



DSSSB TGT

PART (A+B)



MATHS

GROUP THEORY

Part -4



20/07/2024 04:30PM



(ii) Associative $\checkmark$ $U(n); n > 1$ is group under

$$a \times (b \times c) = (a \times b) \times c$$

multiplication modulo?

yes

$U(n); n > 1$ गुणन मॉड्यूलो के अंतर्गत समूह है?

(iii) Identity =

$$e = 1 \checkmark$$

$$a \times e = a$$

$$e = 1$$

$$1 \times 1 = 1$$

$$5 \times 1 = 5$$

$$7 \times 1 = 7$$

$$11 \times 1 = 11$$

(iv) inverse $\rightarrow$

$$a \times a^{-1} = e$$

$$a \times a^{-1} = 1$$

$$1 \times 1 = 1$$

$$5 \times 5 = 25(1)$$

$$7 \times 7 = 49(1)$$

$$11 \times 11 = 121(1)$$

$$1^{-1} = 1$$

$$5^{-1} = 5$$

$$7^{-1} = 7$$

$$11^{-1} = 11$$

(i) closure $\rightarrow$

$$U(12) = \{1, 5, 7, 11\}$$

$$\forall a \in U(12) \\ b \in U(12)$$

$$a \cdot b \in U(12)$$

$$7 \cdot 11 = \frac{77}{12} \Rightarrow 5 \in U(12)$$

* Identity पूरे group की unique होती है।

* Inverse प्रत्येक Element का अपना अलग-अलग होता है

$$U(10) = \{1, 3, 7, 9\}$$

(i) Clause $\rightarrow 7 \times 9 \Rightarrow 63 \Rightarrow 3$

(ii) Associative $\rightarrow a \times (b \times c) = (a \times b) \times c$

(iii) Identity $\rightarrow a \times e = e \times a = a$
 $e = 1$

(iv) Inverse $\rightarrow a \times a^{-1} = e$

$1 \times 1 = 1$	$= 1$	$1^{-1} = 1$
$3 \times 7 \Rightarrow 21 \Rightarrow \frac{21}{10}$	$= 1$	$3^{-1} = 7$
$7 \times 3 \Rightarrow 21 \Rightarrow \frac{21}{10}$	$= 1$	$7^{-1} = 3$
$9 \times 9 \Rightarrow 81 \Rightarrow \frac{81}{10}$	$\Rightarrow 1$	$9^{-1} = 9$

Inverse $\rightarrow$

$$U(15) = \{1, 2, 4, 7, 8, 11, 13, 14\}$$

$$11 \times 11 = \frac{121}{15} = 1 \Rightarrow 11^{-1} = 11$$

$$14 \times 14 = \frac{196}{15} \Rightarrow 1 \Rightarrow 14^{-1} = 14$$

$$1 \times 1 \Rightarrow 1 \Rightarrow 1 \Rightarrow 1^{-1} = 1$$

$$2 \times 8 \Rightarrow 16 \Rightarrow \frac{16}{15} \Rightarrow 1 \Rightarrow 2^{-1} = 8$$

$$4 \times 4 = 16 \Rightarrow \frac{16}{15} \Rightarrow 1 \Rightarrow 4^{-1} = 4$$

$$7 \times 13 = 91 \Rightarrow \frac{91}{15} \Rightarrow 1 \Rightarrow 7^{-1} = 13$$

$$8^{-1} = 2$$

$$13^{-1} = 7$$

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Q. $G = u(12)$, Find identity and inverse of each element of $U(12)$

प्रश्न. $G=u(12)$, $U(12)$ के प्रत्येक तत्व की पहचान और व्युत्क्रम ज्ञात करें

$$e = 1$$

$$\text{Inverse} \rightarrow \begin{array}{ll} 1^{-1} = 1 & 7^{-1} = 7 \\ 5^{-1} = 5 & 11^{-1} = 11 \end{array}$$

Quaternion (Q_4) $\rightarrow O(Q_4) = 8$

$$Q_4 = \{\pm 1, \pm i, \pm j, \pm k\}, \quad i^2 = j^2 = k^2 = -1, \quad i \cdot j = k \neq j \cdot k = i \neq k \cdot i = j$$

