



DSSSB TGT

PART (A+B)



MATHS

GROUP THEORY

ORDER OF GROUP



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**$\Rightarrow$ Symmetric (S_n) is a group under
Composition?**

**Q. Find Identity and inverse of each element
of S_2 ?**

Q. $g = (1 \ 2 \ 3 \ 5) \in S_5$ Then $g^{-1} = ?$

(a) $(5 \ 3 \ 2 \ 1)$

(b) $(1 \ 2 \ 3 \ 5)$

(c) $(2 \ 1 \ 5 \ 3)$

(d) None of these

• Note $\rightarrow$ If $f = (a_1, a_2, a_3, \dots, a_r) \in S_n$ $r \leq n$ then $f^{-1}(a_r, a_{r-1}, a_{r-2} \dots a_2, a_1)$

Q $f = (1 \ 2 \ 3) \in S_3$ Then $f \cdot g = ?$
 $g = (1 \ 3 \ 2) \in S_3$

(a) I

(b) (123)

(c) (132)

(d) None of these

Q. $f = (1 \ 2)ES_3$ Then find $f \cdot g$ and $g \cdot f$
 $g = (1 \ 3) \in S_3$

(a) (132), (123)

(b) (123), (321)

(c) (213), (123)

(d) None of these

Q. In the group $G = \{\{2, 4, 6, 8\}, 10\}$ the identity element is = ?

(a) 2

(b) 4

(c) 6

(d) 8

Q. Let G be a group with binary operation defined by $a * b = a + b + 1 \forall a, b \in G$.

Inverse of element ' c ' of G is -

- (a) $2 + c$
- (b) $2 - c$
- (c) $-2 + c$
- (d) $-2 - c$

**Q. $G = \left\{ A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}; 0 \neq a \in \mathbb{R} \right\}$ is group
with Respect to multiplication?**

- (a) Yes, it is a group**
- (b) No, it is not a group**
- (c) Monoid**
- (d) None of these**

Q. $G = \left\{ A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}; 0 \neq a \in \mathbb{Z} \right\}$ is group?

(a) yes

(b) No

(c) Monoid

(d) None of these

Q. $G = Q^- \rightarrow$ set of all negative rational numbers & $a \cdot b = \frac{ab}{3}$, then $(G, \cdot)$ is group?

- (a) Yes**
- (b) No**
- (c) Monoid**
- (d) None of these**

Q. If N is a set of natural numbers, 1* is an operation such that $a * b = a + b + ab \forall a, b \in N$, then identity element for $*$ is:

यदि N प्राकृत संख्याओं का एक समुच्चय है, 1* एक संक्रिया इस प्रकार है कि $a * b = a + b + ab \forall a, b \in N$, तब $*$ के लिए तत्समक अवयव है:

(a) 1

(b) 0

(c) 2

(d) Non-existent/अस्तित्वहीन

Q. If $(G,*)$ is a group and $x * y = x + 2y - 3 \forall x, y \in G$, then inverse of x in the group is.

यदि $(G,*)$ एक समूह है और $x * y = x + 2y - 3 \forall x, y \in G$, तो x का समूह में व्युत्क्रम है-

(a) $\frac{2x+9}{4}$

(b) $\frac{9-2x}{4}$

(c) $\frac{x-3}{4}$

(d) $\frac{x+2}{4}$

Q. Let 'a' be an element of a group and $O(a) = 30$, $O(a^{18})$ is equal to -

मान लीजिए 'a' एक समूह का अवयव है और $O(a) = 30$, $O(a^{18})$ बराबर है-

(a) 2

(b) 5

(c) 6

(d) 10

Q. Let $*$ be the binary operation defined on the set of positive rational number Q^+ by the rule $a * b = \frac{ab}{3}$ $\forall a, b \in Q^+$ Then, inverse of $4 * 6$ is?

मान लीजिए $*$ धनात्मक परिमेय संख्या Q^+ के समुच्चय पर नियम $a * b = \frac{ab}{3} \forall a, b \in Q^+$ द्वारा परिभाषित द्विआधारी संक्रिया है, तो $4 * 6$ का व्युत्क्रम क्या है?

