DSSSB TGT PART(A+B)

MATHS

GROUP THEORY

(SET THEORY) Part -2

LIVE 16/07/2024 04:30PM

Set Theory

4) Dihedral $\Rightarrow D_n$

Imp $O(D_n) \Rightarrow 2 \times n$

$D_n = \{x^i y^j / x^2 = e, y^n = e, xy^{-1} = yx$

$D_n = \{x^i y^j / x^2 = e, y^n = e, xy^{-1} = yx$

$x \rightarrow$ Reflexion
$y \rightarrow$ Rotation
$xy \rightarrow$ Generator
$x^2 = e, y^n = e \Rightarrow$ Relator
$xy^{-1} = yx =$ Relation

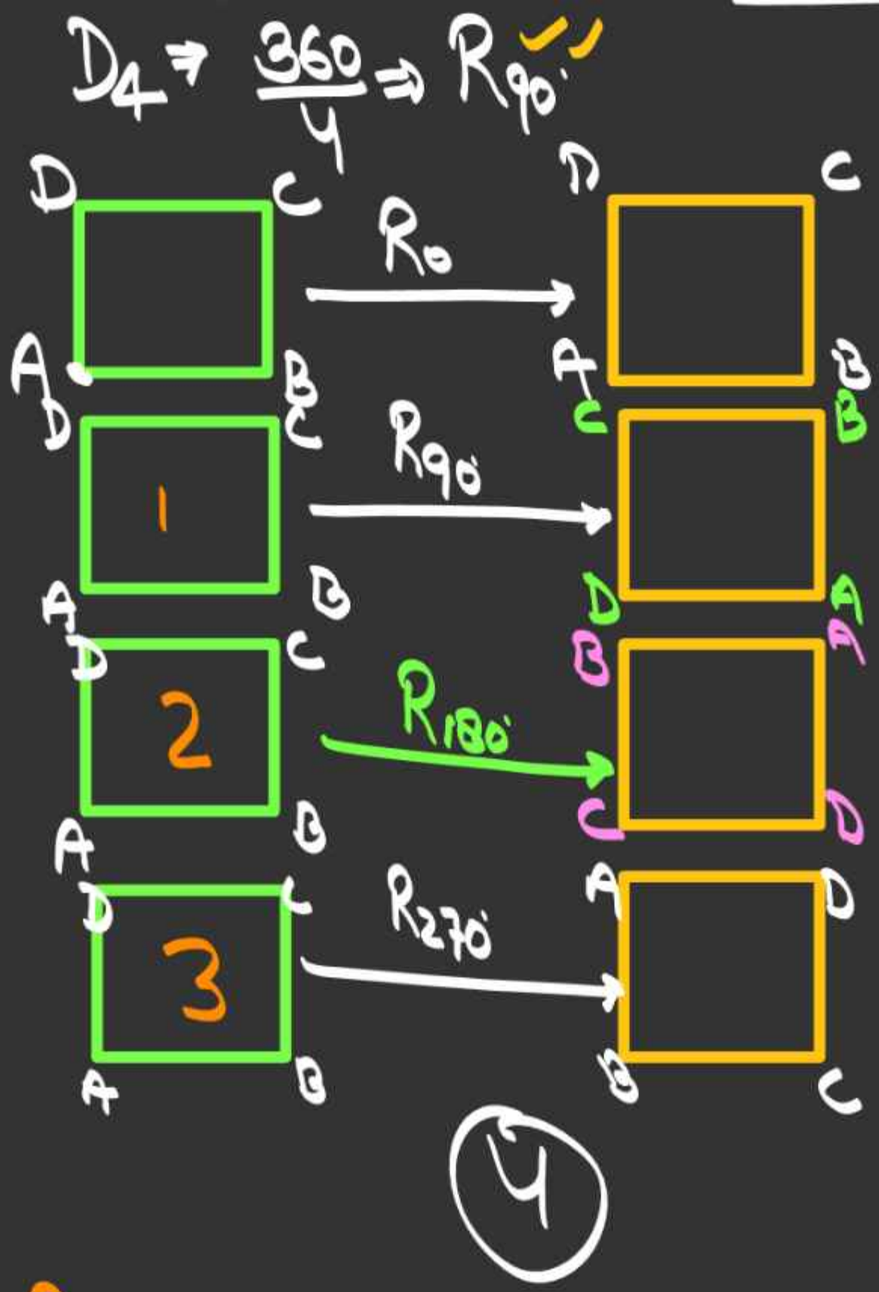
$\theta = \frac{360}{n}$

$y = y$

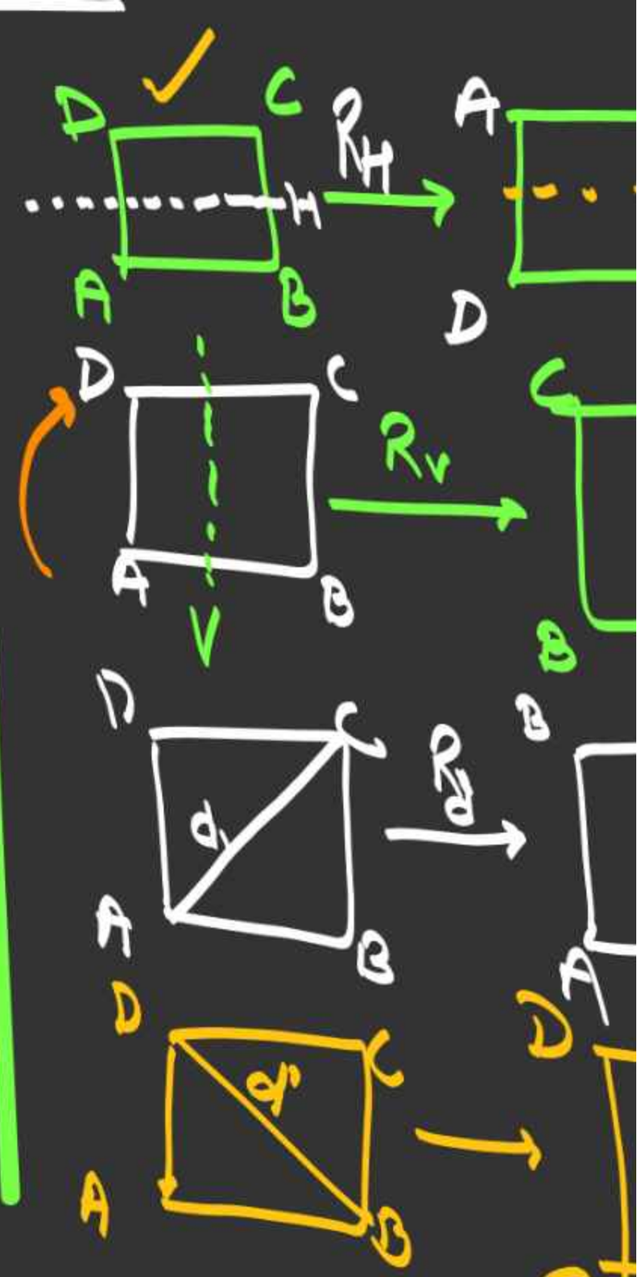
$x = x, y = y$

- Reflexion
+ Rotation

$D_4 \rightarrow \frac{360}{4} \Rightarrow R_{90}$



Rotation



$D_4 = \{ R_0, R_{90}, R_{180}, R_{270}, H, V, D, D' \}$

Reflexion

Q. Order of Ashok Chakar?

1 24

2 35

3 45

4 48

$n = 24$

$O(D_n) = 2 \times n$

$\Rightarrow 2 \times 24$

$\Rightarrow 48$

Q. $x^2 = 1; y^3 = 1 \& (xy)^2 = 1$ **then** $O(G)$ **is**

1 4

2 5

3 6

4 8

$x^2=1, y^3=1 \quad (xy)^2=1$

$x^2=1 \quad y^3=1$

$e=1 \quad n=3$

$D_n = \quad x^2=e \quad y^2=e \quad xy^{-1}=yx$

$x\cdot y\cdot x\cdot y = 1$

$x\cdot y\cdot x = \frac{1}{y} = y^{-1}$

$x\cdot y = y^{-1}\cdot x^{-1}$

$xy = xy^{-1}$

$x^2 = 1$ (Given)

$x\cdot x = 1$

$x = x^{-1}$

Set Theory

Q. How Many Rotation & Order of this Structure?

1. Rotation-3 & Order-10
2. Rotation-6 & Order-10
3. Rotation-6 & Order-12
4. Rotation-2 & Order-12

Rotation = Reflexion

Rotation + Reflexion = Order

