



DSSSB TGT

PART (A+B)



MATHS

GROUP THEORY

Part -3



19/07/2024 04:30PM



1. Closure Property

$$\forall a \in G \ \& \ \forall b \in G \\ a * b \in G$$

$$N = \{1, 2, 3, 4, \dots, \infty\}$$

$(N, +)$

$3 \in N$
 $8 \in N$

$$3 + 8 = 11 \in N \checkmark$$
$$3 \times 8 = 24 \in N \checkmark$$

2. Associative Property

$$\forall a, b, c \in G$$

$$a * (b * c) = (a * b) * c$$

$$a + (b + c) = (a + b) + c$$

$$a + (b + c) = (a + b) + c$$

$$1 + 5 = 3 + 3$$
$$6 = 6$$

$$a = 1$$
$$b = 2$$
$$c = 3$$

3. Existence of Identity

addition $\rightarrow e=0$

identity $\uparrow$

$$a * e = e * a = \underline{a}$$

$\uparrow$

$$\forall a \in G \exists e \in G$$

(identity)

such that $a + e = e + a = a$

$$N = \{1, 2, 3, 4, 5, \dots\}$$

$\downarrow$
 $2 + = 2$

4. Existence of Inverse

multiplication $e=1$

$\times \times \times$
 $N = \{1, 2, 3, 4, \dots, \infty\}$

$$4 + (-4) = (-4) + 4 = 0$$

$$4 \times \frac{1}{4} = \frac{1}{4} \times 4 = 1$$

for all $a \in G \exists a^{-1} \in G$

(identity)

such that $a * a^{-1} = a^{-1} * a = e$

Yes

$\mathbb{Z}$

3. Identity →

$$4 + (0) = 4$$

4. Inverse →

$$a + a^{-1} = 0$$

$$-13 + 13 = 0$$

$(\mathbb{Z}', +)$ is a group?

$\mathbb{Z}$

is a group under addition?

$$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \dots, \pm \infty\}$$

1. Closure → $a=5, b=-7$

$$a+b \Rightarrow 5-7 \Rightarrow -2 \in \mathbb{Z}$$

2. Associative →

$$a+(b+c) = (a+b)+c$$

$$1+(2+3) = (1+2)+3$$

$$6 = 6$$

2. Associative →

$$\begin{aligned} a &= 5 \\ b &= 4 \\ c &= 7 \end{aligned}$$

$$a - (b - c) = (a - b) - c$$

$$5 - (4 - 7) = (5 - 4) - 7$$

$$5 - (-3) = 1 - 7$$

$$8 \neq -6$$

$(\mathbb{Z}, -)$ is a group?

No

$\mathbb{Z}$ is a group under subtraction?

$$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots, \pm \infty\}$$

1. Clauser →

$$\begin{array}{cc} 7 & 4 \end{array}$$

$$7 - 4 = 3 \in \mathbb{Z} \checkmark$$

2. Associative →

$$a = \frac{1}{2}, b = \frac{2}{5}, c = 5$$

$$a \times (b \times c) = (a \times b) \times c$$

~~$$\frac{1}{2} \times \left(\frac{2}{5} \times 5 \right) = \left(\frac{1}{2} \times \frac{2}{5} \right) \times 5$$~~

$$1 = 1 \checkmark$$

3. Identity → $a \times e = e \times a = a$

4. Inverse → $a \times a^{-1} = a^{-1} \times a = e$

~~$$4 \times \frac{1}{4} = \frac{1}{4} \times 4 = 1$$~~

$(\mathbb{Q}, \times)$ is a group?

$\mathbb{Q}$ is a group under multiplication?

Rational ← $\mathbb{Q} = \{ -\infty \dots \dots \dots +\infty \}$
परिमेय

1. Counter → $-\frac{1}{2}, -\frac{7}{5}$
↳ $-\frac{1}{2} \times -\frac{7}{5} = \frac{7}{10}$

Note:-



Groupoid

जब केवल Closure Property ही satisfy करें।

Semi-group

(Closure + Associative)

Monoid

(Closure + Associative + identity)

GROUP



Closure
Associative
Identity
Inverse