(a) $\frac{3}{8}$

(b) $\frac{2}{3}$

(c) $\frac{9}{8}$

(d) $\frac{3}{2}$

Q. In the multiplication group $\{1, -1, i, -i\}$ where $i^2 = -1$; the inverse of i is?

गुणन समूह $\{1, -1, i, -i\}$ में जहाँ $i^2 = -1$; का व्युत्क्रम क्या है?

(a) 1

(b) -1

(c) i

(d) $-i$

Q. Which of the following structures is not group?

निम्नलिखित में से कौन सी संरचना समूह नहीं है?

(a) $\{(1, \omega, \omega^2), 0\}$

(b) $\{\{1, -1\}, 0\}$

(c) $\{\{1, -1\}, +\}$

(d) $(\{1, -1, i, -i\}, \cdot)$

Q. Which of the following is group?

निम्नलिखित में से कौन सा समूह है?

(a) $(\mathbb{Z}, *)$ where $a * b = ab; \forall a, b \in \mathbb{Z}$

(b) $(\mathbb{Z}, *)$ where $a * b = a - b; \forall a, b \in \mathbb{Z}$

(c) $(\mathbb{R}, *)$ where $a * b = ab; \forall a, b \in \mathbb{R}^+$

(d) $(\mathbb{Z}, *)$ where $a * b = a + b; \forall a, b \in \mathbb{Z}^-$

Q. The identity element in the set Z of integers under the binary operation $*$ Defined by $a * b = a + b + 1, \forall a, b \in Z$ is?

$a * b = a + b + 1, \forall a, b \in Z$ द्वारा परिभाषित बाइनरी ऑपरेशन $*$ के अंतर्गत पूर्णांकों के सेट Z में पहचान तत्व क्या है?

(a) 1

(b) -1

(c) 0

(b) 2

Q. In the group $Z_5 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}\}$ of residue classes Modulo 5 , with respect to the operation of addition of residue classes modulo 5 , the inverse of $\bar{2}$ is?

अवशेष वर्गों 5 के समूह $Z_5 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}\}$ में,
अवशेष वर्गों 5 के जोड़ के संचालन के संबंध में, $\bar{2}$
का व्युत्क्रम क्या है?

(a) $\bar{1}$

(b) $\bar{2}$

(c) $\bar{3}$

(d) $\bar{4}$

Q. The set $\{\overline{5}, \overline{15}, \overline{25}, \overline{35}\}$ under the binary operation of multiplication of residue classes modulo 40 is?

अवशेष वर्गों के गुणन के बाइनरी ऑपरेशन के तहत $\{\overline{5}, \overline{15}, \overline{25}, \overline{35}\}$ मॉड्यूलो 40 है?

[UP PG]

- (a) not a group
- (b) a group with identity $\overline{5}$
- (c) A group with identity $\overline{25}$
- (d) A group with identity $\overline{35}$

$$a * b = a + b - ab$$

$$2 * x * 7 = 13$$

$$(2 + x - 2x)$$

$$(2 - x) * 7 = 13$$

$$(2 - x + 7) - (2 - x)7 = 13$$

$$9 - x - 14 + 7x = 13$$

$$6x = 18$$

$$x = 3$$

Q. If S denotes the set of all rational numbers except 1 and $*$ is defined on S as: $a * b = a + b - ab$ Then the solution of the equation $2 * x * 7 = 13$ is?

यदि S , 1 को छोड़कर सभी परिमेय संख्याओं के समुच्चय को दर्शाता है और S पर $*$ को इस प्रकार परिभाषित किया गया है: $a * b = a + b - ab$ तो समीकरण $2 * x * 7 = 13$ का हल क्या है?

(a) 1

(b) 2

(c) 3

(d) 7

[UP SGT]

Order of group $\rightarrow$ Let $(G, *)$ be a group

Number of element in G is called the order of G .

