DSSSB TGT PART(A+B)

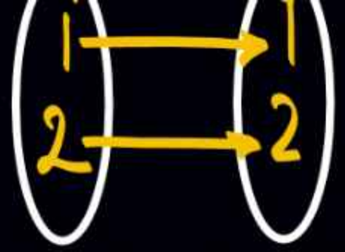
MATHS

GROUP THEORY

Part -6

LIVE 23/07/2024 04:30PM

$S_2 =$



$I \Leftarrow \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \Rightarrow (12)$

$S_2 = \{f_1, f_2\}$

$\Rightarrow \{I, (12)\}$

$I \quad (12)$

* $a \cdot a^{-1} = e \quad I^{-1} = I$

$I \cdot I = I$

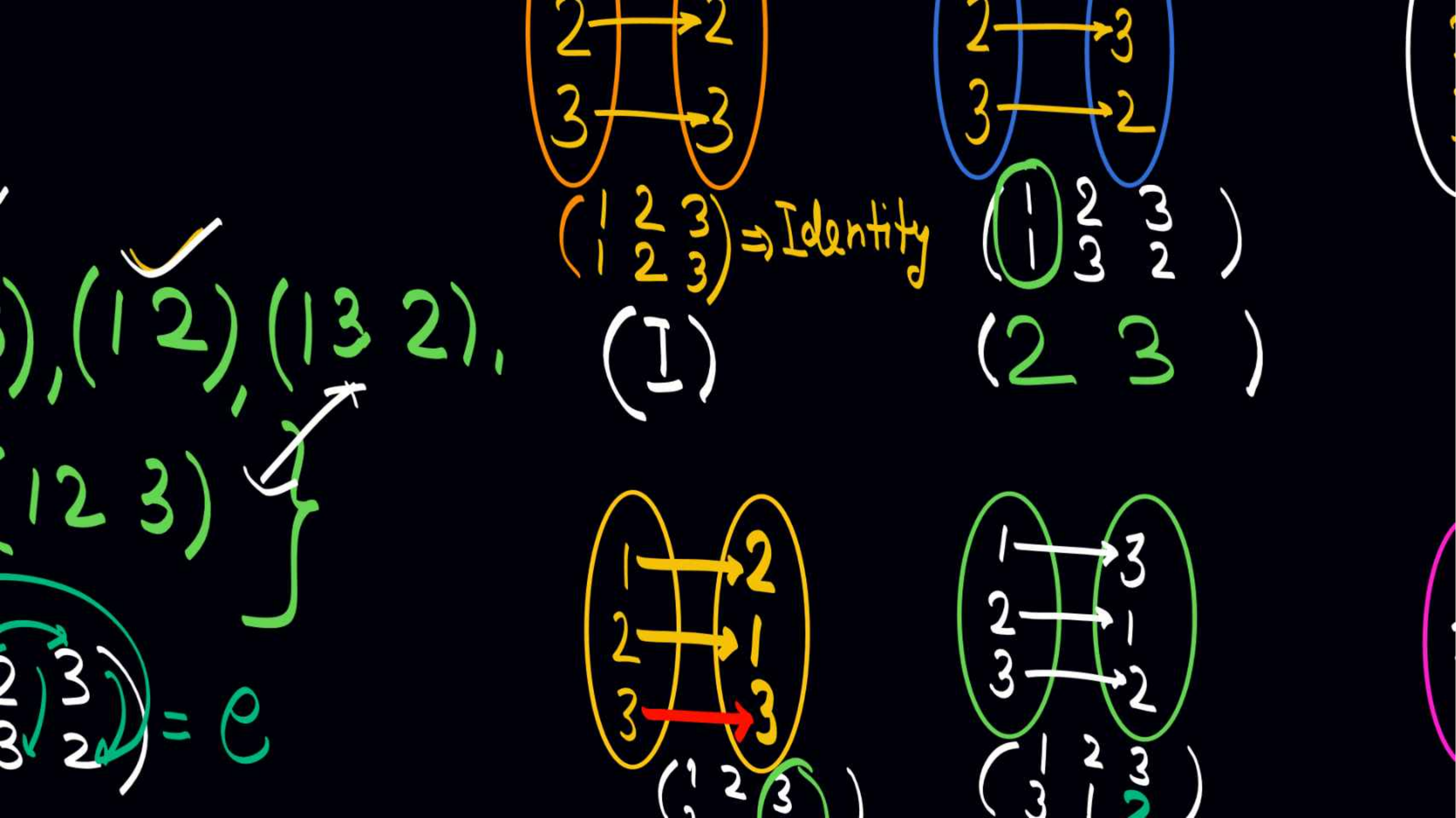
* $a \cdot a^{-1} = e$

$(12) \cdot (12) \Rightarrow$

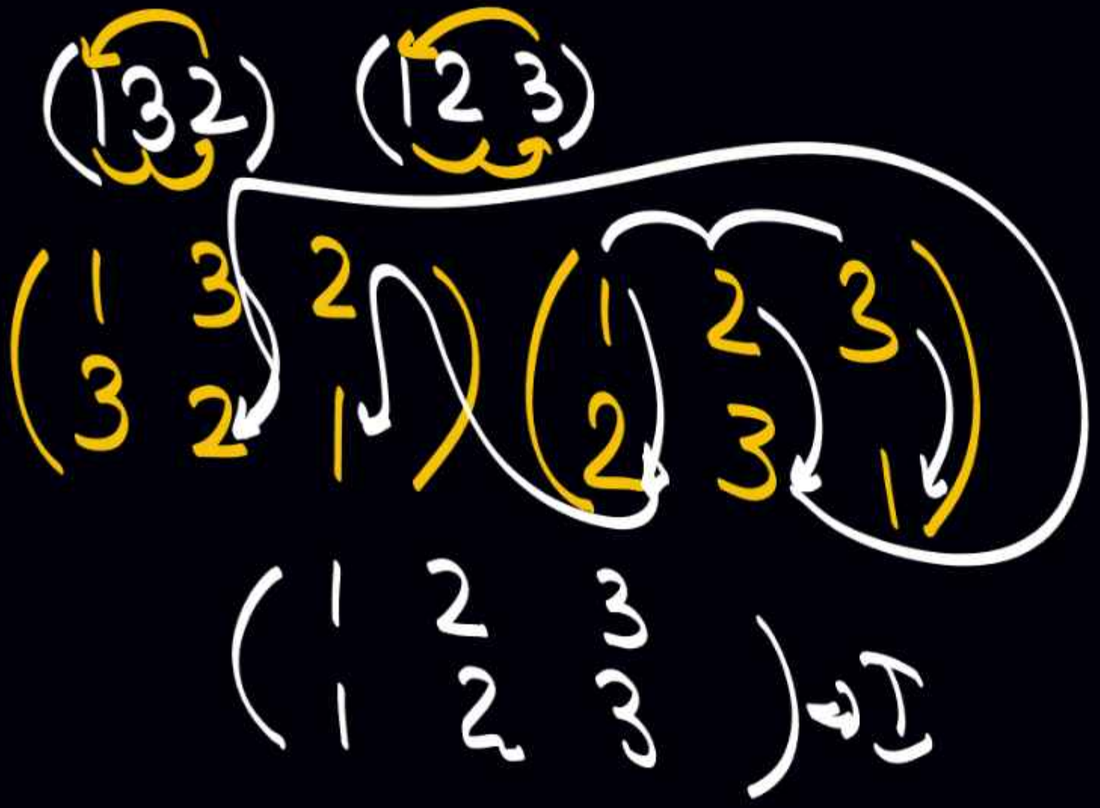
$\Rightarrow \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = I$

$S_1 = \{f\} \Rightarrow \{1\} \Rightarrow$ Identity

(1)


(1 2 3) ⇒ Identity
(I)
(1 2 3)
(2 3)

$$(1\,3\,2)^{-1} = (1\,2\,3)$$



Permutation grp

⟸⟹ Symmetric (S_n) is a group under Composition? — (Yes)

Q. Find Identity and inverse of each element of S_2?

$S_2 = \{I, (12)\}$

Identity → I

$I^{-1} = I$

$(12)^{-1} = (12)$

Find the identity & inverse of each element of S_3?

