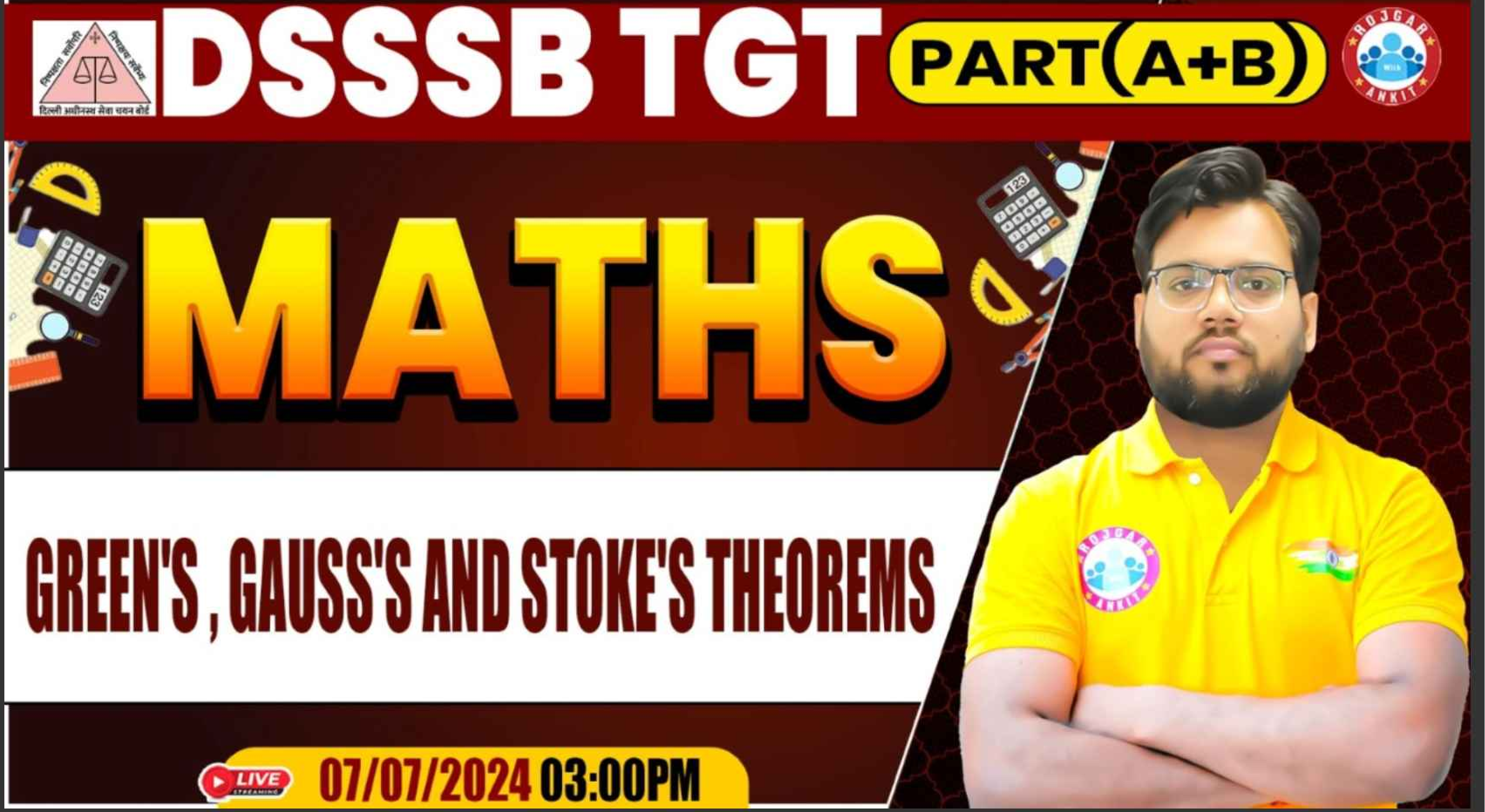

DSSSB TGT
PART(A+B)
ROJGAR
ANKIT
MATHS
GREEN'S , GAUSS'S AND STOKE'S THEOREMS
LIVE
07/07/2024 03:00PM

1. **Evaluate** $\int_C \mathbf{F}\cdot d\mathbf{r}$ **where** $\mathbf{F} = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$ **and** C **is the arc of the curve** $\mathbf{r} = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ **from** $t = 0$ **to** $t = 2\pi$.