$O(G) \Rightarrow n$ (finite) [Finite Group]

$O(G) \Rightarrow \infty$ (Infinite) [Infinite group]

$$(1) O(Z_n) = n$$

$$(2) O(U_n) = \phi n$$

$$(3) O(Z_n^*) = \phi n$$

$$(4) O(K_4) = 4 \quad (a, e, b, ab)$$

$$(5) O(Q_4) = 8 \quad (\pm 1, \pm i, \pm j, \pm k)$$

$$(6) O(D_n) = 2n$$

$$(7) O(S_n) = n!$$

$$(8) O(Z^*) = \infty$$

$$(9) O(Z) = \infty$$

$$(10) O(Q^*) = \infty$$

$$(11) O(R^*) = \infty$$

finite group

* → Not includes zero

Operation → Mult. - plication

$$U(8) = \{1, 3, 5, 7\}$$

$$8 \rightarrow 2^3$$

$$F.N. \Rightarrow \frac{1}{2} \times \frac{1}{2} \rightarrow 4$$

$$\{\pm 1, \pm 2, \pm 3, \dots, \infty\}$$

Infinite gp

$$\begin{aligned} 200 &\Rightarrow 2 \times 100 \\ &\Rightarrow 2 \times 2^2 \times 5^2 \\ 200 &\Rightarrow 2^3 \times 5^2 \end{aligned}$$

$$F.N. \Rightarrow 200 \times \frac{1}{2} \times \frac{1}{2} \Rightarrow 80$$

$$(12) \ O(C^*) = \infty$$

$$(13) \ O(R) = \infty$$

$$(14) \ O(Q) = \infty$$

$$(15) \ O(C) = \infty$$

Infinite gre

$\underbrace{a \times a \times a \times \dots}_{n \text{ times}} = e$

$a^n = e$

$\underbrace{a + a + a + \dots}_{n \text{ times}} = e$

Order of Element

→ Let G be a group and $a \in G$. Then, the order of an

element $a \in G$ is the least positive integer

n such that $a^n = e$.

सबसे छोटा धनात्मक पूर्णांक

If such n does not exist then, order of ' a ' is infinite.

$$Q + e = a$$

$$\# a^n = e$$

$$\underbrace{a + a + a + \dots}_{n \text{ times}} = e$$

$$\underbrace{a \times a \times a + \dots}_{n \text{ times}} = e$$

$$= Z' = \{0, \pm 1, \pm 2, \pm 3, \dots, \infty\}$$

$$(Z', +) \Rightarrow \boxed{\text{Identity} = 0} \quad (e = 0)$$

$$O(0) = 1$$

$$O(1) = 1 + 1 + 1 + 1 + \dots \neq 0 \infty$$

$$O(2) = \infty$$

Q. $G = Z = \{0, \pm 1, \pm 2, \pm 3, \pm \dots \infty\}$ find
order of each element of $(Z, +)$?

(a) 1, 2

Identity = $e = 0$

(b) 1, 7

$O(6) = 1$

(c) 7, ∞

$O(5) = 5 + 5 + 5 + \dots = \infty$

(d) 1, ∞

$O(1, 2, 3, 4, \dots, 69, 99, 700) = \infty$

SIMILARLY $\rightarrow (\mathbf{Q}, +), (\mathbf{R}, +), (\mathbf{C}, +)$ has
elements of order $1 \& \infty$.

$0 \rightarrow \text{Rational}$
 $\downarrow$
 $\frac{p}{q}$ जहाँ $q \neq 0$

$\frac{0}{1}, \frac{0}{970}, \frac{0}{103}$

$(\mathbf{Q}, +), (\mathbf{R}, +), (\mathbf{C}, +)$

$(\mathbf{Q}, +), (\mathbf{R}, +), (\mathbf{C}, +)$
 $\downarrow \quad \downarrow \quad \downarrow$
Rational Real Complex
No

Q. Find ~~number~~ of element finite in Q^* ?

order each

(a) 1

(b) 2

(c) ∞

(d) All of the above

$$O(1) = 1$$

$$O(-1) = 2$$

$Q^* \Rightarrow$ Rational No. except Zero

$\ast \rightarrow$ operation
 $\downarrow$
 multiplication

$$Q^* = Q - \{0\}$$

$$a^n = e$$

$$(1)^1 = e = 1$$

$$(-1)^2 = e = 1$$

least positive int

$$Q^* = \{1, 2, 3, 4, \frac{1}{4}, \frac{2}{5}, \frac{7}{9}, \dots\}$$

$$\text{Identity} = 1$$

$$(-1)^4 = e = 1$$

$$(-1)^6 = e = 1$$

$$(-1)^{1002} = e = 1$$

$$\left(\frac{2}{5}\right)^? = e = 1$$

∞

$(Q, \cdot)$

$$\# Q = \{1, 2, 3, 4, \frac{1}{2}, \frac{5}{3}, \frac{7}{2}, \dots\}$$

$$\textcircled{1} \text{ Identity} = 1$$

$$\textcircled{2} \text{ Inverse} \Rightarrow 4 \times 4^{-1} = 1$$

$$\cancel{4} \times \frac{1}{\cancel{4}} = 1$$

SIMILARLY $\rightarrow$ $\mathbb{R}^*$ has elements of order
1, 2, & ∞ .