Identity $= I$

$(23)^{-1} = (23)$

$(13)^{-1} = (13)$

$(12)^{-1} = (12)$

$(132)^{-1} = (123)$

$(123)^{-1} = (132)$

$S_3 = \{I, (23), (13), (12), (132), (123)\}$

Q. $g = (1 \quad 2 \quad 3 \quad 5) \in S_5$ Then $g^{-1} = ?$

(a) $(5 \quad 3 \quad 2 \quad 1)$

(b) $(1 \quad 2 \quad 3 \quad 5)$

(c) $(2 \quad 1 \quad 5 \quad 3)$

(d) None of these

$r \leq n$

- Note → If $f = (a_1, a_2, a_3, \ldots, a_r) \in S_n r \leq n$ then $f^{-1}(a_r, a_{r-1}, a_{r-2} \ldots a_2, a_1)$

Q $f = (1\ \ 2\ \ 3) \in S_3$

$g = (1\ \ 3\ \ 2) \in S_3$

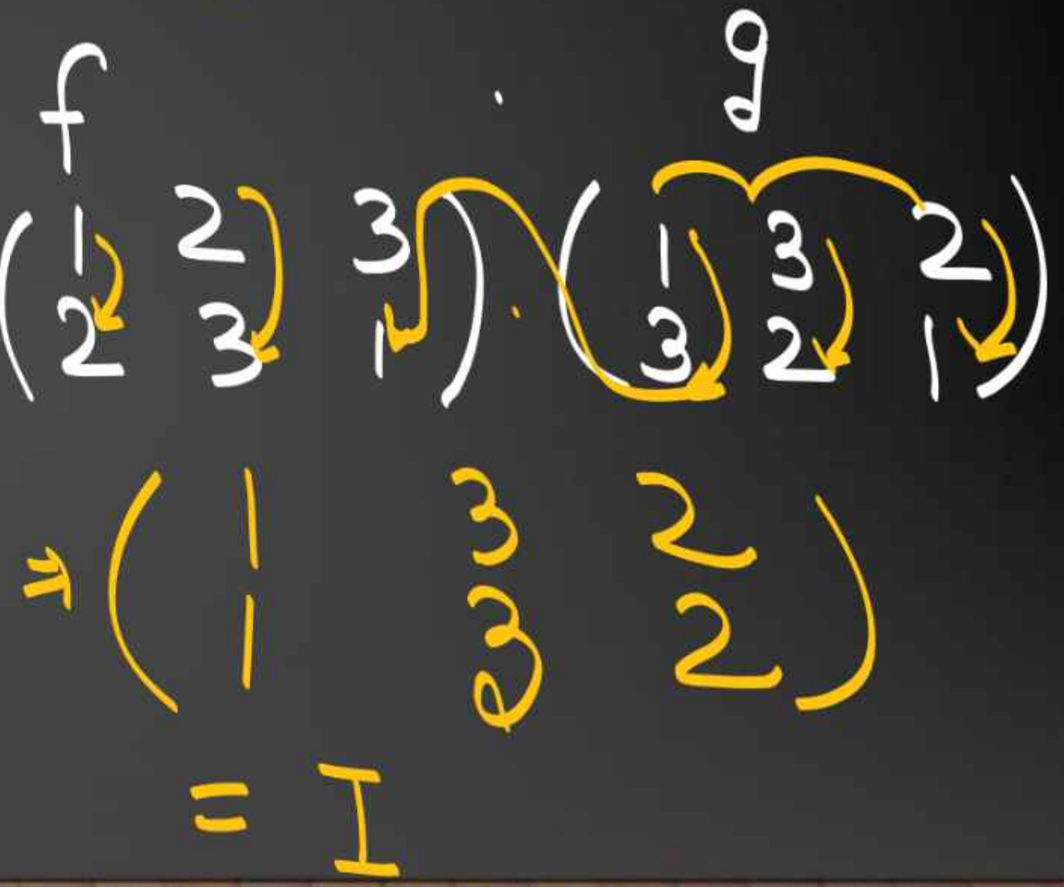
Then $f \cdot g = ?$

(a) I

(b) (123)