$r = xi + yj + zk$

$r = \cos t\, i + \sin t\, j + t k$

$\frac{dr}{dt} = -\sin t\, i + \cos t\, j + k$

$F\cdot\frac{dr}{dt} = -z\sin t + x\cos t + y$

$F\cdot\frac{dr}{dt} = -t\sin t + \cos^2 t + \sin t$

$\int F\left(\frac{dr}{dt}\right) dt$ 1

$\int\left(-t\sin t + \cos^2 t + \sin t\right) dt$ 2

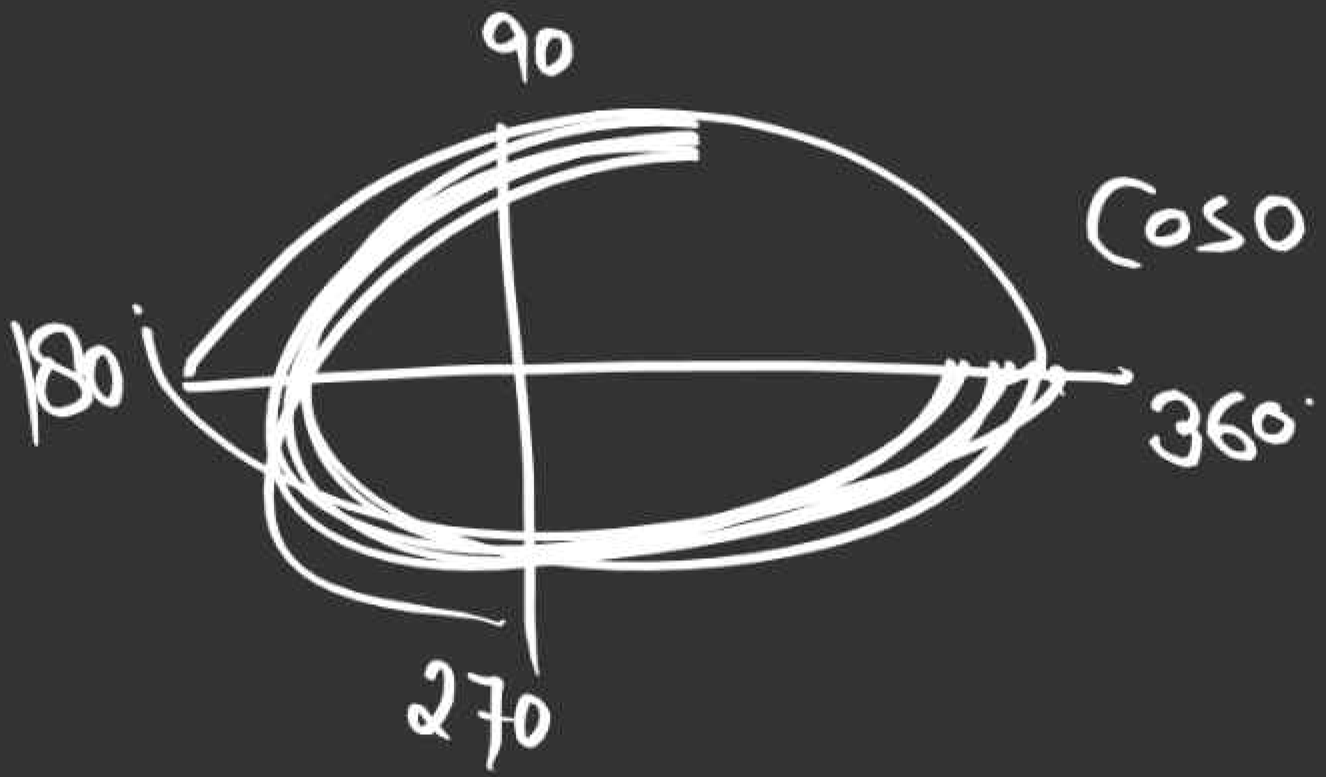
$\int\left(-t\sin t + \frac{1}{2} + \frac{\cos 2t}{2} + \sin t\right) dt$ 3

