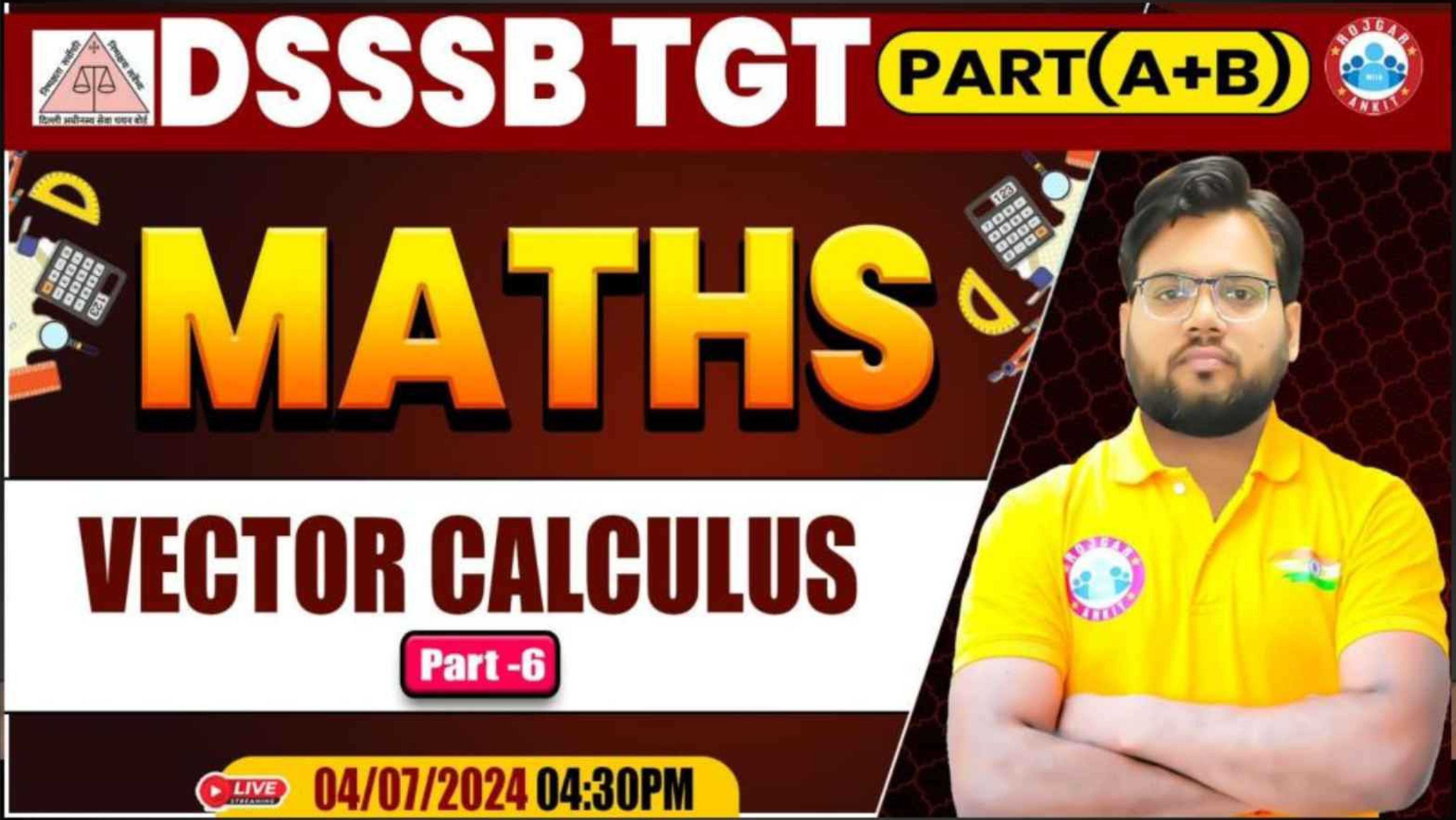
DSSSB TGT
PART(A+B)
MATHS
VECTOR CALCULUS
Part -6
LIVE
04/07/2024 04:30PM

1. If $r(t) = ti - 3j + 2tk$ है तथा $S(t) = i - 2j + 2k$ है तथा $V(t) = 3i + tj - k$ है तो $\int_1^2 r \cdot (s \times v)dt$ का मान क्या होगा?