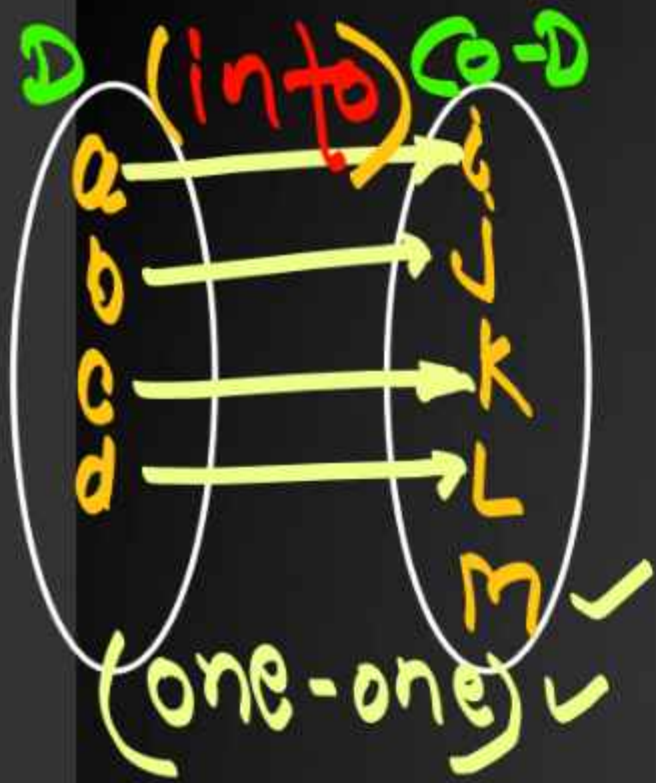
$-j \cdot i = k \neq -k \cdot j = i \neq -i \cdot k = j$