(c) (132)

(d) None of these



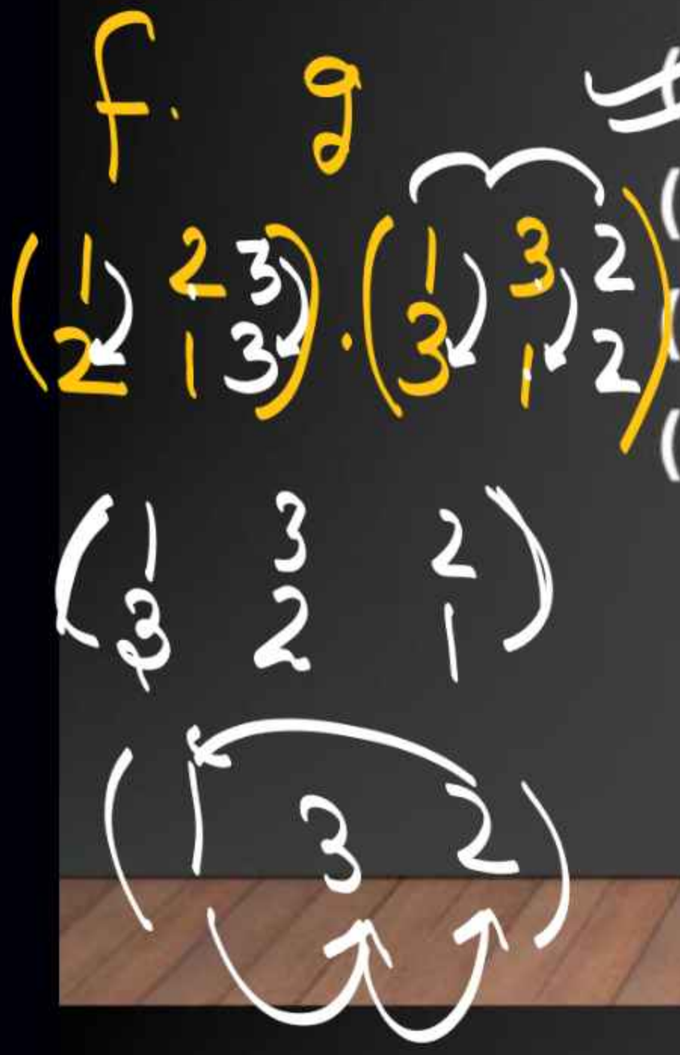
Q. $f = (1 \quad 2) \in S_3$
$g = (1 \quad 3) \in S_3$ **Then find f.g and** $g \cdot f$