(a) 0

(b) 49

(c) 36

(d) -5

$r \cdot (s \times v)$

$\Rightarrow t(2-2t) - 21 + 2t(t+6)$

$\int_1^2 (2t - \cancel{2t^2} - 21 + \cancel{2t^2} + 12t)\,dt$

$\int_1^2 (14t - 21)\,dt$

$\Rightarrow \left[\frac{\cancel{14}^{7} t^2}{\cancel{2}} - 21t\right]_1^2$

$28 - 42 - 7 + 21$

$49 - 49 = 0$

$$S \times V = \begin{vmatrix} i & j & k \\ 1 & -2 & +2 \\ 3 & t & -1 \end{vmatrix}$$

$i(2-2t) - j(-1-6) + k(t+6)$

$S \times V = (2-2t)i + 7j + (t+6)k$

Ans = 0

2. Evaluate $\int_1^2 \mathbf{r} \times \frac{d^2\mathbf{r}}{dt^2}dt$, **where** $\mathbf{r} = 2t^2\mathbf{i} + t\mathbf{j} - 3t^3\mathbf{k}$.

$\frac{dr}{dt} = 4t\,i + j - 9t^2 k$

$\frac{d^2r}{dt^2} = 4i - 18t\,k$

$r \times \frac{d^2r}{dt^2} = \begin{vmatrix} i & j & k \\ 2t^2 & t & -3t^3 \\ 4 & 0 & -18t \end{vmatrix}$

$i(-18t^2) - j(-36t^3 + 12t^3) + k(-4t)$

$-18t^2 i + 24t^3 j - 4tk$

$\int_1^2 \left(r \times \frac{d^2r}{dt^2}\right) dt$

$\int_1^2 (-18t^2 i + 24t^3 j - 4tk)\,dt$

$\left[-6t^3 i + 6t^4 j - 2t^2 k\right]_1^2$

$-48i + 96j - 8k + 6i - 6j + 2k$

$-42i + 90j - 6k$ Ans.

$$= -42i + 90j - 6k$$

$$\underbrace{}_{\text{Answer}}$$

Line Integral (रेखा समाकल)- किसी वक्र के अनुदिशा प्राप्त किए गये समाकल को रेखा समाकल कहते है।

$\int_C \left\{F(r).\frac{dr}{ds}\right\} ds$ या $\int_C F(r).dr$ को सदिश फलन F(r) का वक्र C पर रेखा समाकल कहते है।

$\int \iint \iiint$

→ $\int_C Line\ integral$

→ $\iint_S Surface\ integral$

→ $\iiint_v Volume\ integral$

➢ Cartesian Form of Line Integral (रेखा समाकल का कार्तीय रूप):-

$r = xi + yj + zk$

$dr = dxi + dyj + dzk$

➢ मान लीजिए $F(r) = F_1(t)i + F_2(t)j + F_3(t)k$ तथा $\boxed{dr = dx \cdot i + dy \cdot j + dz \cdot k}$

जहाँ $r = xi + yj + zk$

अतः $\int_C F(r) \cdot dr = \int_C \left(F_1 i + F_j j + F_3 k\right) \cdot (idx + jdy + kdz)$

या $\boxed{\int_c F \cdot dr = \int_C (F_1 dx + F_2 dy + F_3 dz)}$

➢ **Evaluate** $\int_C \mathbf{F}\cdot d\mathbf{r}$, **where** $\mathbf{F} = x^2\mathbf{i} + y^3\mathbf{j}$ **and curve** C **is the arc of the parabola** $y = x^2$ **in the** x**-y plane from** $(0,0)$ **to** $(1,1)$.