$\left[-t(-\cos t) - \int -1(-\cos t)\,dt + \frac{1}{2}t + \frac{\sin 2t}{4} - \cos t\right]_0^{2\pi}$ 4

$\Rightarrow \left[t\cos t - \sin t + \frac{1}{2}t + \frac{\sin 2t}{4} - \cos t\right]_0^{2\pi}$ 5

$\Rightarrow 2\pi\times 1 - 0 + \frac{2\pi}{2} + 0 - \cancel{1} - 0 + 0 - 0 - 0 + \cancel{1}]$ 6

$\Rightarrow 3\pi$ 7



$\cos 0 \Rightarrow 1$

$$\boxed{\int uv\,dx \Rightarrow u\int v\,dx - \int\left[\frac{d}{dx}(u)\int v\,dx\right]dx}$$

$$\sin^2\theta \Rightarrow \frac{1-\cos 2\theta}{2}$$

$$\begin{cases} \cos 2\theta = 1-2\sin^2\theta \\ \cos 2\theta = 2\cos^2\theta - 1 \end{cases}$$

$$\Downarrow$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$=3\pi.$

ANSWER $=3\pi$

2. Find the total work done in moving a particle in a force field given by $\mathbf{F} = 3\,xy\,\mathbf{i} - 5\,z\,\mathbf{j} + 10x\,\mathbf{k}$.
along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$.

$\int F \cdot dr = \int F \cdot \left(\frac{dr}{dt}\right) \cdot dt$

$r = xi + yj + zK$

$\frac{dr}{dt} = 2ti + 4tj + 3t^2 K$

$F \cdot \frac{dr}{dt} = 3xy\,2t - 5z\,4t + 10x\,3t^2$

$\int [3xy\,2t - 5z\,4t + 10x\,3t^2]\,dt$

$\Rightarrow \int [3(t^2+1) \cdot 2t^2 \cdot 2t - 5t^3 \cdot 4t + 10(t^2+1) \cdot 3t^2]\,dt$

