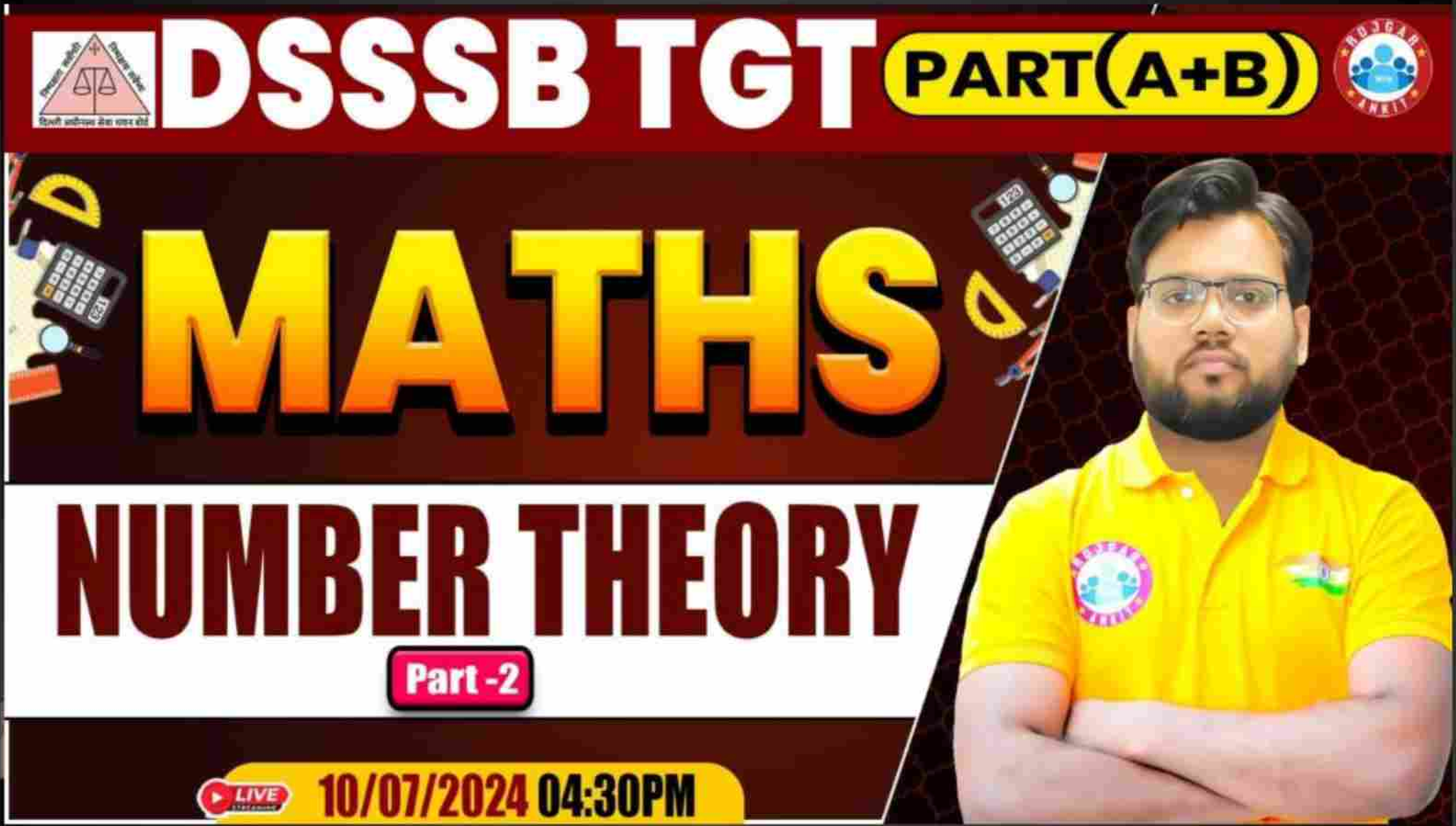
DSSSB TGT
PART(A+B)
MATHS
NUMBER THEORY
Part -2
LIVE
10/07/2024 04:30PM

FERMAT'S THEOREM

Fermat's Theorem

If p is a prime number and a denotes an integer such that $(a, p) = 1$, then

$$a^{p-1} \equiv 1(\bmod p)$$

$$\frac{a^{p-1}}{p} \equiv 1$$

Theorem: If p is a prime number and a_i denotes an integer, then

$$(a_1 + a_2 + a_3 + a_4 + \cdots a_n)^p = a_1^p + a_2^p + a_3^p + a_4^p + \cdots + a_n^p + \mathrm{M}(p)$$

where $\mathrm{M}(p)$ denotes multiple of p.