$r = xi + yj$

$dr = dx\,i + dy\,j$

$F\cdot dr = (x^2 i + y^3 j)\cdot(dx\,i + dy\,j)$

$F\cdot dr = x^2 dx + y^3 dy$

$\int F\cdot dr = \int x^2 dx + \int y^3 dy$

$\left[\frac{x^3}{3}\right]_0^1 + \left[\frac{y^4}{4}\right]_0^1 \Rightarrow \frac{1}{3} + \frac{1}{4} \Rightarrow \frac{7}{12}$ Ans.

$= \frac{7}{12}.$

➢ **Evaluate** $\int_C \mathbf{F}\cdot d\mathbf{r}$, **where** $\mathbf{F} = (x^2 - y^2)\mathbf{i} + xy\mathbf{j}$ **and curve** C **is the arc of the curve** $y = x^3$ **from** $(0, 0)$ **to** $(2, 8)$.

$$\int_C F\cdot dr = \int_0^2 (x^2 - x^6)\,dx + \int_0^8 y^{4/3}\,dy$$

$$\left[\frac{x^3}{3} - \frac{x^7}{7}\right]_0^2 + \left[\frac{3y^{7/3}}{7}\right]_0^8$$

$$\left[\frac{8}{3} - \frac{128}{7}\right] + \left[\frac{3}{7}\times 128\right]$$

$$\frac{8}{3} + \frac{256}{7} \Rightarrow \frac{56+768}{21}$$

$$\Rightarrow \frac{824}{21}$$

$x = y^{1/3}$

$r = x\,i + y\,j$

$dr = dx\,i + dy\,j$

$$F\cdot dr = \left[(x^2 - y^2)\,i + xy\,j\right]\cdot\left[dx\,i + dy\,j\right]$$

$$(x^2 - y^2)\,dx + xy\,dy$$

$$(x^2 - x^6)\,dx + y^{4/3}\,dy$$

$8^{7/3}$

$(2^3)^{7/3}$

$(a^n)^m = a^{n\times n}$

$\frac{1}{3} + 1$

$$= \frac{824}{21}.$$

➢ If $\mathbf{F} = 3xy\mathbf{i} - y^2\mathbf{j}$, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve in the xy-plane, $y = 2x^2$, from $(0,0)$ to $(1,2)$.

$F \cdot dr = 3xy\,dx - y^2\,dy$

$F \cdot dr = 6x^3\,dx - y^2\,dy$

$\int F \cdot dr = \left[\frac{6x^4}{4}\right]_0^1 - \left[\frac{y^3}{3}\right]_0^2$

$\Rightarrow \frac{3}{2} - \frac{8}{3} \Rightarrow \frac{-7}{6}$

$$= -\frac{7}{6}.$$

➢ **Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}$ is $x^2y^2\mathbf{i} + y\mathbf{j}$ and C is $y^2 = 4x$ in the xy-plane from $(0,0)$ to $(4,4)$.**

$F\cdot dr = (x^2y^2)dx + y\,dy$

$F\cdot dr = 4x^3dx + y\,dy$

$\int F\cdot dr = [x^4]_0^4 + \left[\frac{y^2}{2}\right]_0^4$

$\Rightarrow 256 + 8$

$\Rightarrow 264$

14

264

= 264.

- Integrate the function $\mathbf{F} = x^2\mathbf{i} - xy\mathbf{j}$ from the point $(0,0)$ to $(1,1)$ along parabola $y^2 = x$

$$F \cdot dr = x^2 dx - xy\, dy$$

$$= x^2 dx - y^3 dy$$

$$\int F \cdot dr \Rightarrow \left[\frac{x^3}{3}\right]_0^1 - \left[\frac{y^4}{4}\right]_0^1$$

$$\Rightarrow \frac{1}{3} - \frac{1}{4} \Rightarrow \frac{1}{12}$$

$$= \frac{1}{12}.$$

- Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = (x^2 + y^2)\mathbf{i} + xy\mathbf{j}$ and the curve C is the arc of the parabola $y = x^2$ from $(0,0)$ to $(3,9)$ in the xy-plane.

H.W.

$$= \frac{774}{5}.$$