$D_n = \frac{360}{6} \Rightarrow 60$

General Linear → $GL_n(\mathbb{F})$ or $Gl_n(\mathbb{F})$ or GX

$\mathbb{F}$ → field

$n \times n$ matrix

Case → I → (Infinite) →

$O(GL_n(\mathbb{F})) = \infty$

Case-II → Finite

$\mathbb{F} = \mathbb{F}_q$

$O(GL_n(\mathbb{F}_q)) \Rightarrow (q^n - q^{n-1})(q^n - q^{n-2})$ $(q^n -$

OR $\Rightarrow O(GL_n(\mathbb{F}_q)) \Rightarrow (q^n - q^0)(q^n - q^1)(q^n$

Q. Find order of $GL_2(F_2)$ $\equiv GL_n(\mathbb{F}_q)$

$n=2 \quad q=2$

1.) 6

2.) 8

3.) 2^2

4.) 4^5

$$O(GL_2(F_2)) \Rightarrow (q^2-q^0)(q^2-q^1)$$

$$(4-1)(4-2)$$

$$3 \times 2 = 6$$

$$O(GL_n(\mathbb{F}_q)) \Rightarrow (q^n-q^0)(q^n-q^1)\cdots(q^n-q^{n-1})$$

Q. Find order of $GL_3(F_2)$

1.) 168

2.) 384

3.) 2^3

4.) 3^2

$GL_n(\mathbb{F}_q)$

$n=3,\ q=2$

$O(GL_n(\mathbb{F}_q)) \Rightarrow (q^n-q^0)(q^n-q^1)\cdots(q^n-q^{n-1})$

$O(GL_3(\mathbb{F}_2)) = (q^3-q^0)(q^3-q^1)(q^3-q^2)$

$(8-1)(8-2)(8-4)$

$7 \times 6 \times 4$

$\rightarrow 168$

$O(SL_3(\mathbb{F}_2)) \rightarrow \frac{168}{q-1}$

$\Rightarrow 168$

Special Linear $\Rightarrow SL_n(\mathbb{F}_q)$

$$o\left(SL_n(\mathbb{F}_q)\right) \Rightarrow \frac{(q^n-q^0)(q^n-q^1)(q^n-q^2)\cdots\cdots(q^n-q^{n-1})}{(q-1)}$$

Q. Find order of $SL_2(F_2)$

$SL_n(F_q)$

$n=2 \quad q=2$

1.) 8

2.) 6

3.) 2^2

4.) 3

$$O(SL_2(F_2)) \Rightarrow \frac{(2^2-2^0)(2^2-2^1)}{2-1}$$

$$\Rightarrow \frac{(4-1)(4-2)}{1} \Rightarrow 6$$

$$O(GL_2(F_2)) \Rightarrow 6$$