



DSSSB TGT

PART (A+B)



MATHS

GROUP THEORY



15/07/2024 04:30PM



Eular Thm →

$$\Downarrow \begin{array}{r} 404 \\ (903) \\ \hline 100 \end{array}$$

$$\begin{array}{r} (404) \\ 3 \\ \hline 100 \end{array}$$

$$\Rightarrow \begin{array}{r} 34 \\ 100 \end{array} \Rightarrow \frac{81}{100} = (R=81)$$

$$60 \rightarrow 2 \times 3 \times 2 \times 5$$

$$\Rightarrow 2^2 \times 3 \times 5 \Rightarrow \text{E.N.} \Rightarrow \frac{2}{60} \times \frac{1}{2} \times \frac{2}{3} \times \frac{4}{5} = \frac{16}{15}$$

Fermat Thm →

$$\frac{a^{p-1}}{p}, (R=1)$$

↓ prime No.
only for prime No.

$$100 \notin \text{E.N.} \rightarrow 100 = 2 \times 5^2$$

$$\Rightarrow \frac{100}{2} \times \frac{1}{5} \times \frac{4}{5} = \frac{4}{1}$$

$$\Rightarrow 40$$

Remainder Thm →

- Basic
- Binomial
- ✓ → Fermat
- ✓ → Euler
- ✓ → Wilson
- ✓ → Chinese
- Special Rule

$$\frac{4^n}{6} \Rightarrow R=4$$

$$\frac{10^n}{6} \Rightarrow R=4$$

Abstract Algebra | Modern Algebra

TOPIC-Group Theory

1. Set Theory — $\mathbb{Z}_n$, $U(n)$, $\mathbb{Z}_n^*$, Gaussian integers, $GL_n(\mathbb{F})$ or General Linear, $SL_n(\mathbb{F})$ or Special Linear, Klein's 4 Group, Q_4 or Quaternion, D_n (Dihedral), S_n (symmetric)

● Group Theory

1. Order of Group

2. Subgroups

3. Abelian Group

4. Lagrange's theorem

5. Normal subgroups and quotient groups

- ✓ 6. Cyclic groups
- ✓ 7. Centre of Group
- 8. Centralizer of a Group
- 9. Cosets
- ✓ 10. Permutation groups
- ✓ 11. Sylow theorem
- 12. Symmetry groups
- 13. Alternating Group
- 14. Simple Group
- 15. Cayley's Theorem
- 16. Isomorphism theorems
- 17. Zassenhaus Lemma

- 18. Jordan Theorem**
- 19. Solvable group**
- 20. Nilpotent Group**
- 21. Homomorphism**
- 22. Automorphism**
- 23. Epimorphism**
- 24. Monomorphism, endomorphism**

Set Theory

✓ 1) Z_n - Set of Residue Class modulo n

↳ set of remainders when divided by n

$$Z_n = \{0, 1, 2, 3, \dots, (n-1)\}$$

$$Z_n = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \dots, \overline{(n-1)}\}$$

$$Z_n = \{(0), (1), (2), \dots, (n-1)\}$$

$$\text{Order } Z_n = O(Z_n) = n$$

$$n=7$$

$$\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7},$$
$$\frac{7}{7}, \frac{8}{7}, \frac{9}{7}, \frac{10}{7}, \dots$$

$$Z_7 = \{0, 1, 2, 3, 4, 5, 6\}$$

$$11 \rightarrow 11' \\ \rightarrow \cancel{11} \times \frac{10}{\cancel{11}}$$

$$2) U(n) = \{x \in \mathbb{N} / 1 \leq \check{x} \leq \check{n}; \text{ and } \gcd(x, n) = 1\}; n > 1$$

$\square$ $\underline{1 < x < n}$

$\begin{matrix} (x, n) \\ \text{HCF} \end{matrix} \Rightarrow 1 \} : n > 1$

$$U(11) \rightarrow \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$U(\check{6}) = 1, 2, 3, 4, 5, 6$$

$$(1, 6) \rightarrow \text{HCF} = 1 \checkmark$$

$$(2, 6) \rightarrow \times$$

$$(3, 6) \rightarrow \times$$

$$(4, 6) \rightarrow \times$$

$$(5, 6) \rightarrow \text{HCF} = 1$$

$$(6, 6) \rightarrow \times$$

$$U(6) = \{1, 5\}$$

$$U(100) = \{40\}$$

Totient fun ϕ
= Euler No

Set Theory

3) $\checkmark$ $Z_n^* = \{x \in Z_n / x^{-1} \text{ exist w.r.t. multiplicative modulo } n\}$

$$\Rightarrow \underline{x \in \mathbb{N} \mid 1 \leq x \leq n}$$

$$(x, n) = 1 \mid n > 1$$

$U(n)$

$$Z_6^* = \{0, 1, 2, 3, 4, 5\}$$

$$Z_6^* = \{1, 5\}$$

$$O(Z_n^*) = O(U(n))$$

$$Z_n^* = U(n) ; n > 1$$

$$\downarrow x \cdot x^{-1} = 1$$

$$0x \equiv 1 \pmod{6} \times$$

$$1x \equiv 1 \pmod{6} \checkmark$$

$$\times 2x \equiv 1 \pmod{6}$$

$$\times 3x \equiv 1 \pmod{6}$$

$$\times 4x \equiv 1 \pmod{6}$$

$$\checkmark 5x \equiv 1 \pmod{6}$$