$\int [12t^5 + 12t^3 - \underbrace{20t^4 + 30t^4}_{+10t^4} + 30t^2]\,dt$

$\left[2t^6 + 3t^4 + 2t^5 + 10t^3\right]_1^2$

$\Rightarrow 128 + 48 + 64 + 80 - 2 - 3 - 2 - 10$

2^2

128
48
64
80
320

$\Rightarrow 320 - 17 \Rightarrow 303$ Ans

$= 303.$

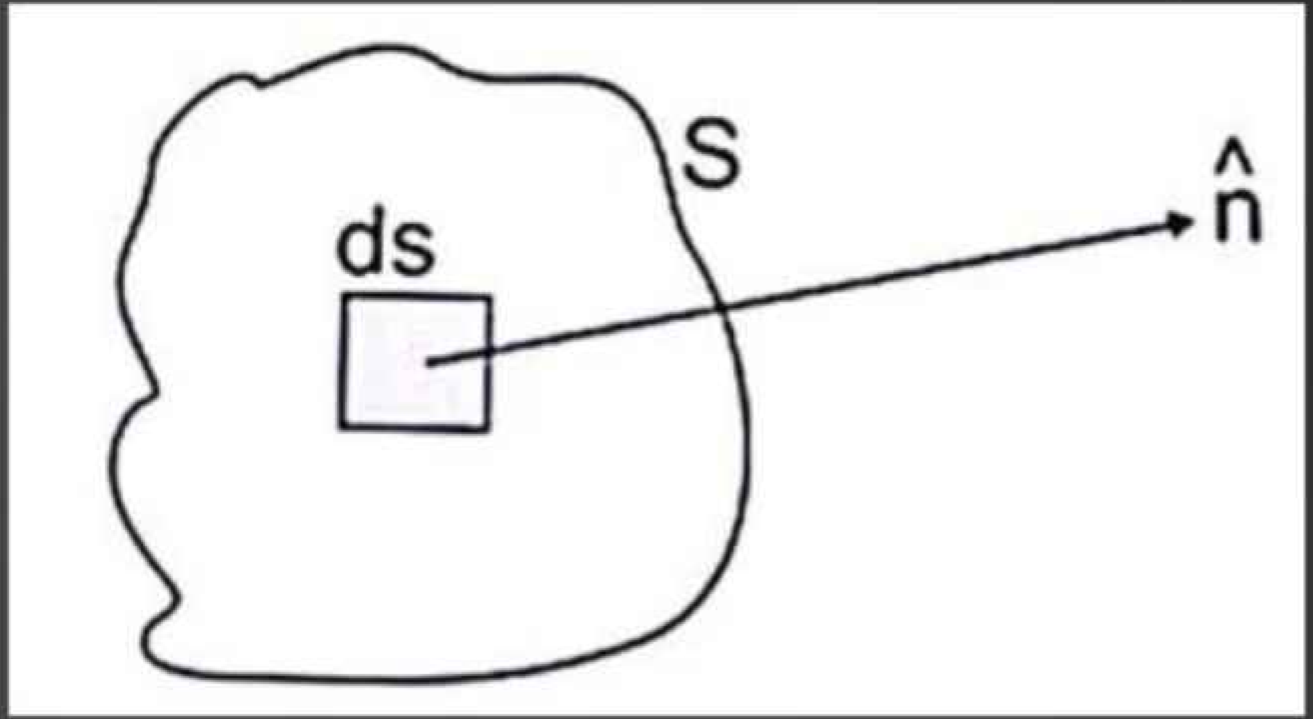
$\boxed{303}$

SURFACE INTEGRAL

$\vec{F}$

$\hat{n} = \frac{\text{grad } f}{|\text{grad } f|}$

3. Any integral which is to be evaluated over a surface is called a surface integral.



$$\boxed{\iint_s F.ds} = \iint_s (F.\hat{n})ds$$

NOTE :

i j

1. If R be projection of S on xy-plane then

$$ds = \frac{dxdy}{|\hat{n}\cdot\hat{k}|}$$

j k

2. If R be projection of S on yz-plane then

$$ds = \frac{dydz}{|\dot{n}\cdot i|}$$

i k

3. If R be projection of S on xz-plane then

$$ds = \frac{dxdz}{|\dot{n}.j|}$$

- **Flux across a surface (किसी पृष्ठ के आरपार फ्लक्स या अभिवाह)→**

किसी सतत् सदिश बिन्दु फलन F का किसी बन्द पृष्ठ S पर अभिलम्बीय पृष्ठ समाकलन $\int_S F.dS$ पृष्ठ S के आर-पार F का फ्लक्स या अभिवाह कहलाता है।

$\int F.ds$

$\nabla \cdot F$

THE DIVERGENCE THEOREM OF GAUSS.

The surface integral of the normal component of a vector F taken over a closed surface is equal to the integral of the divergence of F taken over the volume enclosed by the surface.

$$\iiint_V \overset{\text{div } F}{\nabla \cdot \mathbb{F}}\, \underline{dV} = \iint_S \mathbf{F} \cdot \hat{\mathbf{n}}\, dS$$

where $\mathbf{n}$ is the outwards drawn unit normal vector to S.

4. Cartesian Foam,

$$\iint F_1 dydz + F_2 dzdx + F_3 dxdy$$

$$= \iiint_V \left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right) dV$$

x=
y=
z=

partial
Diffn

dxdydz

5. Evaluate (मान ज्ञात काजए) :

$$\iint_S F\cdot\hat{n}\,dS,\ \text{जहाँ } F = 4xz\,i - y^2 j + yz\,k$$

S is the surface of the cube bounded by the planes :

(S उस घन का पृष्ठ हैं जो कि निम्न समतलों में परिबद्ध है)

$$x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$$

$$\nabla\cdot f = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot\left(4xz\,i - y^2 j + yz\,k\right)$$

$$(4z - 2y + y)$$

$$= \iiint_V \nabla\cdot f\,dV$$

$$= \iiint_{z=0}^{1} (4z - y)\,dx\,dy\,dz$$

$$\Rightarrow \iint_{y=0}^{1}\left[2z^2 - yz\right]_0^1 dx\,dy$$

$$\Rightarrow \int\int_{y=0}^{y=1} (2 - y)\,dx\,dy$$

$$\Rightarrow \int_{x=0}^{1}\left[2y - \frac{y^2}{2}\right]_0^1 dx$$

$$\Rightarrow \int_0^1\left[2 - \frac{1}{2}\right]dx = \left[\frac{3}{2}x\right]_0^1 \Rightarrow \frac{3}{2}\ \text{Ans.}$$