$$\mathbb{R}^* = \mathbb{R} - \{0\}$$

Q. Find order of each element of $(Z^*, \cdot)$?

~~(a) 1~~

~~(b) 2~~

(c) both a & b

(d) None of these

$\downarrow$
 $\{1, -1\}$

Order of elements $\rightarrow$

$$(1)^1 = e = 1$$

$$(-1)^2 = e = 1$$

1, 2

$$(Z^*, \cdot) \Rightarrow \text{Identity} = 1$$

$$Z^* = \{1, -1\}$$

Q. The order of the element i of the multiplicative group $G = \{1, -1, i, -i\}$.

(a) 4

(b) 3

(c) 2

(d) 1

$$i^4 = e$$

$$i^? = e = 1$$

$$i \times i = i^2 = -1$$
$$i \times i = i^2 = -1$$

[UP PGT, DSSSB TGT]

$$i^2 \times i^2 = -1 \times (-1)$$
$$i^4 = 1$$

Q. $G = (\{0, 1, 2, 3, 4, 5\}, \underbrace{+}_6)$ is a group.

Order of 5 is:

↓ addition

$G = (\{0, 1, 2, 3, 4, 5\}, +_6)$ एक समूह है।

अवयव 5 की कोटि (order) है:

$$5^n = 5 + 5 + 5 + 5 + 5 + 5$$

(a) 1

(b) 3

(c) 5

(d) 6

$$G = (\{0, 1, 2, 3, 4, 5\}, +_6)$$

$$e = 0$$

$$0(0) \Rightarrow 1$$

$$0(1) \Rightarrow 6$$

$$0(2) \Rightarrow 3$$

$$0(3) \Rightarrow 2$$

$$0(4) \Rightarrow 4$$

$$1+1+1+1$$

$$2+2+2$$

$$3+2$$

$$\frac{30}{6} = 5$$

$$n = 6$$

Theorem $\Rightarrow$ No of elements of ~~order~~
order
'n' in C^* is $\phi(n)$.

Q. How many elements of order' 3 in C^* ?

(a) 1

(b) 5

(c) 2

(d) 4

$$\underline{\underline{C^* = C - \{0\}}}$$

3
↓
અભાજ્ય સંખ્યા
↓
2

Q. How many elements of order 8 in C^* ?

(a) 2

(b) 4

(c) 3

(d) 1

$$\begin{aligned} \phi(n) &\Rightarrow \text{Euler } \phi^n \\ &\Downarrow \\ 8 &\Rightarrow 2^3 \\ \text{E.N.} &\rightarrow 8 \times \frac{1}{2} = \textcircled{4} \end{aligned}$$

Q. How many elements of order 49 in C^* ?

- (a) 40
(b) 6
(c) 42
(d) 7

$49 \rightarrow 7$
 $\xrightarrow{\text{E.N.}} \frac{49}{7} \times \frac{6}{7}$
 $\Rightarrow 42$

Q. The Set $(1, -1, i, -i)$ is a multiplicative finite group of order?

[UP PGT]

$$(1, -1, i, -i)$$

(a) 3

(b) 4

(c) 1

(d) 2

Q. The order of identity element in a additive group of integers is?

[DSSSB TGT]

$$a^n = e$$

$$0^1 = e = 0$$

- (a) zero
- (b) infinite
- (c) one
- (d) Two

$$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots, \infty\}$$

Additive

$$\Rightarrow e = 0$$

Q. The order of 3 in Z_6 ?

(a) 1

(b) 2

(c) 3

(d) 6

$$Z_6 = \{0, 1, 2, 3, 4, 5\}$$

$$\boxed{0=0}$$

$$Z_n = \{0, 1, 2, 3, \dots, \underline{n-1}\}$$

$$3^2 = \frac{3+3}{6} = \underline{0=0}$$