Order (Q_4) = 8

(i) Clause $\rightarrow$

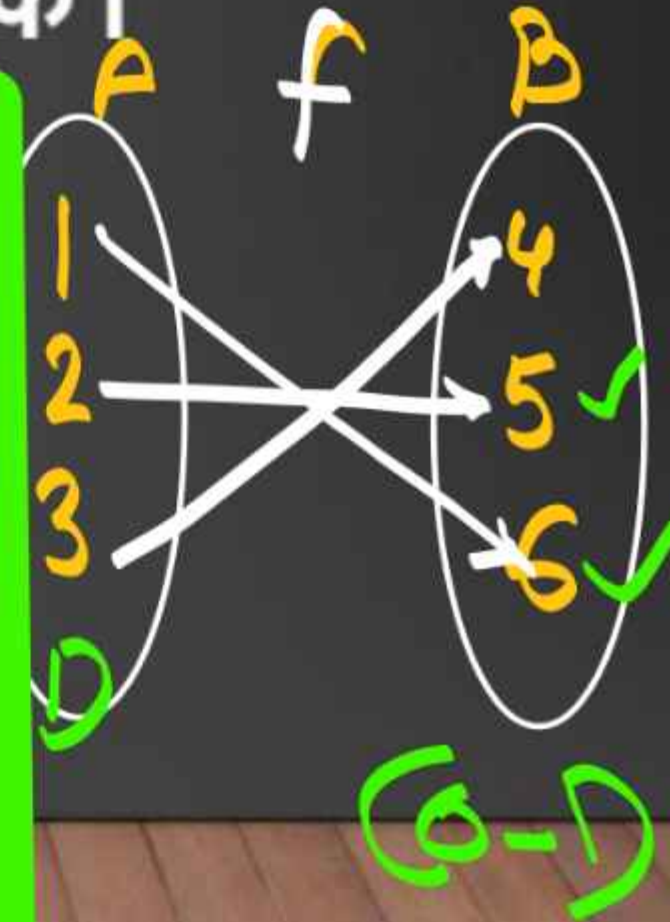
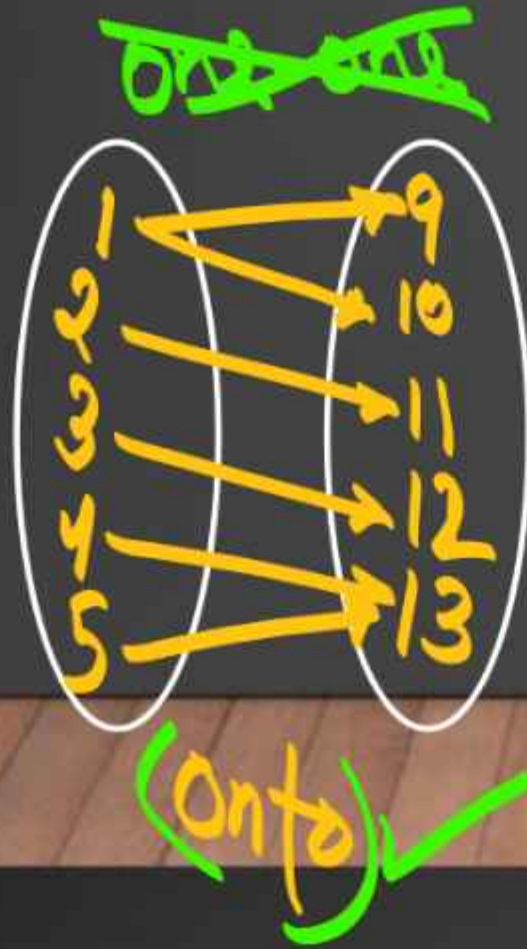
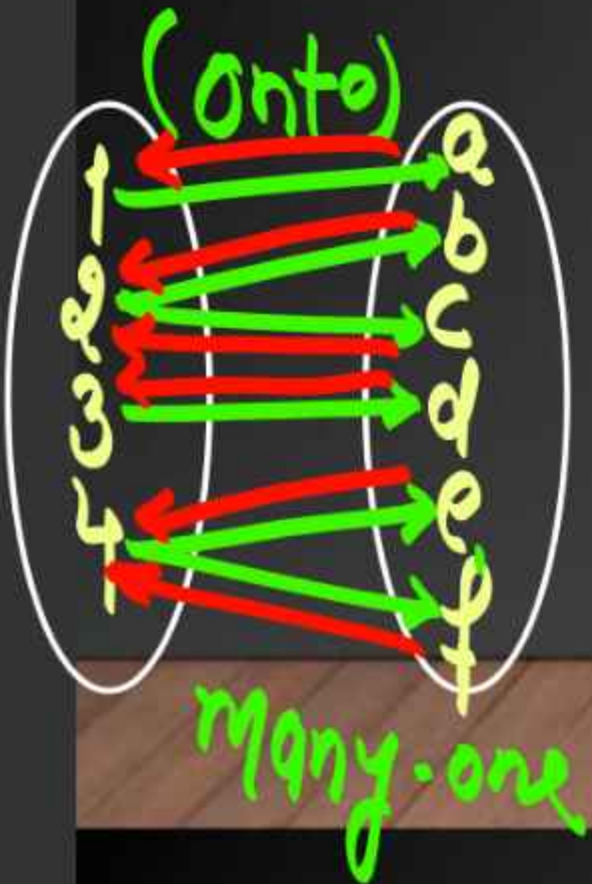
Klein's - (K_4)

$$\text{Klein's-four group} \Rightarrow (K_4) = \{a, b, e, ab \mid a^2=e, b^2=e, \forall ab=ba\}$$
$$\Rightarrow \{a, b \mid a^2=b^2=(ab)^2=1\}$$



S_n → Set of all one-one & onto Mappings
From set containing n -elements to itself.

S_n - सभी एक-एक और एक-पर मैपिंग का सेट,
 n -तत्वों वाले सेट से स्वयं तक।



➤ Q_4 (Quaternion group): is a group under multiplication modulo? **YES ✓**

Q_4 (क्वाटर्नियन समूह): गुणन मॉड्यूलो के अंतर्गत एक समूह है?

➤ (K_4) क्लेन – चार समूह $\Rightarrow \{a, b, e, ab / a^2 = e, b^2 = e, ab = ba\}$ गुणन मॉड्यूलो के अंतर्गत समूह है?

(K_4) klein-four group $\Rightarrow \{a, b, e, ab / a^2 = e, b^2 = e, ab = ba\}$ is group under multiplication modulo?

हिंदू

➤ D_n is a group under Multiplication modulo? YES

D_n गुणन मॉड्यूलो के अंतर्गत एक समूह है?

