \pmod{40} = 5$

$15 \times 25 \Rightarrow 375 \pmod{40} = 15$

$25 \times 25 = 625 \pmod{40} = 25$

$35 \times 25 \Rightarrow 875 \pmod{40}$

$\Rightarrow 35$

Q. If S denotes the set of all rational numbers except 1 and $*$ is defined on S as: $a * b = a + b - ab$ Then the solution of the equation $2 * x * 7 = 13$ is?

H.W.

यदि S, 1 को छोड़कर सभी परिमेय संख्याओं के समुच्चय को दर्शाता है और S पर * को इस प्रकार परिभाषित किया गया है: $a * b = a + b - ab$ तो समीकरण $2 * x * 7 = 13$ का हल क्या है?

(a) 1 (b) 2

(c) 3 (d) 7

[UP SGT]