$$\text{Answer} = \frac{3}{2}$$

6. **Use Gauss's divergence theorem i to show that :**

(गॉस प्रमेय की सहायता से प्रदर्शित कीजिए) :

$$\iint_S (\overset{F_1}{x}dydz + \overset{F_2}{y}dzdx + \overset{F_3}{z}dxdy) = 4\pi a^3,$$

where the surface S is the sphere $x^2 + y^2 + z^2 = a^2$.(जहाँ सतह S, गोला $x^2 + y^2 + z^2 = a^2$ है)

$$\iint_S F_1 dydz + F_2 dzdx + F_3 dxdy$$

$$\equiv \iiint\left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right)dv$$

$$\iiint_v (1+1+1)\,dv$$

$$\iiint_v 3\,dv$$

$$\Rightarrow 3\iiint_v dv$$

$$\Rightarrow 3 \times \frac{4}{3}\pi a^3 = \boxed{4\pi a^3}$$

Stoke's Theorem (स्टॉर्क्रमेयेय) :'Statement:

The line integral of a vector function F around any closed curve is equal to the surface integral of curl Fiaken over any surface of which the curve is a boundary edge.

(किसी सदिश फलन F का किसी बंद वक्र के चारों ओर लिया गया रेखा समाकल के curl F के पृष् समाकल के ब्रयबर होता हैं यह समांकल उस पृष्ष पर लिया जाता है जिसकी सीमा वह बंद वक्र बनाती है

7. interior. Then

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS = \iint_S (\mathbf{curl}\ \mathbf{F}) \cdot dS$$

Cartesian form (कार्तीय रूप):

$$\int_C F\cdot dr = \iint_C (iF_1 + jF_2 + kF_3)\cdot(idx + jdy + kdz)$$

$$= \int_C F_1 dx + F_2 dy \qquad [\because dz = 0]$$

चूँकि $\hat{n} = k$, अत:

$$\int_C (\nabla\times F)\cdot \hat{n} dS = \int (\nabla\times F)\cdot k dS$$

$$= \iint_S \left(\frac{\partial F_1}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\, dy$$

8. Using Stoke's theorem, evaluate:

(स्ट्रोक्स प्रमेय का उपयोग करके मान ज्ञात कीजिए) :

$$\int_C xy\,dx + xy^2\,dy,$$

where C is the square in the xy plane with vertices respectively :

(जहाँ C, xy समतल में एक वर्ग हैं जिसके शीर्ष क्रमशः हैं):

$$(1,0), (-1,0), (0,1); (0,1)$$

Answer = $\frac{4}{3}$

GREEN'S THEOREM IN THE PLANE.

Let R be a closed bounded region in the x-y plane whose boundary C consists of finitely many smooth curves. Let M and N be continuous functions of x and y having continuous partial derivatives $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$ in R. Then

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dxdy = \oint_C (Mdx + Ndy),$$

the line integral being taken along the entire boundary C of R such that R is on the left as one advances in the direction of integration.

9. Verify Green's theorem in the plane for

$$\int_C (x^2 - xy^3)dx + (y^2 - 2xy)dy$$

where C **is the square with vertices** $(0,0), (2,0), (2,2), (0,2)$.

$= 8$

10. Evaluate by Green's theorem in plane

$$\int_C \left(e^{-x}\sin y\,dx + e^{-x}\cos y\,dy\right)$$

where C is the rectangle with vertices $(0,0), (\pi, 0), \left(\pi, \frac{1}{2}\pi\right), \left(0, \frac{1}{2}\pi\right)$.

$$= 2(e^{-\pi} - 1)$$

11. **If** $\mathbf{F} = (x^2 - y^2)\mathbf{i} + 2xy\mathbf{j}$ **and** $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, **find the value of** $\int \mathbf{F} \cdot d\mathbf{r}$ **around the rectangular boundary** $x = 0, x = a, y = 0, y = b$.

$$= 2ab^2.$$