1. If $(n, 17) = 1$ where k is any positive integer then $n^{16k} - 1$ is divisible by

a. 18

b. 17

c. 15

d. 20

2. If $13^{10} (\mathrm{mod} 11)$ then remainder is?

a. 2^{10}

b. 1

c. 0

d. 2

co prime $\left[\frac{13^{10}}{11} \right.$, $R = ?$

$11 \rightarrow$ prime

$\frac{13^{10}}{11}$ $\frac{a^{p-1}}{p}$, $R = 1$

3. If $(102)^{502}(\text{mod}11)$ then remainder is?

a. 3

b. 1

c. 3^{10}

d. 9

$a^{n^m} \Rightarrow a^{n \times n \times n \times n \cdots m \text{ बार}}$

$(a^n)^m \Rightarrow a^{n \times m}$

HCF=1 $\dfrac{102^{502}}{11}$ 11 → prime

$\dfrac{(102^{10})^{50} \times 102^2}{11}$

$\dfrac{102^{10}}{11}$: R=1

$\Rightarrow \dfrac{\overset{3}{102} \times \overset{3}{102}}{11} \Rightarrow 9$

$$\frac{20^{300}}{19} \Rightarrow \frac{(19+1)^{300}}{19}$$

$$\Rightarrow \frac{1^{300}}{19} \Rightarrow R=1$$

$$\Rightarrow \frac{103^3}{11} \Rightarrow \frac{4^3}{11} \Rightarrow \frac{64}{11} \Rightarrow \overset{R}{9}$$

द्विपद प्रमेय -

$$\frac{(x+a)^n}{x}, \quad \frac{a^n}{x} \checkmark$$

$$\frac{123^{28}}{29},\ R=1$$

$$\Rightarrow \text{Co-prime}\left[\frac{123^{281}}{29}\right],\ R=?$$

29 → prime

$29\times4 \Rightarrow 116$

$$\Rightarrow \frac{\left(123^{28}\right)^{10}\times 123}{29} \Rightarrow \frac{123}{29} \Rightarrow R=7$$

1^{10}

$$\frac{2^{306}}{31}, R = ?$$

If $2^{306} \pmod{31}$ then Remainder $= ?$

$$\frac{(2^{30})^{10} \times 2^6}{31}$$

$$\Rightarrow \frac{2^6}{31} \Rightarrow \frac{64}{31}$$

$$\Rightarrow R = 2$$

$(12\times 10)+1 \quad (12\times 20+1) \quad (12\times 30+1)$

$$\# \quad \frac{24^{121}\times 23^{241}\times 25^{361}}{13 - \text{prime}}, \quad R=?$$

$$\frac{a^{p-1}}{p}, \quad R=1$$

$$\frac{\overset{-2}{24^1}\times \overset{-3}{23^1}\times \overset{-1}{25^1}}{13} \Rightarrow \frac{-6}{13}$$

$$\Rightarrow \quad R = 13-6 = 7$$

TOPIC - NUMBER THEORY

1. Divisibility (Theorems On Divisibility, G.C.D. & L.C.M.)
2. Possible Solutions or Integer Solutions
3. Relatively prime Number
4. Congruences
5. Residue System (mod m)
6. Chinese Remainder Theorem
7. Euler's ϕ Function
8. Fermat's Theorems
9. Euler's Theorems

Wilson Theorem

10. Wilson's Theorems
11. Some Functions of Number Theory
12. Positive Divisors
13. Number of Divisors
14. Sum of divisors

Congruences

$$\Rightarrow \quad M \overline{) N} (q$$
$$\underline{\quad r \quad}$$

$$N = Mq + r$$

(modulus)

Division Algorithm

$$a \overline{) b} (q$$
$$\underline{\quad r \quad}$$

$$\boxed{0 \leq r < a}$$

$$\boxed{b = aq + r}$$

Congruences →

a & b be two integers

m is (+ve) Integer.

If m divides (a-b)
then m is a divisor of (a-b)

$$\left(\frac{a-b}{m}\right)$$

$$15 \equiv 3 \pmod 4$$

$$\frac{15-3}{4} \checkmark$$

$$a \equiv b \pmod m$$

$a - b = mk$ जहाँ $k \in Z$

CONGRUENCES

THEOREM ON CONGRUENCE

$\boxed{\textbf{Theorem 1}}$ **An integer is congruent to its remainder.**

$m\overline{)a}(q$

$\underline{r}$

$a = mq + r$

$a - r = mq \rightarrow q \in Z'$

$\frac{a-r}{m} = q$

$a \equiv r \pmod{m}$

$\frac{20}{7} \Rightarrow R \Rightarrow 2$ $\frac{3}{7} = R = 3$

Theorem 2. If $a \equiv b(\bmod m)$, then a and b leave the same remainder when divided by m and conversely.

$\frac{a-b}{m}, R =$

$\frac{a}{m} - \frac{b}{m} = R =$

$$25 \equiv 3 \pmod{4}$$

$$\frac{25-3}{4} \Rightarrow \frac{22}{4} \Rightarrow 2$$

$$\frac{\overset{1}{25}}{4} - \frac{\overset{3}{3}}{4} \Rightarrow \frac{-2}{4} \Rightarrow 4-2 \Rightarrow 2$$

CONGRUENCES

Q. Find 'a' such that $a \equiv 7(\text{mod}5)$.

5

10

8

-8

$\frac{-8-7}{5}$

$\Rightarrow -3$

$\boxed{r=0}$

Q. Find the least positive integer (mod11) to which 282 is congruent.

A. 25

B. 7

C. 11

D. None of these

$$\frac{282 - x}{11}$$

$$\Rightarrow 282 - 7 \Rightarrow \frac{275}{11}$$