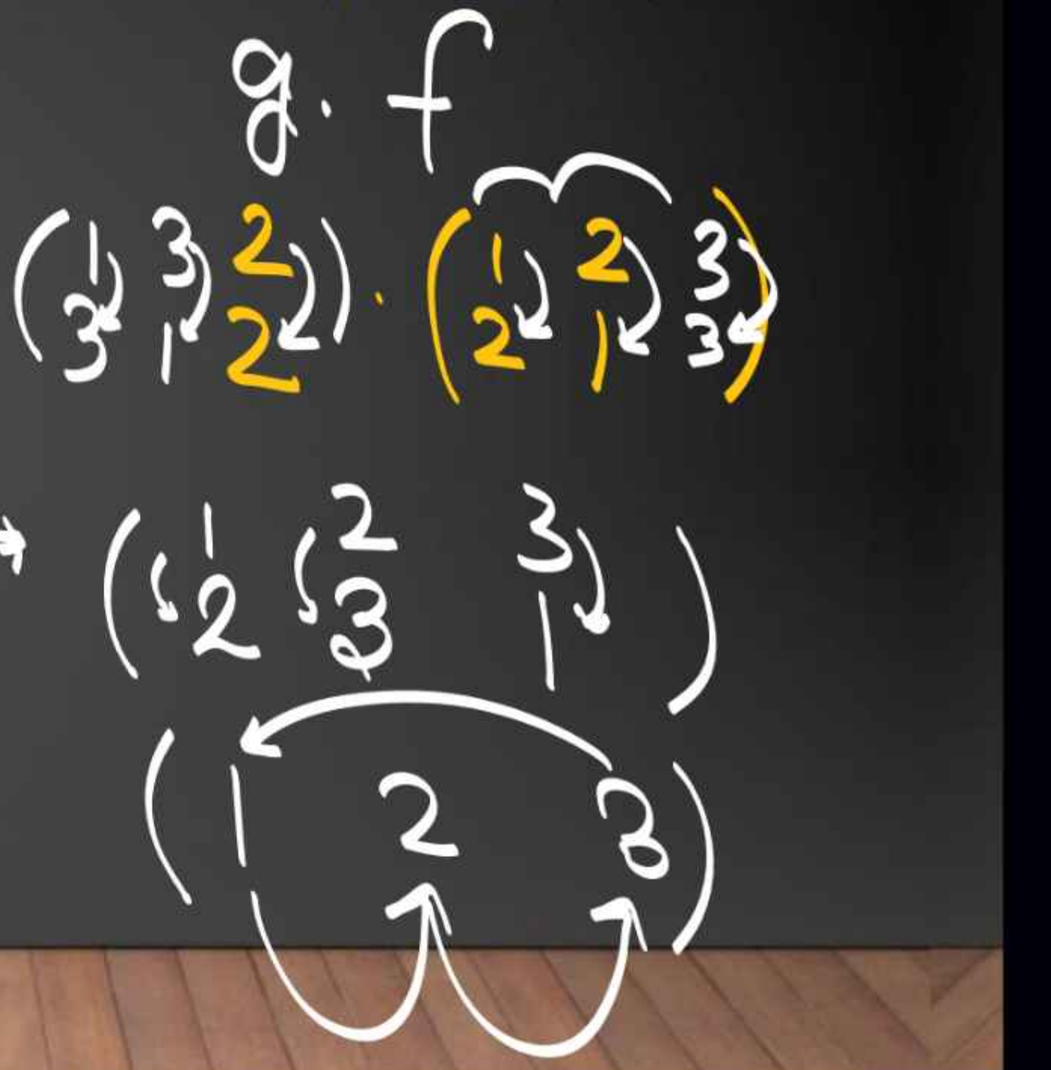
(a) (132),(123)

(b) (123), (321)

(c) (213), (123)

(d) None of these





Q. In the group $G = \{\{2,4,6,8\},10\}$ the identity element is = ?

(a) 2

(b) 4

(c) 6

(d) 8

$2 \times 6 \Rightarrow 12 \rightarrow 2$

$4 \times 6 \Rightarrow 24 \rightarrow 4$

$6 \times 6 \Rightarrow 36 \Rightarrow 6$

$8 \times 6 \rightarrow 48 \Rightarrow 8$

$I = 6$

$G = \{\{2,4,6,8\}, \cdot 10\}$

$G = \{\{2,4,6,8\}, \times 10\}$

Multiplication modulo 10

$2 \times 2 \Rightarrow 4 \pmod{10} = 4$

$2 \times 4 \Rightarrow 8 \pmod{10} = 8$

$2 \times 6 \Rightarrow 12 \pmod{10} \Rightarrow 2$

$2 \times 8 \Rightarrow 16 \pmod{10} \Rightarrow 6$

$\forall a \in G, \exists e \in G$

$St: \Rightarrow a \times e = e \times a = a$

Q. Let G be a group with binary operation defined by $a * b = a + b + 1 \; \forall ab \in G$. Inverse of element '$c$' of G is -

(a) $2 + c$

(b) $2 - c$

(c) $-2 + c$

(d) $-2 - c$

Identity $\Rightarrow$ $a * e = a$

$a + e + 1 = a$

$e = -1$

Inverse $\Rightarrow$ $a * a^{-1} = e = -1$

$a + a^{-1} + 1 = -1$

$a + a^{-1} = -2$

$a^{-1} = -a - 2$